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ENGR 12L
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Final Project Circuit Demo

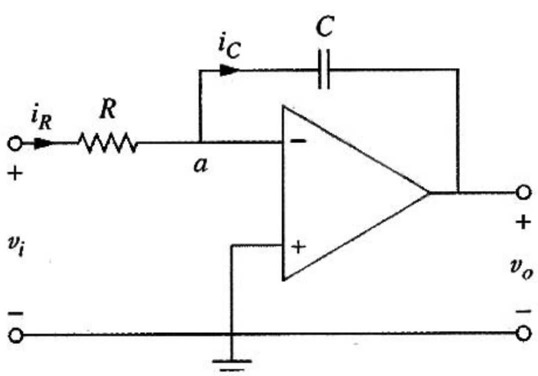
# Analog Differential Equation Solver Circuit

The AKAT-1 (1959) was the first differential analyzer of it's kind to use transistors instead of vacuum tubes. Just as the microchip did for digital computer, this allowed for a much more compact design.

Before the widespread availability of digital computers, analog computers were commonly built in order to solve very specific kinds of problems. One such type of computer, is a differential equation solver. While the model built in this demo will be nowhere near as sophisticated as commercial models from years past, it will operate around the same general principles.

Before constructing an entire circuit, it is wise to first construct and test it's smaller components. The most vital part of an analog DE solver is the integrator that will take whatever voltage function fed into it and return the integral of that function through it's output. An integrator is easily constructed using an OP-AMP and a capacitor.

Let's examine how it actually works. The sum of the current flowing through node a must sum up to zero, so they must be equal to each other. Writing the currents in terms of their voltages and solving for the output voltage reveals that the output voltage is the negative of the integrated input voltage divided by the resistance and capacitance.




$$i_R = i_C$$

$$i_R = \frac{v_i}{R}, \qquad i_C = -C\frac{dv_o}{dt}$$

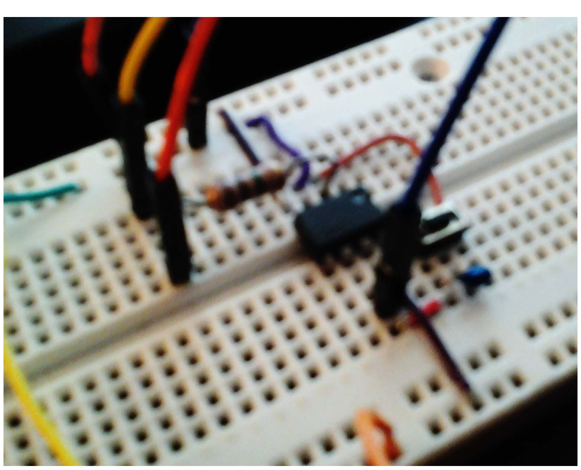
$$\frac{v_i}{R} = -C\frac{dv_o}{dt}$$

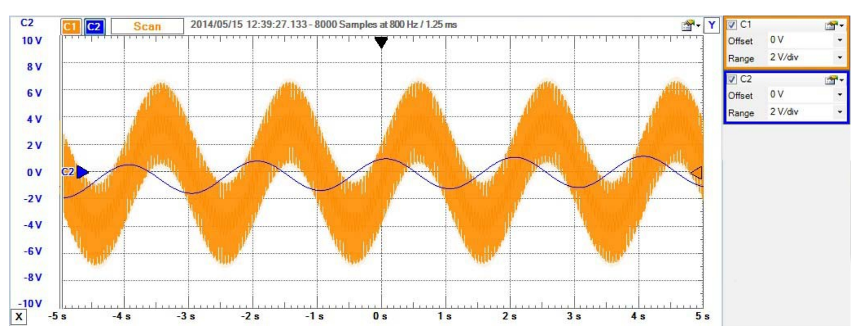
$$dv_o = -\frac{1}{RC}v_i\,dt$$

$$v_o(t) - v_o(0) = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

$$v_o = -\frac{1}{RC}\int_0^t v_i(\tau)\,d\tau$$

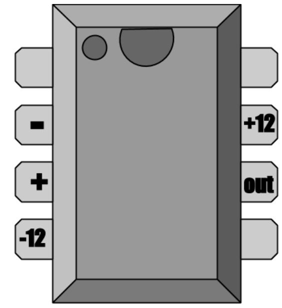
Now that we have a good idea about how an integrator should work and how to build one, we set one up with the circuit kit and test it with a simple input function, a sin function in this case.

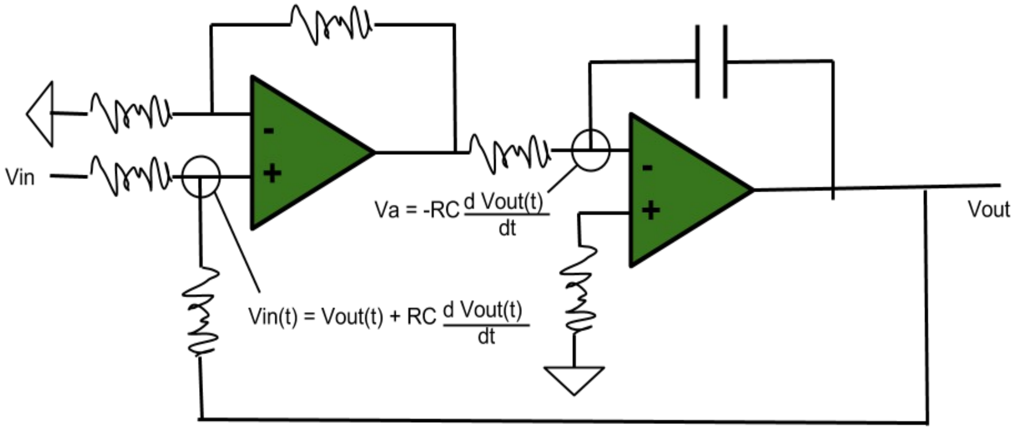




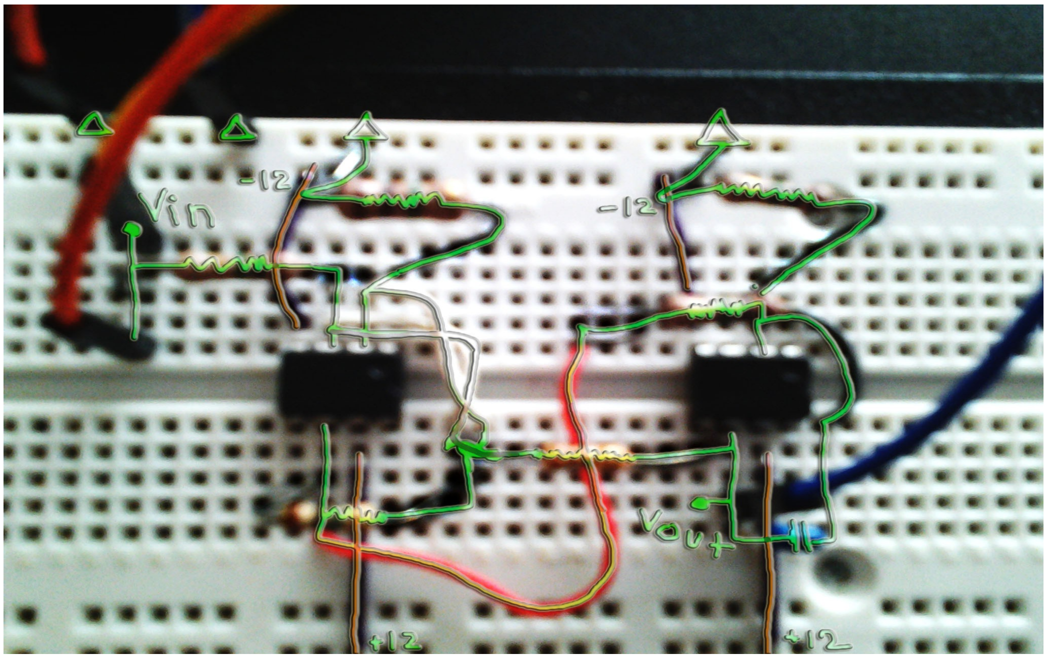

If a sin(t) function is fed into the circuit (shown in orange), a cos(t) function is read at the output (blue). Note that the integral of sin(t) is -cos(t) which is inverted by the integrator. Notice the phase shift of 90°. This indicates that the integrator is functioning properly.

OP-AMP CIRCUIT CHIP




Now that we have a fully functioning integrator, we can combine it to a non-inverting op amp to create a complete circuit that will model a differential equation. The output voltage will be the solution to the equation in which the driving term is defined by the input voltage.



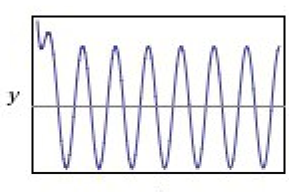

Just look at that mess of a circuit! To make it a bit easier to follow, I went ahead and traced over the bread board to better illustrate the connections of all the individual components. After careful examination, you will see that it matches up with the schematic above.

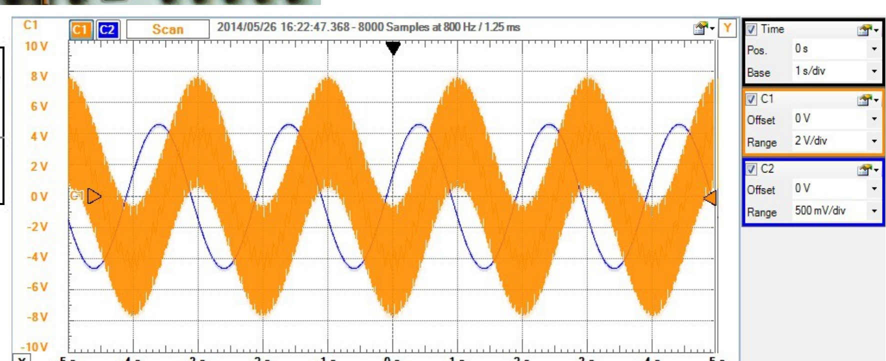
Now that we have a complete DE solver, it is again necessary to test it. We do this by feeding in a set of experimental inputs and comparing the results to what we expected.

sin(t) = y + y'

Differential equation solution:

$$y(t) = c_1\, e^{-t} + \frac{\sin(t)}{2} - \frac{\cos(t)}{2}$$

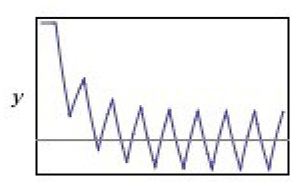


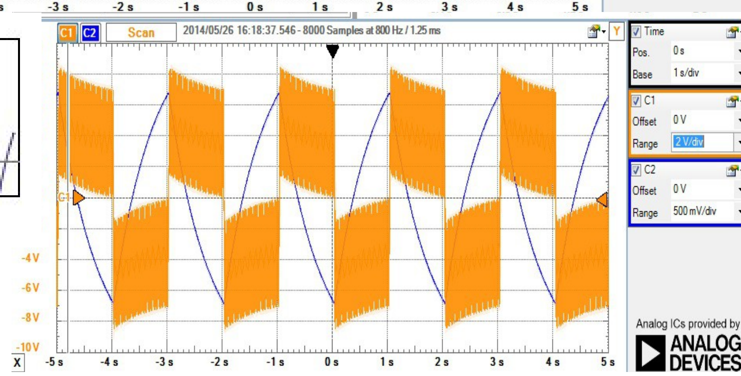

SquareWave[t] = y + y'

Differential equation solution:

$$y(t) = \left( \begin{cases} -1 & \frac{1}{2} \le t \bmod 1 < 1 \\ 1 & \text{(otherwise)} \end{cases} \right) + c_1\, e^{-t}$$




While it isn't quite practical to aim for a numerically precise analysis of the results, we see that the graph of the solutions and the output voltage seem to agree with each other. So we can conclude that the circuit is working.



Computers like these were some of the first machines built to assist man's mind and solve problems. They were eventually replaced in favor of digital computers. Analog computers had to be rewired specifically for each individual problem that it would solve. With digital computers, data and the algorithm that it is run through are physically speaking the same thing, so the computer doesn’t have to be rewired for it to solve a wide variety of problems even at the same time. Digital circuits tend to be more energy efficient and compact, which is why digital circuits are prevalent in the modern world, even among otherwise completely analog oriented technology. Still, this project is a treat to try for any student interested in computer hardware.