Lab 15 Names _______________
Frequency Selective Circuits

Objectives – in this lab you will

- Measure the frequency response of a circuit
- Determine the Q of a resonant circuit
- Build a filter and apply it to an audio signal

Key Prerequisites

- Chapter 14

Required Resources

- Laptop, Lab Kits, DMM, Analog Discovery

References

1. Wikepedia: Analog Filter Development
2. Tim Storm, Lowest Male Voice
3. High Pitched Girl's Voice

In this lab we explore the frequency dependent nature of AC circuits. Since the impedance of both capacitors and inductors is a function of frequency, we can use these components to amplify or suppress different frequency bands of a signal. This forms the basis of analog filter design, a deep and important topic essential to modern communication systems and many other technologies. Filters allow the combining and later separation of multiple telephone conversations on a single channel, the selection of a chosen radio station in a radio receiver, and the enhancement of music signals prior to application to bass, mid-range, and tweeter loudspeakers[1].

A typical filter is described by its frequency response, an example of which is shown in Figure 1.

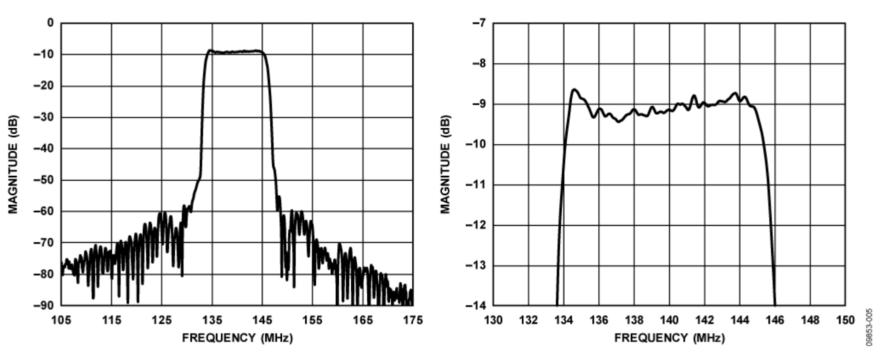


Figure 1 A 140 MHz SAW Filter [2]

Because of the prominence of filters in communication systems (a critical human technology), filter theory has been studied in great depth since the late 1800's when they were applied to telegraph signals. The filter in Figure 1, taken from a modern communication system, is made with Surface Acoustic Wave (SAW) technology, a completely different phenomena from what we will be exploring today with resistors, capacitors and inductors, but the basic properties will be the same.

The plot in Figure 1 shows the filter gain in decibels (dB) as a function of frequency. The passband ranges from about 133 to 147 MHz. The rejected frequency band is attenuated by 50 – 60 decibels or higher. Applied to voltages, a decibel is 20 $\log_{10}(V_{out}/V_{in})$, so this means a gain (or suppression in this case) of about 0.001 for these undesired frequency components. The passband is also attenuated—by 10 dB—which equates to a voltage gain of about 0.3.

The filters we develop in this lab will have less dramatic features, but we will apply them on an audio feed composed of two signals: a very deep voice and a very high pitched voice, and we'll be able to use our filters to suppress the unwanted signal so we can hear the desired signal.

Vocabulary

All key vocabulary used in this lab are listed below, with closely related words listed together:

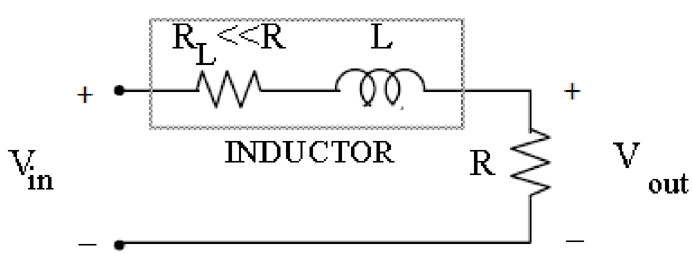
Gain, Passband, Decibels, Resonance

Discussion and Procedure

Parts:	R = 470 Ω	Measured	______________
	L = 0.033 H (black cylinder)	Measured R_L	______________
	C = 0.1 uF (code 104 stamped on side)	Assume ideal	0.1 uF

Part 1 RL Low-Pass Filter

Before building the RL Low-Pass Filter in Figure 2, measure the internal winding resistance R_L of the inductor, as well as the 470 Ω resistor. You can assume the capacitance of C is exactly 0.1 uF. The inductor resistance will affect the frequency response of the filter slightly, so we'll check to see that it is much less than (say, 5% of) the resistor.



Figuer 2. RL Low-Pass Filter

The filter forms a voltage divider between Vout and Vin. Therefore, the formula for Vout, and by extension, the filter Gain, G, is:

$$\mathbf{V_{out}} = \frac{\mathbf{V_{in}}(\mathrm{R})}{\mathrm{R_L} + \mathrm{R} + \mathrm{j}\omega \mathrm{L}} \approx \frac{\mathbf{V_{in}}(\mathrm{R})}{\mathrm{R} + \mathrm{j}\omega \mathrm{L}}$$

$$\mathbf{G}(\boldsymbol{\omega}) \equiv \frac{\mathbf{V_{out}}}{\mathbf{V_{in}}} = \frac{\mathrm{R}}{\mathrm{R} + \mathrm{j}\omega \mathrm{L}} \quad \text{or} \quad \mathbf{G}(\mathbf{f}) = \frac{\mathrm{R}}{\mathrm{R} + \mathrm{j}(2\pi \mathrm{f})\mathrm{L}}$$

Figure 3. Output voltage and gain of a series RL low-pass filter

If this was a phasor problem with a single sinusoid for Vin, we would plug in the single value for ω and calculate the single value for Vout and G. However, today we are going to sweep the value of ω to determine how signals of varying frequency will be processed.

We can start by looking at the extremes. We can see that when ω is close to zero, the value of Vout approaches Vin, and when ω approaches infinitity, the value of Vout approaches 0. This circuit is called a low-pass filter, because the output voltage is equal to the input voltage for low frequencies and the output voltage is reduced from the input voltage as the frequency is increased.

If we substitute $2\pi f$ for ω, we can express these functions in terms of frequency f in cycles/second or Hz (to us a more reasonable unit). A key operating point in a filter is the frequency at which the gain drops to .7071 (or $\frac{\sqrt{2}}{2}$) of the value at $f = 0$. This is called the cutoff, or corner, frequency, since beyond this point the gain falls off rapidly. It occurs when $2\pi f_c L = R$, or $fc = \frac{R}{2\pi L}$. If we include R_L in the formula, it's a little more complicated. Calculate the corner frequency both ways now and enter into your datasheet.

Approximate Corner frequency $fc = \dfrac{R}{2\pi L}$ ____________________________

Corner frequency including R_L $fc = \dfrac{\sqrt{R^2 - 2RR_L + R_L^2}}{2\pi L}$ ____________________________

% difference ____________________________

1. Build the RL low-pass filter using the resistor and inductor you've already measured. This is very similar to the RC circuit you built in Lab 13, except the capacitor is replaced with an inductor. You will be making the same connections with the Analog Discovery as you did in that lab. Here are two figures to help you with the construction.

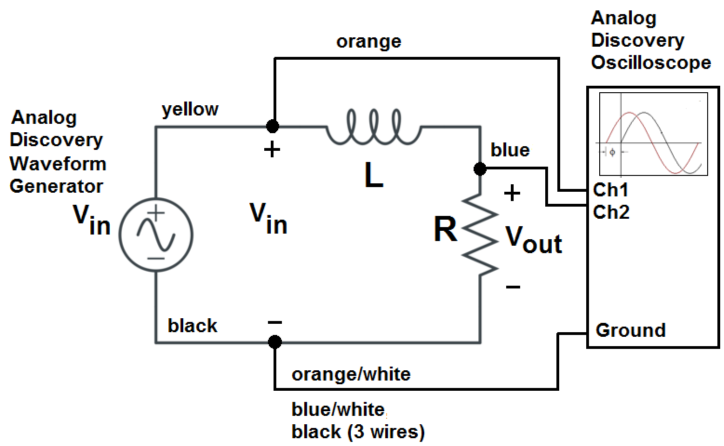


Fig 4. Circuit for determining the frequency response of an RL low-pass filter

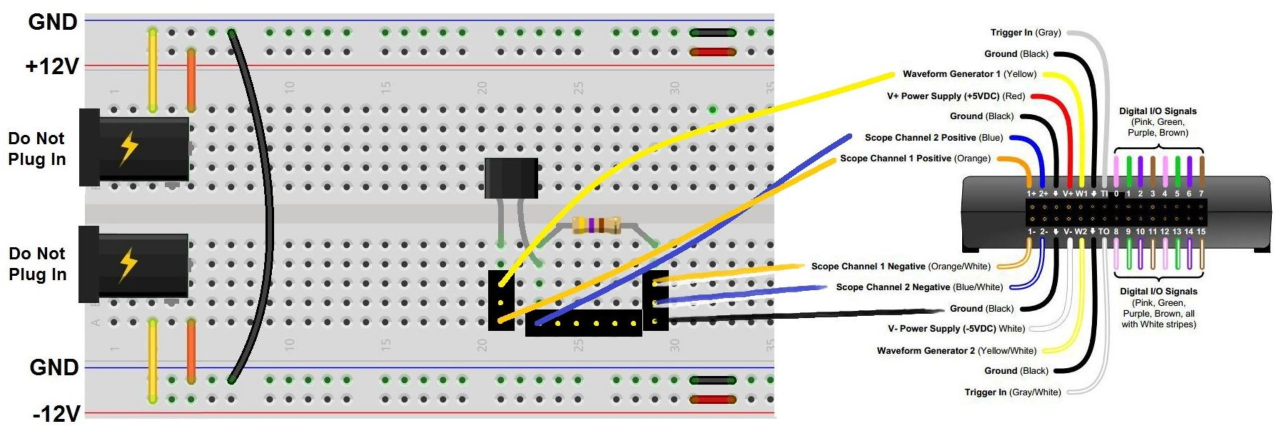


Fig 5. One possible breadboard layout for the circuit in Fig 4.

2. Use the Analog Discovery WaveGen function generator to provide Vin, a 5V sinewave at 100 Hz.

3. Use the Analog Discovery oscilloscope to verify you've got the function generator set to the correct frequency (100 Hz), and amplitude (10 Volts peak-to-peak). Use the measurement tool (yellow drafting triangle) to show the peak-to-peak voltage of channel 1 and channel 2, and the average frequency for channel 1 on the measurements pane.

4. Although we will not be changing the input voltage amplitude, make sure you record the input peak-to-peak measurement value at each frequency, since the function generator may have its own frequency response.

5. Complete Table I for your Low-pass filter. In the table, where it says "calculated fc," use the second fc you calculated above, since that is likely to be more accurate. Then adjust the input frequency until you have a gain of 0.707, and record the values of $\mathbf{V_{in}}$ and $\mathbf{V_{out}}$ for that frequency.

Table I RL Low-Pass filter measurements (complete in datasheet)

Frequency, f	$\mathbf{V_{in,\ pk\text{-}pk}}$	$\mathbf{V_{out,\ pk\text{-}pk}}$	**Gain ($\mathbf{V_{out}/V_{in}}$)**
100 Hz			
200 Hz			
500 Hz			
1000 Hz			
calculated fc = _______			
measured fc = _______ (adjust f for gain of 0.707)			
2000 Hz			
5000 Hz			
10000 Hz			
20000 Hz			
50000 Hz			

Note that the corner frequency should have a gain of -3 dB. This frequency is also called the 3dB frequency or the half-power frequency, since the power, which is proportional to V^2,will be reduced by half from the passband at that frequency.

6. Use FreeMat to plot the Gain (linear ratio) vs frequency using linear axes. Enter your test data into two vectors f and G in FreeMat and use these commands to plot the data.

```
plot(f,G,'+-'); xlabel('freq, Hz'); ylabel('Gain, Vout/Vin');
title('Low-Pass Filter Frequency Response (Linear Scale)');
```

Copy and paste the plot into your datasheet.

7. Repeat the plot, only now make a new vector GDB for Gain in decibels, and plot frequency on a logarithmic scale. The semilogx command will do that, since the Gain GDB will already be in decibels (a logarithmic unit).

```
GDB = 20*log10(G);
semilogx(f,GDB,'+-'); xlabel('freq, Hz'); ylabel('Gain, dB');
title('Low-Pass Filter Frequency Response (Log Scale)');
```

Copy and paste the plot into your datasheet. As a unit, decibels are helpful to convey the behavior of the gain over a broad range of magnitudes.

8. Since the frequency response of filters and other circuits is of such importance in many electronic systems, a special tool called a Network Analyzer is typically used to measure this curve. While a good network analyzer can cost tens of thousands of dollars, we have a software network analyzer built into our Analog Discovery. In Waveforms, on the main instrument page, choose “more instruments” and then “Network Analyzer”.

9. Set up the sweep parameters. Choose the start frequency of 100 Hz, stop frequency of 50 kHz, and the Max Gain of 1X. Then click run. The network analyzer will show not only the gain but also the phase (θ_{out} - θ_{in}) of the response.

10. When the plot is stable and seems correct, click on “Single” to just grab a sweep and freeze the curve. Drag an X cursor onto the plot and position it at the corner frequency, such that the difference in dB between C1 and C2 is 3 dB. Record the frequency of the X cursor in your datasheet.

    Corner frequency meaured by Network Analyzer, fc = ________________

11. Then Capture a copy of this plot and paste it into your datasheet.

12. Compare this automated plot from the Network Analyzer to your manual measurement and MATLAB plot and comment on any similarities or differences you observe, in particular, between your two measurements of the corner frequency of the filter.

13. For reference, the theoretical shape of the plot, in logarithmic units, should look something like the following. The filter should be flat for most of the passband. After passing the corner frequency, the gain should roll off by 6 dB per octave (frequency doubling), or 20 dB per decade. That means a 6 dB drop every time the frequency doubles, and a 20 dB drop every time the frequency increases by a factor of 10.

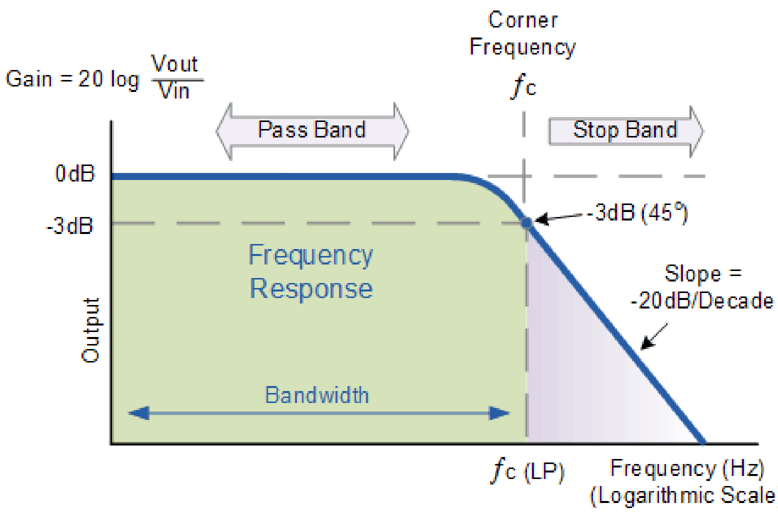


Figure 4. Theoretical frequency response for a Low-Pass Filter on logarithmic scales

Part 2 RC High-Pass Filter

We can change our low-pass filter into a high-pass filter by swapping the inductor with our 0.1 uF capacitor, as shown in figure 5.

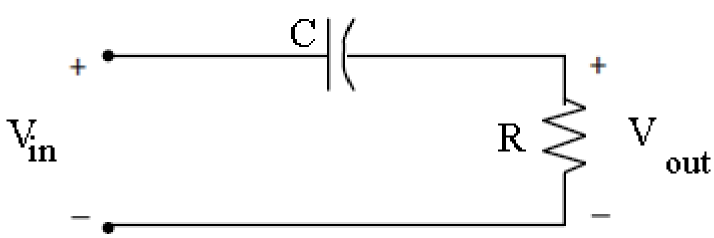


Figure 5. RC High-Pass Filter

As before, the filter forms a voltage divider between Vout and Vin. Therefore, the formula for Vout, and by extension, the filter Gain, G, is now:

$$V_{out} = \frac{V_{in}(R)}{R + \frac{1}{j\omega C}}$$

$$G(\omega) = \frac{R}{R + \frac{1}{j\omega C}} \quad \text{or} \quad G(f) = \frac{1}{1 - j\frac{1}{2\pi fCR}} = \frac{1}{1 - j\frac{f_c}{f}}$$

Figure 6. Output voltage and Gain of a series RC high-pass filter

where the corner frequency fc is now $fc = \frac{1}{2\pi RC}$

14. Build the RC high-pass filter using R = 470, C = 0.1 uF.

    Calculate corner frequency   fc=1/(2πRC)   ______________________

15. For this filter we will jump straight to using the Network Analyzer. Perform another frequency sweep using the same parameters as your first sweep, drag an X cursor onto the plot to locate the measured corner frequency, then capture the gain and phase plot from the display and paste into your datasheet.

16. Record the frequency of the X cursor in your datasheet.

    Corner frequency meaured by Network Analyzer, fc = ________________

17. Comment on any similarities or differences you observe, between your Network Analyzer measurement of the corner frequency and the theoretical estimate of the corner frequency from step 14.

18. For comparison, the theoretical shape of the plot, in logarithmic units, should look something like the following. In the stop band, the filter response should climb by 6 dB per octave, or 20 dB per decade. That means a 6 dB increase every time the frequency doubles, and a 20 dB increase every time the frequency increases by a factor of 10. After passing the corner frequency the response should level out at about 0 dB.

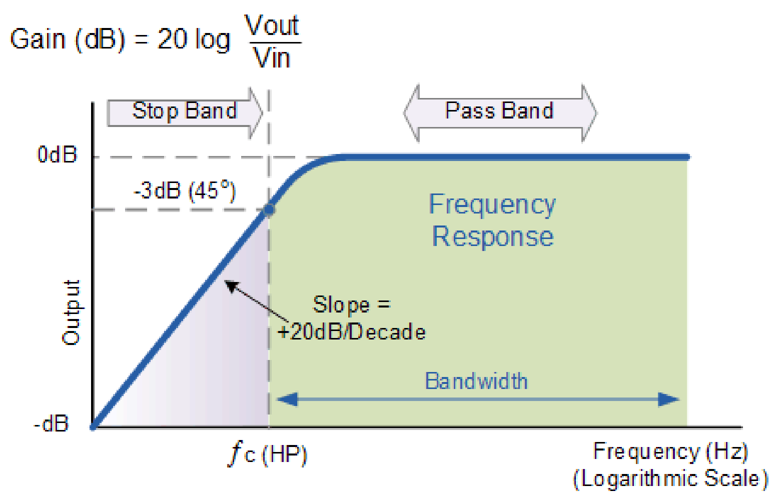


Figure 7. Theoretical frequency response for a High-Pass Filter on logarithmic scales

Part 3 Band-Pass Filter

Finally, we can change our high-pass filter into a band-pass filter by adding the inductor back to the circuit, as shown in figure 8.

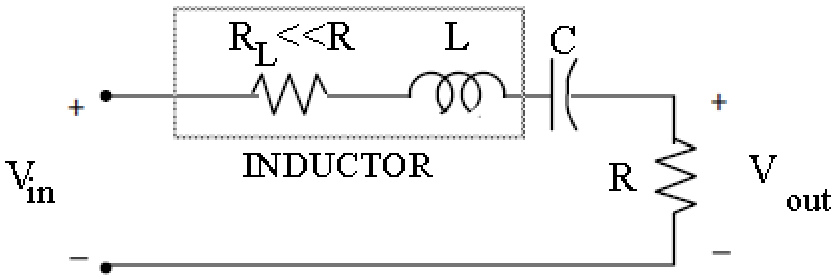


Figure 8. Series RLC Band-Pass Filter

The filter again forms a voltage divider between Vout and Vin. The formula for the filter Gain, G, is now:

$$G(\omega) = \frac{R}{R + \frac{1}{j\omega C} + j\omega L} = \frac{1}{1 + j\left(\omega L - \frac{1}{\omega C}\right)\frac{1}{R}}.$$

Figure 9. Gain of a series RLC band-pass filter

Note that in the previous equation, the complex term (reactance) will cancel when $\omega L = \frac{1}{\omega C}$, resulting in a circuit gain of 1. When this occurs, the voltage and current are in-phase, and the circuit is said to be in <u>resonance</u>. This special frequency is known as the <u>resonant</u> frequency, and is defined by:

$$\omega_r = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

Figure 10. Resonant frequency of an RLC circuit

The band-pass filter also has two corner or half-power frequencies at the low and high ends of the pass-band. These are defined by:

$$f_{c1} = \frac{1}{4\pi L}\left[-R + \sqrt{R^2 + \frac{4L}{C}}\right] \text{ for the low frequency}$$

$$f_{c2} = \frac{1}{4\pi L}\left[+R + \sqrt{R^2 + \frac{4L}{C}}\right] \text{ for the high frequency}$$

Figure 11. A band-pass filter has two corner frequencies

Another interesting parameter of a resonant circuit is the value Q. This value indicates how sharp the resonance (and how narrow the filter) is. The larger the Q, the greater the resonance, and the narrower the bandwidth of the filter in proportion to the resonant frequency. This is captured in the following equation:

$$Q = \frac{\text{resonant frequency}}{\text{width of pass-band}} = \frac{f_r}{f_{c2} - f_{c1}}$$

19. Build the series RLC BandPass filter. Using the formulas above,

    Calculate fc1 = ________________

    Calculate fc1 = ________________

    Calculate fr = ________________

    Calculate Q = ________________

20. Again, for this filter we will jump straight to using the Network Analyzer. Perform another frequency sweep using the same parameters as your first sweep. This time, use two X cursors to locate the two corner frequencies, fc1 and fc2 on the response curve. Then capture the gain and phase plot from the display and paste into your datasheet.

21. Record the corner frequencies measured by Network Analyzer,

    fc1 = ________________

    fc2 = ________________

22. Comment on any similarities or differences you observe, between your Network Analyzer measurement of the corner frequency and the theoretical estimate of the corner frequency from step 19.

Part 4 (Optional) Filter an Audio Signal

You can use your low-pass and high-pass filters in a practical application by playing the following two YouTube videos at the same time (which combines their audio onto a single channel) and feed the audio signal into your filter as Vin using the 3.5mm audio jack in your lab kit. Then take Vout and run it into the Radio Shack loudspeakers in the lab (online students: just plug earbuds into the audio jack on the side of the Analog Discovery while monitoring the output with the scope).

YouTube Videos with high and low pitched voices. Play these at the same time:

1. Tim Storm, Lowest Male Voice
2. High Pitched Girl's Voice

With the low pass filter, you should be able to filter out the high pitched girl (to lower the cutoff frequency, choose a smaller resistor such as 220 or 100 Ohms).

When you are finished, turn in this document to the ENGR12L uploader.