

Lab 13

AC Circuit Measurements

Objectives

- *concepts*
 1. what is impedance, really?
 2. function generator and oscilloscope
 3. RMS vs magnitude vs Peak-to-Peak voltage
 4. phase between sinusoids
- *skills*
 1. measuring RMS voltage and converting to other forms
 2. measuring phase difference between sinusoids

Key Prerequisites

- Chapter 9 and 10 (Sinusoids and Phasors)

Required Resources

- Circuit Kits, DMM
- Digilent Analog Discovery
- Laptop

What *is* Impedance, Actually?

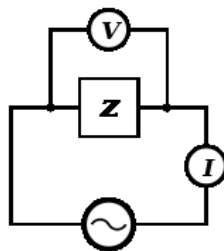


Fig 1. Impedance Z is the ratio of phasor V divided by phasor I

We have worked with the notion of a "complex resistance," also known as **impedance**, for several weeks now. At this point, you know that impedance involves some rather

difficult calculations involving complex numbers. But can we get an intuitive sense of what impedance means? Can we physically measure the equivalent impedance of a network of resistors, capacitors and inductors, just like we did for a simple network of resistors?

In lab 4, you measured the resistance of several resistors in various combinations, the simplest being a series combination:

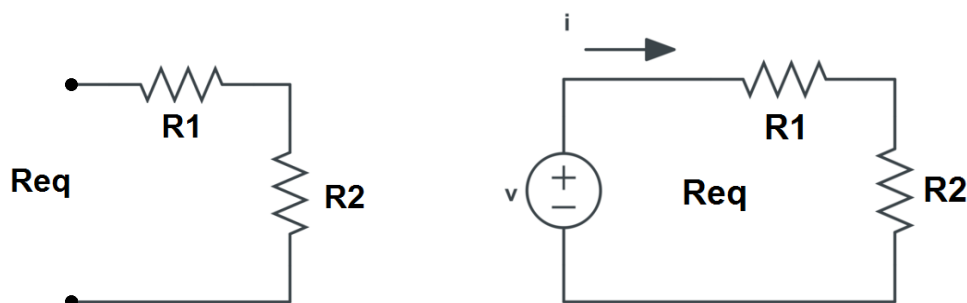


Fig 2. Two resistors in series. $R_{eq} = R1 + R2$. Also, $R_{eq} = v/i$

There are two different ways you can determine R_{eq} experimentally:

- 1) Connect an Ohmmeter to the terminals and measure R_{eq} .
- 2) Connect a Voltage source to the terminals and measure the current. $R_{eq} = v/i$.

But what if the first resistor is a capacitor, and the voltage source is a sinusoid?

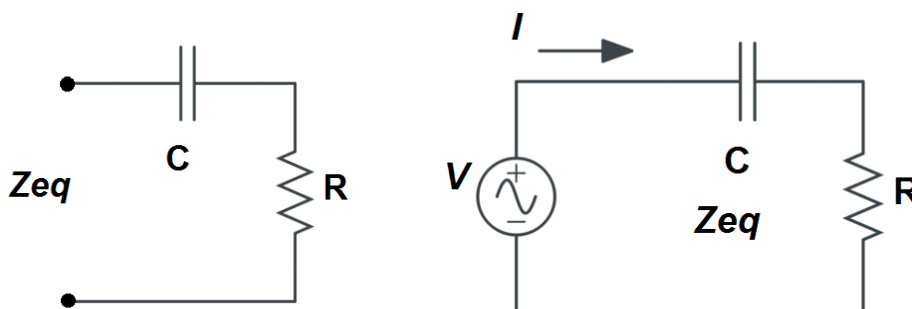


Fig 3. Two circuit elements in series. $Z_{eq} = R + j/(\omega C)$. Also, $Z_{eq} = V/I$

The symbol for impedance, Z , defines the "Ohm's Law" relationship between voltage and current, when these are sinusoidal (Alternating Current, or AC) signals:

$$Z = \frac{V}{I}$$

In this case, V , and I are written in boldface italics to indicate they are **phasors**. Unfortunately, in the lab, we cannot measure impedance using an ohmmeter. We can

only determine impedance experimentally by measuring the phasors V and I and then dividing them. The result will be a complex number.

We will start the lab by calculating the theoretical impedance of a series RC network at 3 different frequencies. Then we will build the networks and measure their impedance experimentally, by connecting an sinusoidal (AC) voltage source to them, measuring the voltage source V and current I as phasors, then dividing V/I . Finally, we will repeat the procedure on a series RLC circuit.

Discussion and Procedure

Materials:

- Analog Discovery Module
- Digilent Scope in WaveForms application.
- Digilent WaveGen in WaveForms application

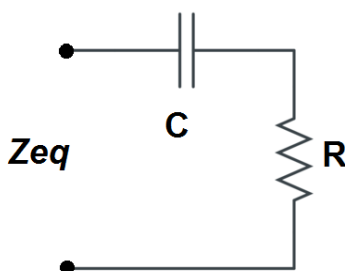
Parts List

- 0.1 μF capacitor (104J)
- 470 Ohm Resistor
- 33 mH inductor



PART 1 Series RC circuit

We are going to measure the equivalent impedance of the following network, using an R of 470 Ω and a C of 0.1 μF (100 nF).



1. Find the capacitor in your component box. It should be green, and it will be marked with the code "104J," which is an indication of the capacitance value. The number 104J means 10×10^4 , or 100,000 pF (pico-Farads) or 100 nF or 0.1 μF . This capacitor is a mylar capacitor. It is not polarized, meaning you can put it in the circuit in either direction without concern.

- Then find the 470 Ω resistor and measure its resistance with an ohmmeter and record this value in your datasheet. We will assume the capacitor has the nominal value of 0.1 uF, and the inductor has the nominal value of 33 mH.

Theoretical Calculations

- Now complete the table of theoretical impedance calculations provided below and in your datasheet, for three different values of ω. Recall that ω = 2πf.

Table I. Impedance calculations for a series RC network at 3 different values of ω. Recall that ω = 2πf.

Frequency, f	Frequency, ω	$Z_{eq} = R - \frac{j}{\omega C}$	$Z_{eq} = Z_m < \theta$
1000 Hz			
4500 Hz			
10000 Hz			

Experimental Measurements

Measuring impedance is fairly complicated. We are going to need an AC signal source, and a way of observing the amplitude and phase of the voltage and current. Fortunately we have our Analog Discovery multi-instrument to help us with this task.

Here is the circuit we are going to work on in lab to determine Z_{eq}:

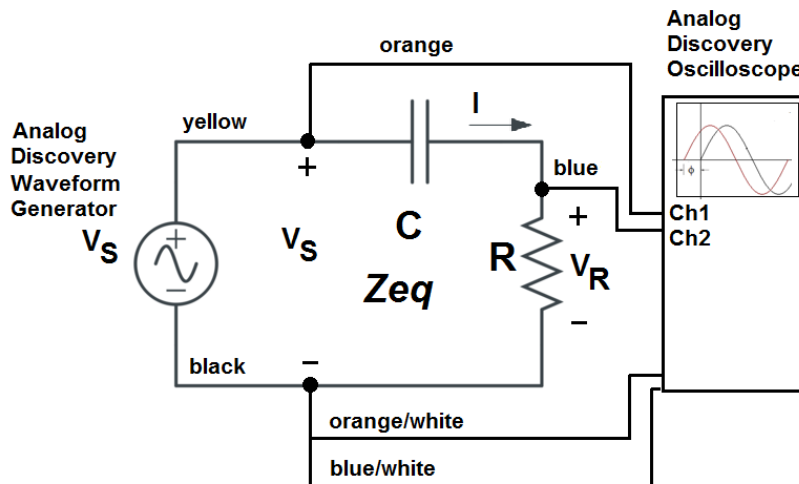


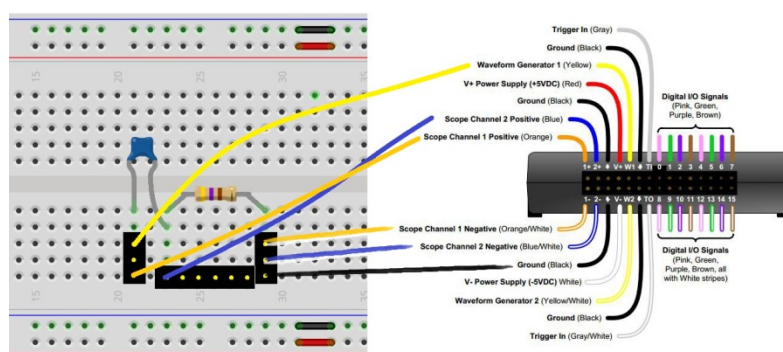
Fig 5. Circuit for determining Z_{eq}, showing **wire colors** for Analog Discovery connections. Note that I can be determined by V_R/R.

As shown in Figure 5, the Analog Discovery Waveform Generator provides the AC Voltage source V_S , and the Analog Discovery Oscilloscope provides the means of viewing the voltage V_S and current I so that we can find the phase difference between them. However, the scope cannot measure I directly, so instead we measure V_R , the voltage across the resistor, and use that to determine I through Ohm's Law, $I = V_R/R$.

Practically speaking, the physical circuit you will be wiring will basically consist of a capacitor and resistor on your breadboard, with various wires connecting to the Analog Discovery. The color of the wires you will be using is shown on Figure 5.

NOTE: Do NOT connect a 12V DC source to the circuit. The source voltage comes from the Analog Discovery unit through the yellow and black wires (Waveform Generator).

4. There are many ways to wire the breadboard for this circuit, but if you are stuck, use the following figure to help you. You may need to break one of your 6-pin headers in half to build this circuit.



Some additional notes about the circuit setup:

- a. V_S is taken from the Analog Discovery WaveGen waveform generator and will supply AC power to the circuit. To connect, attach the Yellow Wire (W1) from the Analog Discovery to the left of the capacitor. Attach the Black Wire to the bottom ground node of your circuit. The WaveGen should be STOPPED or IDLE while you are setting up your circuit.
- b. The two scope channels are also provided by the Analog Discovery. Channel 1 will measure the input voltage coming from the WaveGen output, so it should be connected to the same two points as described above. In other words, connect the Orange wire for CH1+ to the left of the capacitor (where the Yellow wire is), and the Orange/White wire for CH1- to the bottom ground node of your circuit (where the Black Analog Discovery wire is).

Channel 2 will measure the voltage across the resistor. Connect the Blue wire for CH2+ from the Analog Discovery to the top of the resistor, and the Blue/White wire for CH2- to the same ground node as before.

Once you have the circuit constructed,

- In WaveForms, select the Wavegen instrument. Set the WaveGen output amplitude (V_s) to 2 V (this will give you 4 V peak-to-peak). Set the Channel 1 WaveGen Frequency to 1 kHz. Press the **Run** button for Channel 1 to start the output. The green Running label should display above the sinewave, as seen below in Figure 6.

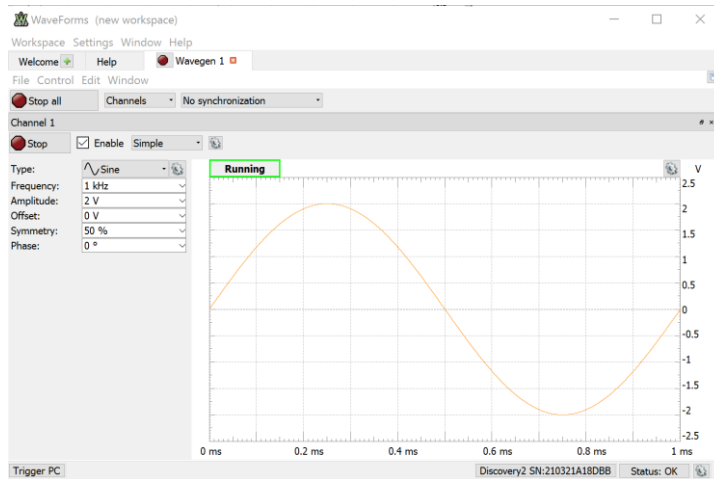


Fig 6. WaveGen display from WaveForms application

- Click on the Welcome tab to open up the scope window. In the scope window, make sure in the right panel under Time, the timebase is 1 ms/div, and under C1, the Offset is set to 0V and the Range is set to 1V/div. Use the scope channel 1 to confirm the peak-to-peak voltage. Also, to avoid distractions initially, uncheck the C2 box on the right to turn off Channel 2. If necessary, you can use Auto Set (click the green down arrow on the right side of display then Auto Set) to initially lock on to the signal, then change the Time Base and Range accordingly. See Figure 7.

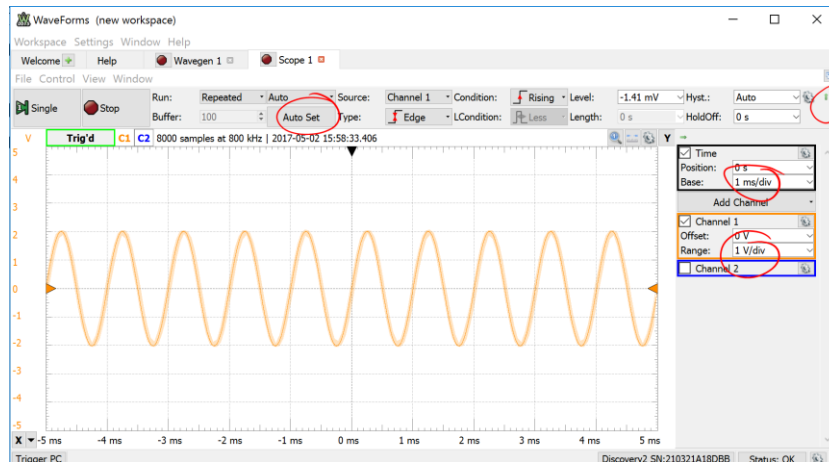


Fig 7. Oscilloscope display from WaveForms application

7. To use the measurement feature, click on the “**View**” menu, then choose “Measure.” Then **Add** a **Defined, Channel 1, Vertical** measurement for the amplitude Peak-to-Peak voltage for Vs or Channel 1 (under Vertical). Record this value in Table II of your datasheet.
8. Also verify the frequency is correct. In “Measure” add a measurement for the period of Channel 1’s waveform (under Horizontal). It should be 1 millisecond. If the number is fluctuating too rapidly, you should be able to show the average period by choosing **Show** (under Measurements) and then pick **Average**. If this doesn’t work, you can also measure the period using two vertical markers (click X in the lower left corner and position them at successive rising zero crossings and record the Δt value (time between the X1 and X2 markers).

Comparing Oscilloscope and DMM Voltage Readings

It can be interesting to compare the measurements of the two voltages taken using the oscilloscope and the digital multimeter (DMM). To do so, we’ll switch the DMM to read AC Voltage (\tilde{V}), and probe the circuit at the same points where the Analog Discovery Oscilloscope channels 1 and 2 are connected. This can be tricky. You may want to clip some short wires to the alligator clips so you can insert them into the breadboard, or attach an alligator clip to the wire from the capacitor.

However, when you read the DMM in AC mode, the measurement is expressed in Volts RMS, or Root Mean Square, which is also called the “effective” voltage. What RMS indicates is the equivalent or “effective” DC Voltage that would provide the same power to a resistor as the AC voltage it is measuring.

For a sinewave, RMS voltage is defined as $V_{rms} = \frac{V_m}{\sqrt{2}}$, where V_m is the magnitude of the sinewave, which is 1/2 of the peak-to-peak voltage that the oscilloscope measures.

9. Calculate the RMS Voltage for Vs. Remember that V_{pp} is twice the value of V_m , and that $V_{rms} = \frac{V_m}{\sqrt{2}}$. Enter this calculated RMS voltage for Vs in the next row of Table II. Also use your digital multimeter (DMM) to measure $V_{s,rms}$, and record in the bottom row of Table II.
10. Now check the C2 box and repeat the measurements for V_R , using Ch2 of the scope. Record in Table II.
11. For I_R , the last column in Table II, just take V_R and divide by R for all three rows

Table II. Comparing Peak-to-Peak and RMS Values from the Oscilloscope and your DMM. Remember that V_{pp} is twice the value of V_m , and that $V_{rms} = \frac{V_m}{\sqrt{2}}$.

Value Determination	V_S	V_R	$I (= V_R / R)$
Scope Peak-to-Peak			replace with V_R / R here
Calculated RMS Value			replace with V_R / R here
DMM RMS Value			replace with V_R / R here

Practically speaking, although it is easier to use the DMM to measure RMS voltage, we will stick with the oscilloscope peak-to-peak value for all future measurements and calculations, since that gives us more visibility into what is happening with our signals. Also, the DMM accuracy breaks down after the frequency increases beyond a certain point.

Phase Measurements

We will assume the phase angle of our voltage source, $\theta_v = 0$, since we are making the voltage source V_S our reference. Since V_R is directly proportional to I , we can measure the phase of the current circulating around the circuit, relative to V_S , by measuring the phase offset between V_S and V_R .

Q: How do we measure phase in degrees when the oscilloscope reports only time on the horizontal axis?

A: We will use the fact that in one cycle or period T of the sinewave, the phase changes 360 degrees. So if we measure the time Δt between zero crossings of the two sinewaves, we can scale that value (multiply) by $360/T$ to get the phase angle in degrees.

Refer to Figure 8 below for an image of what the oscilloscope display should look like during the following steps.

12. First measure the period T of your input sinewave. Use the Measure tab (Tools>Measure) to bring up the measurement selection. Choose Channel 1 > Horizontal and measure the Average Signal Period between positive zero crossings. Record this in Table III of your datasheet.
13. In the Scope display, change the time base to 200 us/div so you can zoom in on the signals. Measure Δt by dragging two vertical cursors from the black X in the lower left hand corner, and arranging them on a successive pair of Channel 2 / Channel 1 zero crossings. The exact difference between your two cursors will be displayed by the "X2 Δ 1" at the bottom of the scope display. This is your Δt measurement. Record in Table III of your datasheet.

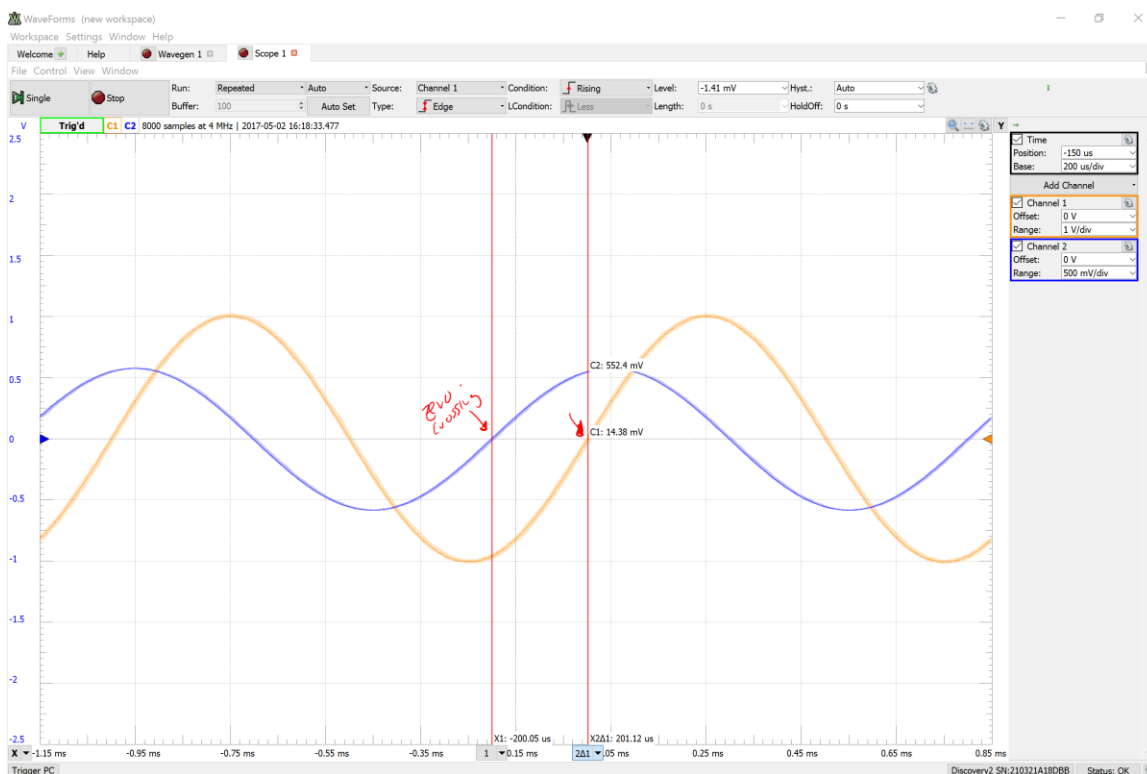


Fig 8. Measuring Phase by finding the time between zero crossings
(Red arrows added for emphasis)

14. Multiply this time offset Δt by $360/T$ (the period) to get phase angle in degrees. The sign of the phase angle (θ_i) should be positive if the V_R waveform peaks to the left of the V_S waveform. This is an example of how the current "leads" the voltage and is the norm for capacitive circuits. (Note: if we were doing power calculations the angular expression $\theta_v - \theta_i$ would be a negative number when the current leads the voltage).

Summarizing, $\theta_i = \Delta t \frac{360^\circ}{T}$ and should be positive if the current peaks before the voltage (as in this case). Add this calculation of θ_i to Table II of your datasheet.

Table III Current phase measurement (θ_i) at 1000 Hz. The phase angle should be close to 72 degrees.

Frequency, f	T, ms	Δt , ms	θ_i , degrees
1000 Hz			

Determining Z_{eq} Experimentally

We now have all of the information needed to determine the equivalent impedance Z_{eq} of our series RC connection from our experimental data, so we can compare it to our theoretical calculations in Table I.

First, we need to express the voltage V_S and current I as phasors. For consistency, we will just use the peak-to-peak values, although in our theory work we will typically use the magnitude or RMS values.

15. Enter the phasor representation of V_S and I in your datasheet. Use the peak-to-peak values for the amplitude. The phase angle of V_S is zero and the phase angle of I is determined in Table III.

$$\begin{aligned} \text{Phasor } V_S &= \underline{\hspace{2cm}} < 0^\circ \\ \text{Phasor } I &= \underline{\hspace{2cm}} < \underline{\hspace{1cm}}^\circ \end{aligned}$$

16. Now find the experimental Z_{eq} for the RC network by dividing the two quantities above. Remember, when dividing complex numbers in polar form, the magnitudes divide, but the phase angle of the denominator is subtracted from the phase angle of the numerator.

$$\begin{aligned} \text{Experimental } Z_{eq} = \mathbf{V_S/I} &= \underline{\hspace{2cm}} < \underline{\hspace{1cm}}^\circ \text{ (polar)} \\ &= \underline{\hspace{2cm}} \text{ (rect)} \end{aligned}$$

Add these calculations to your datasheet.

17. Now repeat these measurements at two additional frequencies, 4500 Hz and 10,000 Hz, and enter all of your results in Table IV in your datasheet.

Table IV Experimental RC Impedance Calculation at 3 frequencies

Frequency, f	$V_{S\ pp}$	$V_{R\ pp}$	I_{pp}	T	Δt for V_R	θ_i	Z_{EQ} expermt (rect)	Z_{EQ} expermt (polar)
1000 Hz								
4500 Hz								
10000 Hz								

Reflecting on your Results

Please write a brief answer to the following questions in your datasheet.

1. How do the experimental results in Table IV compare to the theoretical results in Table I?
2. If there are errors, do they appear to be in the amplitude or phase angle of the impedance, or in the real or imaginary part?
3. Which do you think is the larger contribution to error when measuring impedance, the magnitude or the phase angle measurements?
4. How would you describe what impedance is, in your own words?

PART 2 Series RLC circuit [Optional, for 3 points Extra Credit]

Insert the 33 mH inductor in series with the capacitor as shown in Figure 7 at right.

Then repeat the steps you followed above and complete Table V and VI with your results.

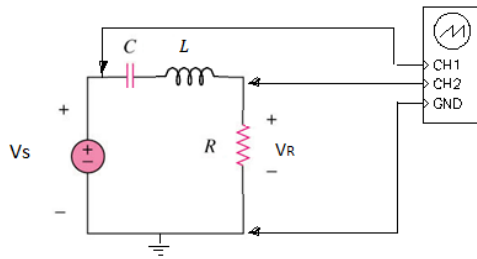


Figure 7: Series RLC circuit

Table V. Theoretical Impedance calculations for a series RLC network at 3 different values of ω . Recall that $\omega = 2\pi f$. Show Z_{eq} in rectangular and polar form.

Frequency, f	Frequency, ω	$Z_{eq} = R + j\omega L - \frac{j}{\omega C}$	$Z_{eq} = Z_m \angle \theta$
1000 Hz			
4500 Hz			
10000 Hz			

Table VI. Experimental RLC Impedance Calculation at 3 frequencies

Frequency, f	V _{S pp}	V _{R pp}	I _{pp}	T	Δt for V _R	θ_i	Z _{EQ} expermt (rect)	Z _{EQ} expermt (polar)
1000 Hz								
4500 Hz								
10000 Hz								

Reflecting on your Results

Please write a brief answer to the following questions in your datasheet.

1. How do the experimental results in Table VI compare to the theoretical results in Table V?
2. The 33 mH inductor has an internal winding resistance of about 100 Ω , which you can determine by connecting an ohmmeter to it. This internal resistance would add to the overall resistance of the circuit, diminishing the value of current I. If you incorporated this extra resistance into your value for R that you measured in step 1, would that bring your theoretical results closer to your experimental results, or farther away?