Lab 11
Phasors and MATLAB

Objectives – in this lab you will

- Learn how to perform arithmetic with complex numbers in Matlab
- Learn how to transform sinusoids into phasor values
- Calculate Complex Impedance and Admittance
- Plot two sinusoids with different phase on the same plot

Key Prerequisites

- Chapter 9

Required Resources

- PC with internet access

This lab introduces the kinds of mathematical computations you will need to perform Phasor Analysis on circuits driven by sinusoidal sources. It shows how MATLAB or FreeMat can make these computations a lot easier to perform, and shows how to plot the time domain representation of two phasors to help visualize the phase properaty of a signal.

Vocabulary

All key vocabulary used in this lab are listed below, with closely related words listed together:

Magnitude and Phase of a Complex Number
Phasor Transform
Impedance and Admittance

Discussion and Procedure

Part 1. Complex Numbers in MATLAB / FreeMat

MATLAB uses the letters `i` or `j` to represent the square root of -1. Complex numbers can be entered directly at the command line in rectangular form and stored in variables for further computation.

```
--> z = 10 - 12j
z =
  10.0000 - 12.0000i
```

HOWEVER, BEWARE of how order of operations works when complex numbers are involved.

For example, the expression `1/2i` in MATLAB evaluates to `−0.5i` or $\left(\frac{1}{2i}\right)$ because the value of `i` is factored in before the division.

However, on my calculator, the express `1/2i` evaluates to `0.5i` or $\left(\frac{1}{2}i\right)$ because the value of i is factored in after the division.

To enter a complex value in polar form, for example, w = 10<45, you'll need to do the conversion to rectangular form as you type:

```
--> w = 10*cosd(45) + j*10*sind(45)
w =
   7.0711 +  7.0711i
```

BE CAREFUL to use the **<u>`sind`</u>** and **<u>`cosd`</u>** functions when using units of degrees for the angles. The regular **<u>`sin`</u>** and **<u>`cos`</u>** functions assume the angles are in radians.

Once a value is entered into matlab in can be used in complex arithmetic. For example:

```
--> u = w * z
u =
 155.5635 - 14.1421i
```

and

```
--> t = w + z
t =
  17.0711 -  4.9289i
```

To convert a value from rectangular form to polar form use the abs command and the angle command

```
--> abs(w)
ans =
 10
--> angle(w)
ans =
    0.7854
--> angle(w)*180/pi
ans =
 45
```

NOTE: Angle returns angle in radians and must be converted to degrees

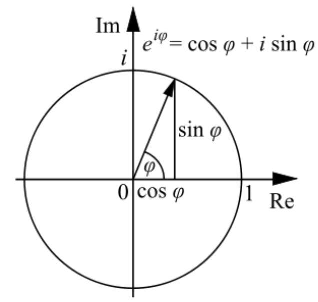
Complex Drill Exercises

Complete the exercises now under Part 1 of the Lab 11 Datasheet.

Part 2. Phasor Transforms

The phasor transform comes from Euler's identity, namely,

$$e^{\pm j\theta} = \cos(\theta) \pm jsin(\theta)$$



We can therefore think of $\cos(\theta)$ as the Real part of $e^{j\theta}$

For a sinewave, $\theta = \omega t + \varphi$, but Euler's identity still applies.

In fact we can consider

$$\mathrm{v} = \mathrm{V_m}\cos(\omega t + \varphi) = Re\{V_m\, e^{j\omega t + \varphi}\}$$

$$= Re\{V_m e^{j\omega t} e^{j\varphi}\}$$
$$= Re\{V_m e^{j\varphi} e^{j\omega t}\}$$

In the previous equation $V_m e^{j\varphi}$ is a complex number that carries the amplitude and phase angle of the original sinusoid. This complex number is the definition of the **phasor transform** of the given sinusoid. In other words, the phasor

$$\mathbf{V} = V_m\, e^{j\varphi} = P\{\mathrm{V_m}\cos(\omega t + \varphi)\},$$

where $P\{\ \}$ indicates the phasor transform. Note that **V** does not have a time component (ωt has been removed). However, you can think of it as representing the amplitude and phase angle of the sinusoid (two quantities rolled into one complex number) when t = 0.

Note also that as a complex numbe, **V** can also be represented in polar form:

$$\mathbf{V} = V_m \cos(\varphi) + j\, V_m \sin(\varphi)$$

We use the phasor to solve various operations on sinusoids of different magnitudes and phases, as well as complex valued circuit elements. When we are finished, we take our solution back to the time domain with the *inverse* phasor transform:

$$P^{-1}\{V_m e^{j\varphi}\} = Re\{V_m e^{j\omega t} e^{j\varphi}\}$$

As an example, suppose we have a sinusoid v1 = 10 cos(wt + 45°) and we wish to add to it the sinewave v2 = 5 cos(wt + 60 °). We can find v = v1 + v2 either with trig identities (very messy) or phasors. Here we'll use phasors:

```
--> v1 = 10*cosd(45) + j*10*sind(45)

v1 =   7.0711 +   7.0711i

--> v2 = 5*cosd(60)+j*5*sind(60)

v2 =   2.5000 +   4.3301i

--> v = v1 + v2
```

```
v =   9.5711 + 11.4012i

--> Vm = abs(v)

Vm =   14.8860

--> phi = angle(v)*180/pi

phi =   49.9872
```

Therefore our solution to v = v1 + v2 is v = 14.8 cos(wt + 49.9°)

Phasor Drill Exercises

Complete the exercises now under Part 2 of the Lab 11 Datasheet

Part 3. Basic Circuit Elements in the AC (Phasor) Doma

In order to obtain a solution to a circuit with applied sinewave signals, we need to formulate the behavior of our familiar Resistor, Inductor and Capacitor circuit elements in the phasor domain.

Whereas a resistor behaves in the phasor domain the same as in the time domain, inductors and capacitors now have "complex resistance" also called "impedance".

The V-I Relationship for an Inductor

We derive the relationship between the phasor current and phasor voltage at the terminals of an inductor by assuming a sinusoidal current and using $L\,di/dt$ to establish the corresponding voltage. Thus, for $i = I_m \cos(\omega t + \theta_i)$, the expression for the voltage is

$$v = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \theta_i). \qquad (7.28)$$

We now rewrite Eq. (7.28) using the cosine function:

$$v = -\omega L I_m \cos(\omega t + \theta_i - 90°). \qquad (7.29)$$

The phasor representation of the voltage given by Eq. (7.29) is

$$\begin{aligned}\mathbf{V} &= -\omega L I_m e^{j(\theta_i - 90°)}\\ &= -\omega L I_m e^{j\theta_i} e^{-j90°}\\ &= j\omega L I_m e^{j\theta_i}\\ &= j\omega L \mathbf{I}. \qquad (7.30)\end{aligned}$$

Note that in deriving Eq. (7.30) we used the identity

$$e^{-j90°} = \cos 90° - j\sin 90° = -j.$$

Equation (7.30) states that the phasor voltage at the terminals

The V-I Relationship for a Capacitor

We obtain the relationship between the phasor current and phasor voltage at the terminals of a capacitor from the derivation of Eq. (7.30). In other words, if we note that for a capacitor that

$$i = C\frac{dv}{dt},$$

and assume that

$$v = V_m \cos(\omega t + \theta_v),$$

then

$$\mathbf{I} = j\omega C \mathbf{V}. \qquad (7.32)$$

Now if we solve Eq. (7.32) for the voltage as a function of the current, we get

$$\mathbf{V} = \frac{1}{j\omega C}\mathbf{I}. \qquad (7.33)$$

Or, Ohm's Law for Inductors is $V = jwL\, I$ and Capacitors is $V = \frac{-j}{wC} I$

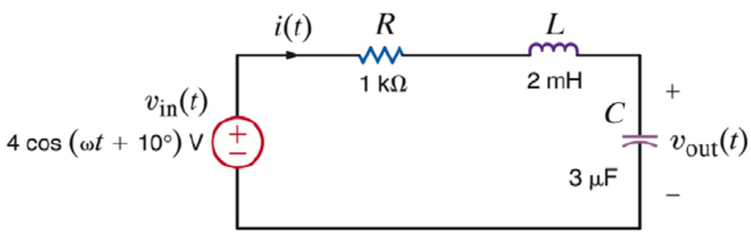
Impedance is the complex valued relationship between phasor voltage and phasor current, while **reactance** is just the imaginary part of the impedance. We summarize these relations in the table below:

TABLE 7.1 Impedance and reactance values

CIRCUIT ELEMENT	IMPEDANCE	REACTANCE
Resistor	R	—
Inductor	$j\omega L$	ωL
Capacitor	$j(-1/\omega C)$	$-1/\omega C$

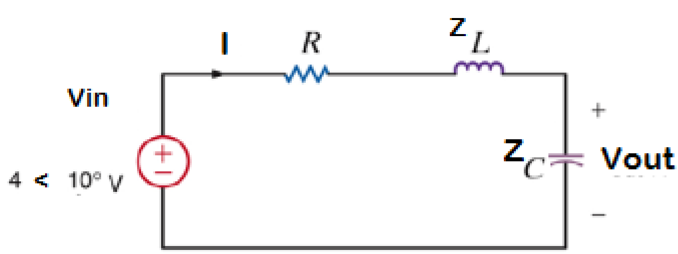
Phasor Drill Exercises

Complete the exercises now under Part 3 of the Lab 11 Datasheet, which pertain to this circuit. Assume that the sinewave frequency is 60 Hz, in which case the angular frequency $\omega = 2\pi f$, which is ~377 radians/sec.



Part 4. AC Solution by Phasor Analysis (assume f still = 60 Hz, or ω = 377 radians/sec)

Having transformed the circuit above into the phasor domain in the previous section, we are now ready to find phasor Vout in the circuit above. It can be helpful to redraw the circuit with all the component values expressed in phasor form, such as



where, in general, Vin, Vout, ZL and ZC are expressed as complex numbers.

If we think of the impedances ZL and ZC as complex "resistances", then the relationship between Vout and Vin can be seen as an example of a voltage divider, only using complex numbers. Therefore, our solution in Matlab for Vout, having already computed the above variables, will be similar to the voltage divider formula we developed for DC circuits:

```
Vout = Vin*Zc / (1000 + ZL + ZC)
```

where ZL and ZC are values computed in Part 3.

Of course, we still need to convert the result back into the time domain, which you can do in the datasheet.

After finishing the example problem, [OPTIONAL] complete the other two problems listed in the datasheet, showing your steps at arriving at an answer.