EQUILIBRIUM OF A PARTICLE, THE FREE-BODY DIAGRAM & COPLANAR FORCE SYSTEMS

Today's Objectives:

Students will be able to :

a) Draw a free body diagram (FBD), and,

b) Apply equations of equilibrium to solve a 2-D problem.

In-Class Activities:

- Reading Quiz
- Applications
- What, Why and How of a FBD
- Equations of Equilibrium
- Analysis of Spring and Pulleys
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1) When a particle is in equilibrium, the sum of forces acting on it equals ___ . (Choose the most appropriate answer)

A) a constant B) a positive number C) zero
D) a negative number E) an integer.

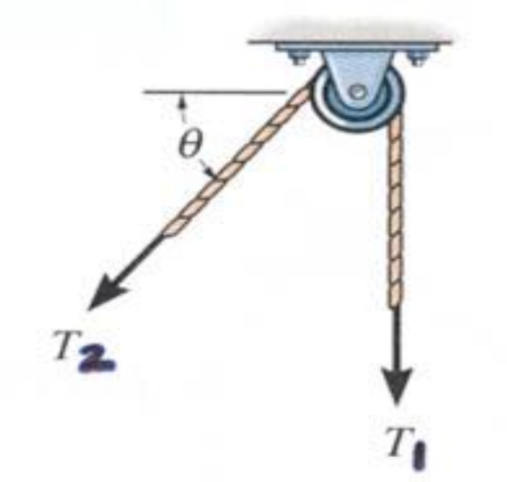
2) For a frictionless pulley and cable, tensions in the cable (T_1 and T_2) are related as _____ .

A) $T_1 > T_2$

B) $T_1 = T_2$

C) $T_1 < T_2$

D) $T_1 = T_2 \sin \theta$

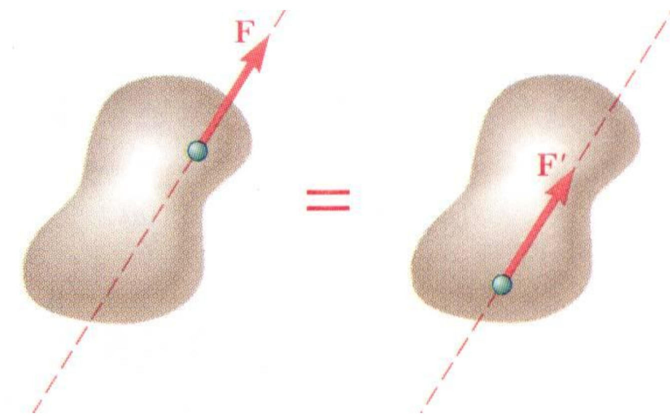


Cable is in tension

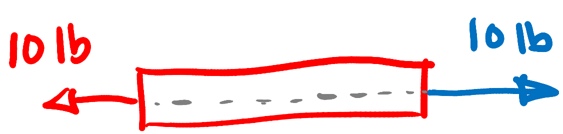
Chapter 3 – Equilibrium of Particles

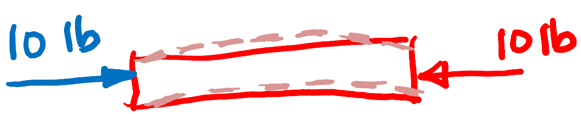
Force – action of one body on another which changes or produces a tendency to change the state of rest or motion of the body acted on.

Vector Quantity (Sliding Vector)
a) Magnitude
b) Direction
c) Line of action



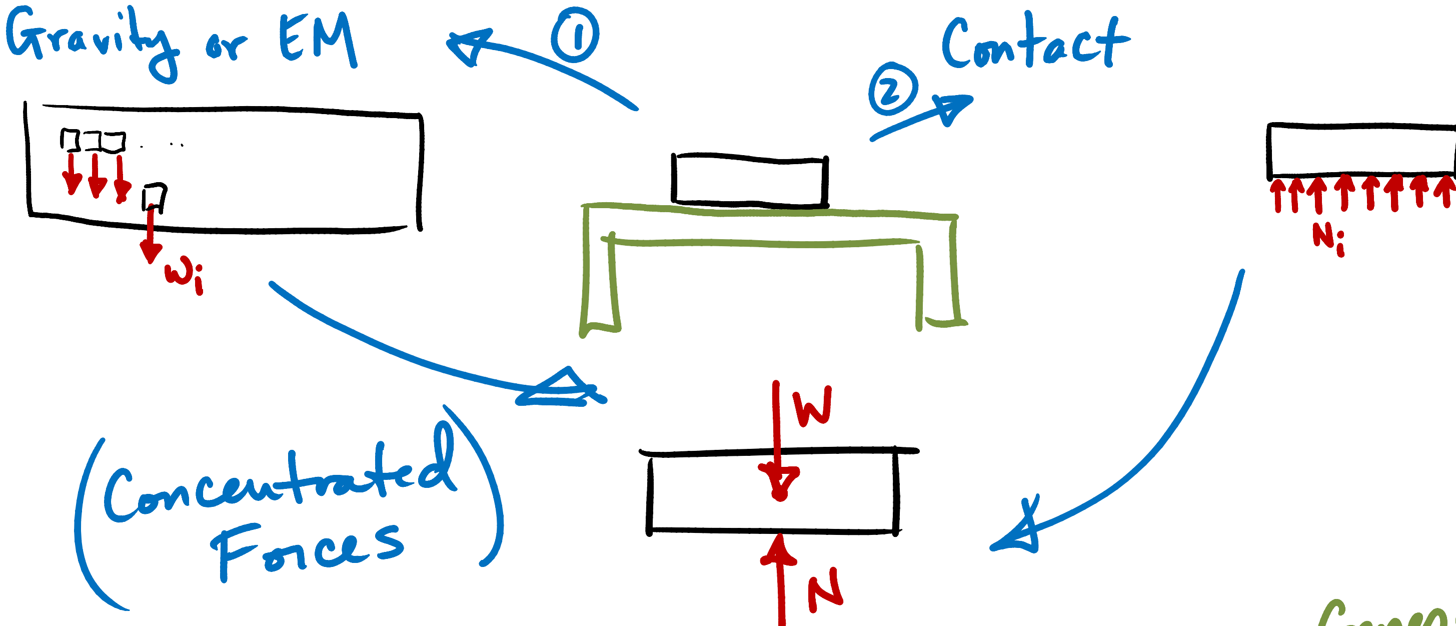
• *Principle of Transmissibility*
for Rigid Bodies





(Distributed Forces)

Body Forces vs. Surface Forces

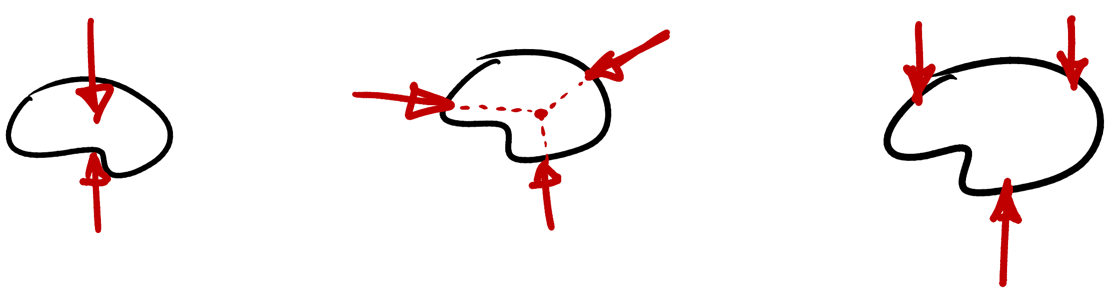


Force System Classification:

A. 2-D (coplanar) vs. 3-D

General Ch 5

B. Collinear vs. Concurrent vs. Parallel vs. Neither



Equilibrium Particle: Concurrent Forces

Equilibrium – all points of the body are at rest or have the same constant velocity.

$$\sum \vec{F}_{\text{Ext}} = 0$$

-Equilibrium equation
-Can be used to find unknown forces
-A **vector equation!**

2D ⇒ 2 eqns, 2 unk

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum F_z = 0$$

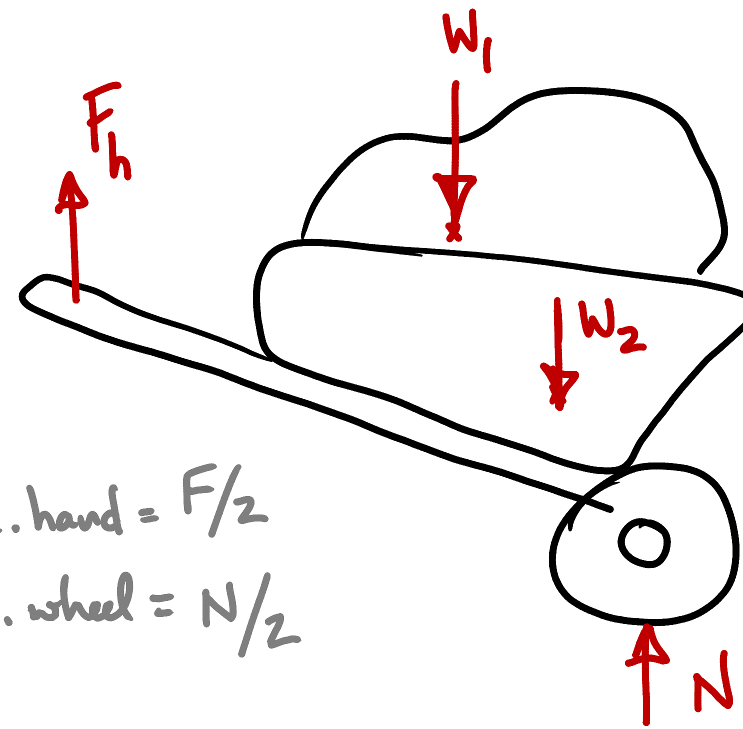
3D ⇒ 3 eqns, 3 unk

To facilitate the application of the vector equation, we use a graphical representation.

Free-Body Diagram (FBD)

- Drawing of an object (or group of objects) showing **all external** forces acting on it.

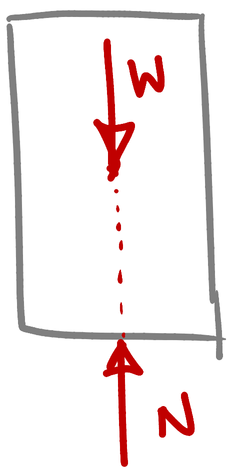
1. Isolate body
2. Show Forces (contact, body, active, reactive)
3. Identify Forces

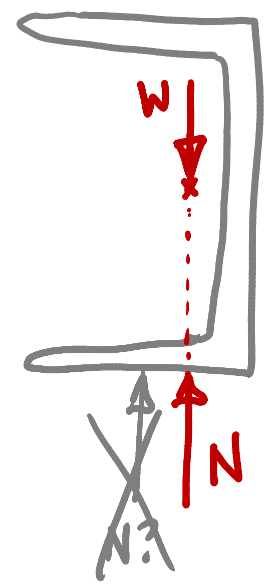


$*$ ea. hand = $F/2$

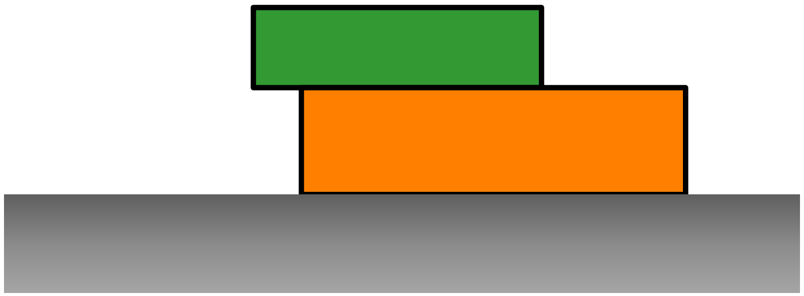
ea. wheel = $N/2$

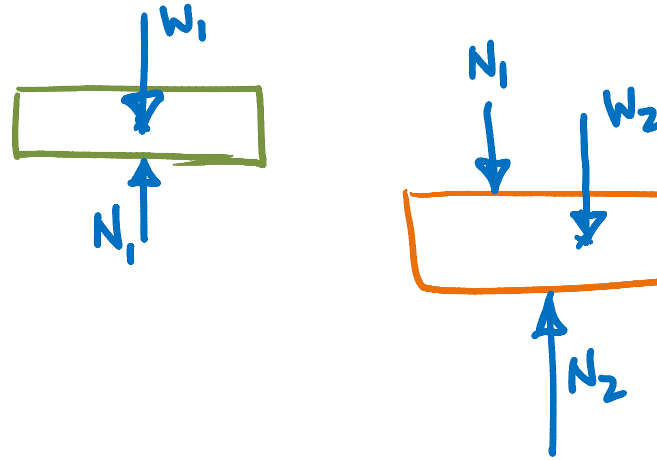
Weight and Normal Force

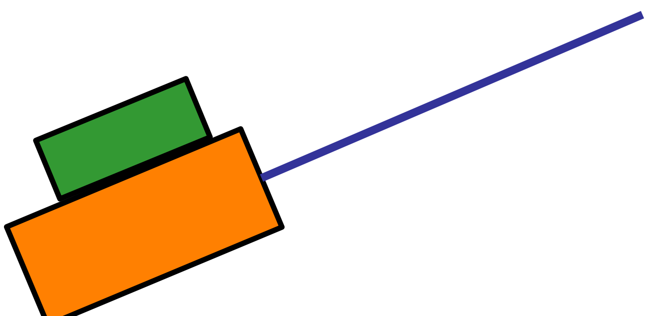


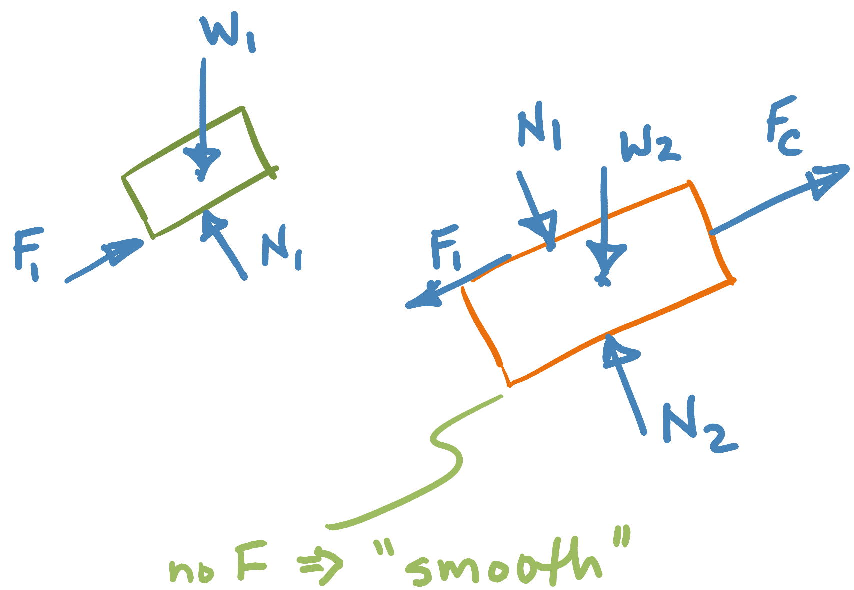


Multiple Bodies & Friction

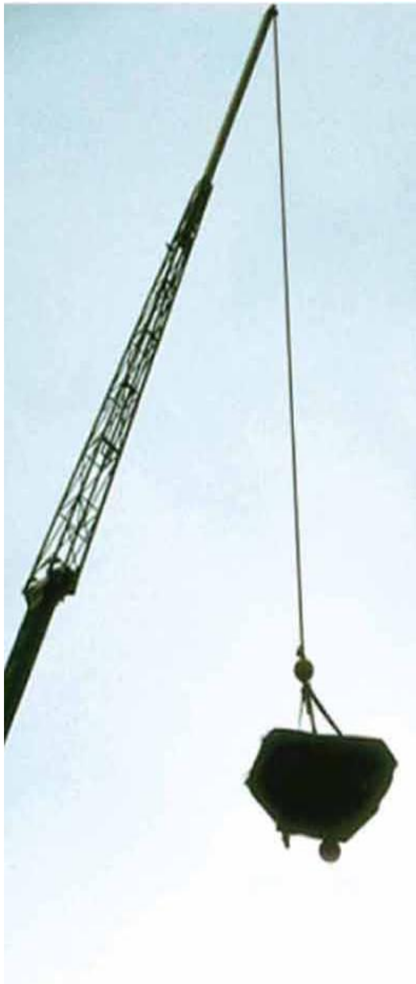


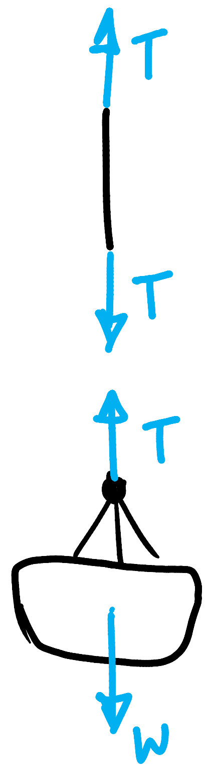


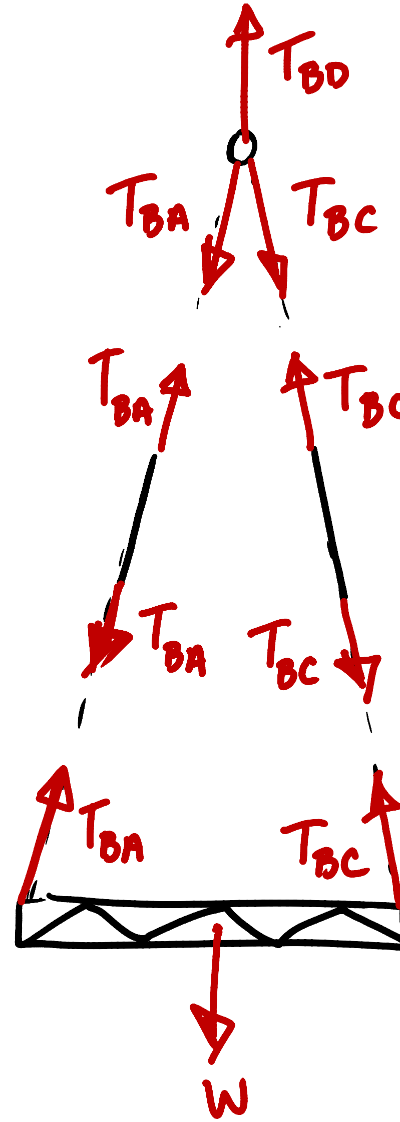




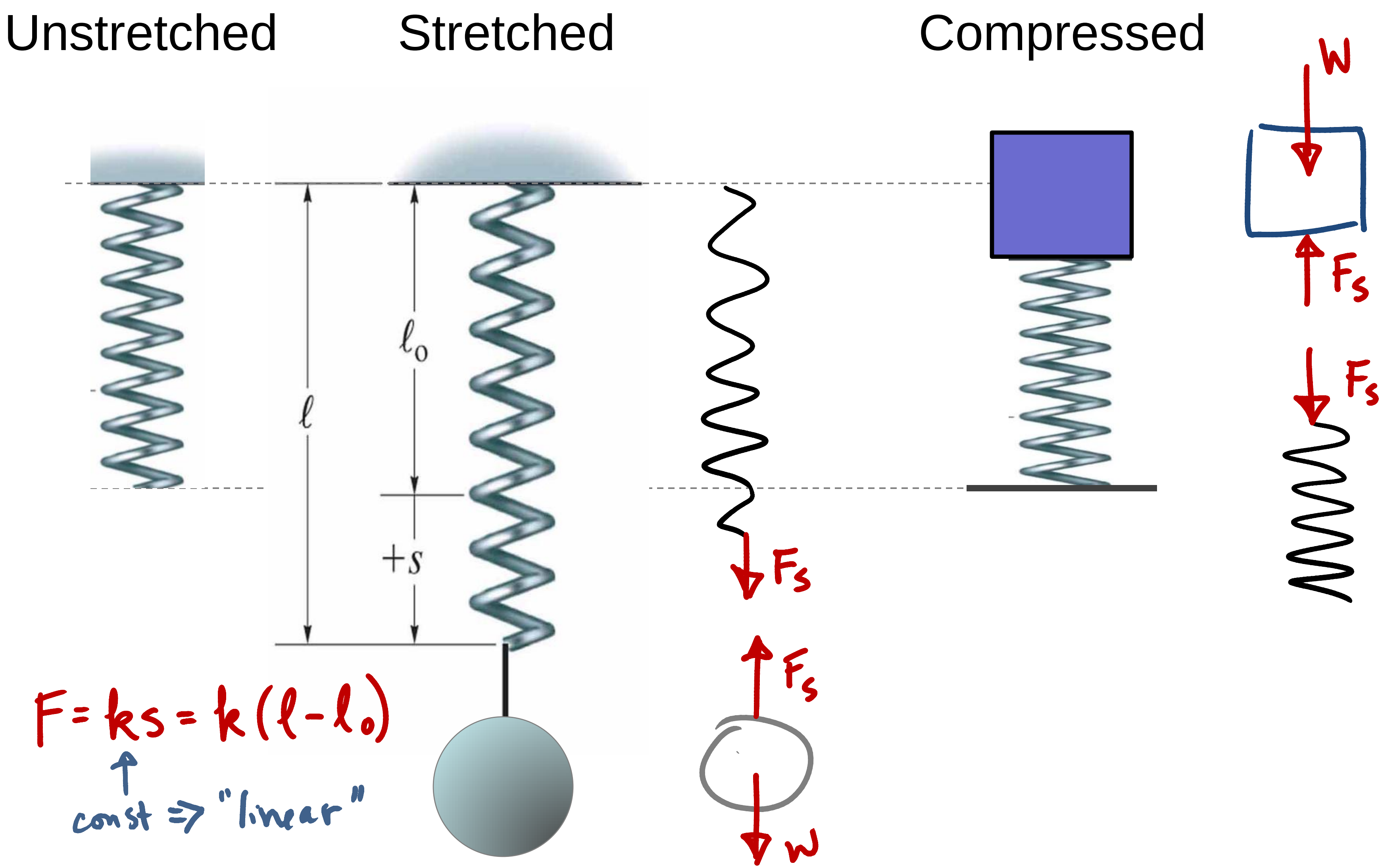
Cables







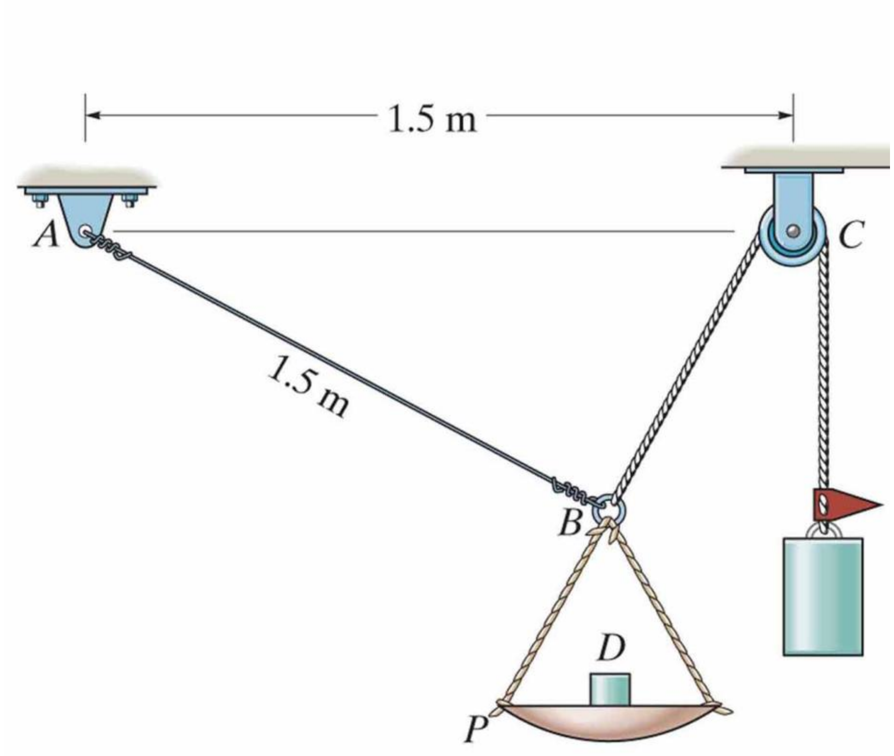
Springs

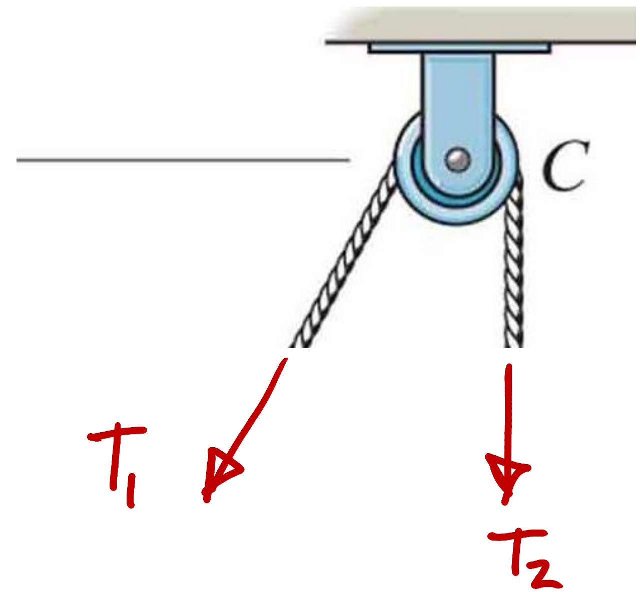


$F = ks = k(\ell - \ell_0)$

const ⇒ "linear"

Pulleys

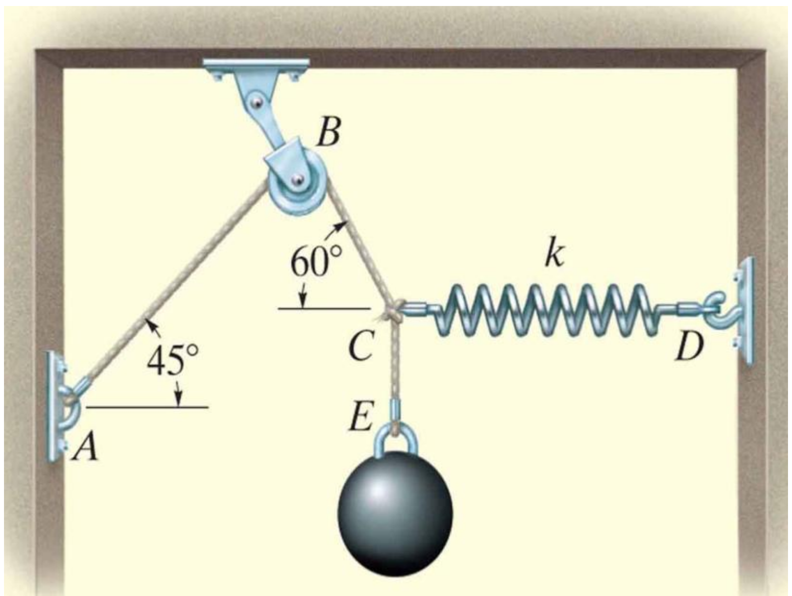


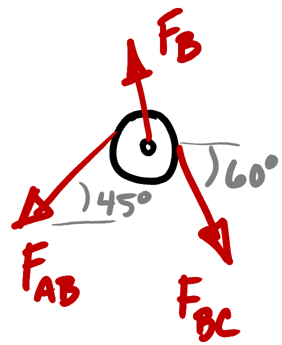


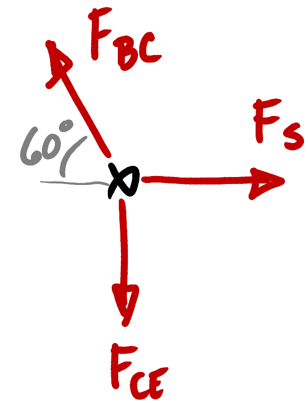
smooth/frictionless pulley

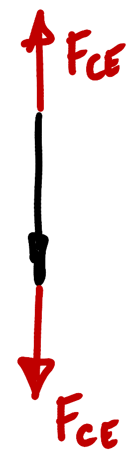
$\Rightarrow$ $\underline{T_1 = T_2}$

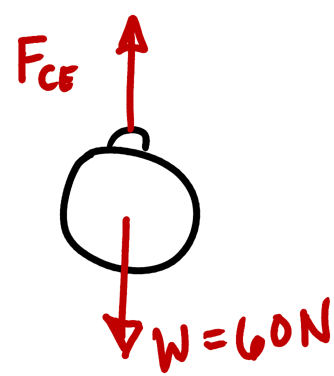
Practice: The sphere has a weight of 60N. Draw the FBD of the sphere, the cord CE, the knot C, and the pulley B.







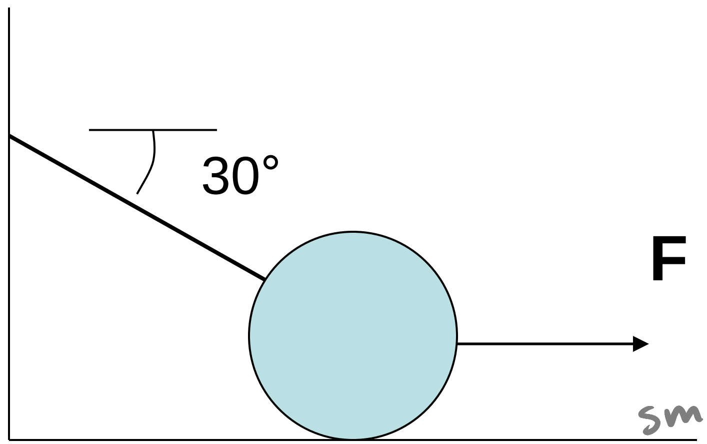


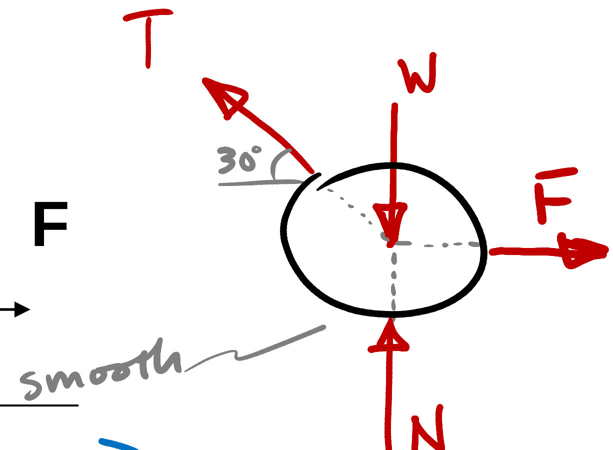


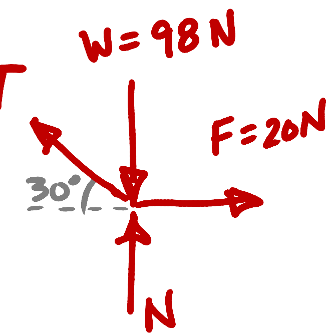
Example 1. The 10-kg sphere is at rest on the smooth horizontal surface. A) Determine the normal force on the floor and the tension in the cable if F = 20 N.

Find: N, T

Given: $F = 20N$, $W = mg = 98N$







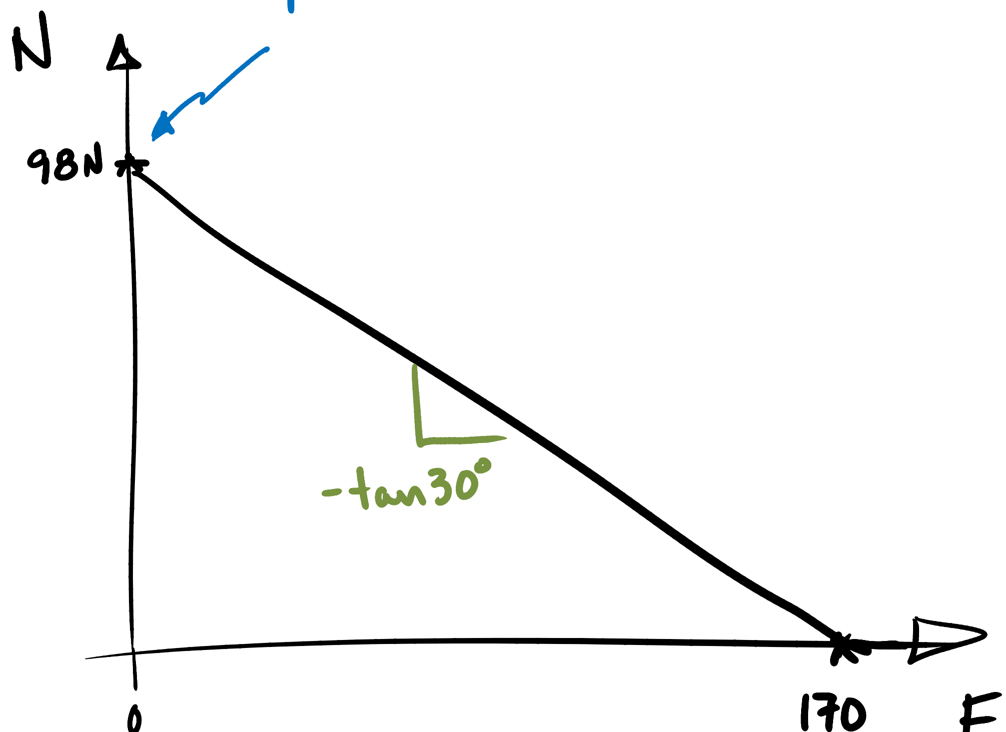
At rest ⇒ Equil.
All F's concurrent ⇒ Particle ⇒ 1 (vector) eqn.

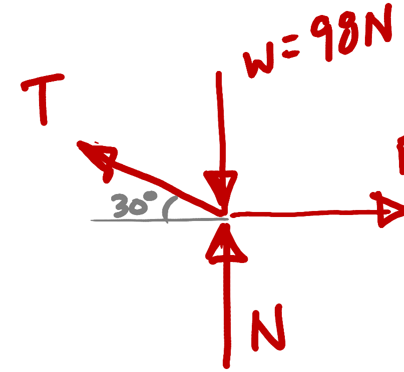
$$\sum \vec{F} = 0 \quad \begin{cases} \sum F_x = 0 = F - T\cos 30° \Rightarrow T = \dfrac{20\,N}{\cos 30°} = \underline{23.1\,N} \\ \sum F_y = 0 = N + T\sin 30° - W \\ \quad = N + (23.1\,N)\sin 30° - 98\,N \Rightarrow \underline{N = 86.5\,N} \end{cases}$$

2D ⇒ 2 scalar eqns
⇒ 2 unks.
T, N

B) Draw graph of N as a function of F.

$F=0 \Rightarrow T=0 \Rightarrow N=W$





$\Sigma F_x \Rightarrow F = T\cos 30°$

as $F\uparrow \Rightarrow T\uparrow$

$\Sigma F_y \Rightarrow N = W - T\sin 30°$

as $T\uparrow \Rightarrow N\downarrow$

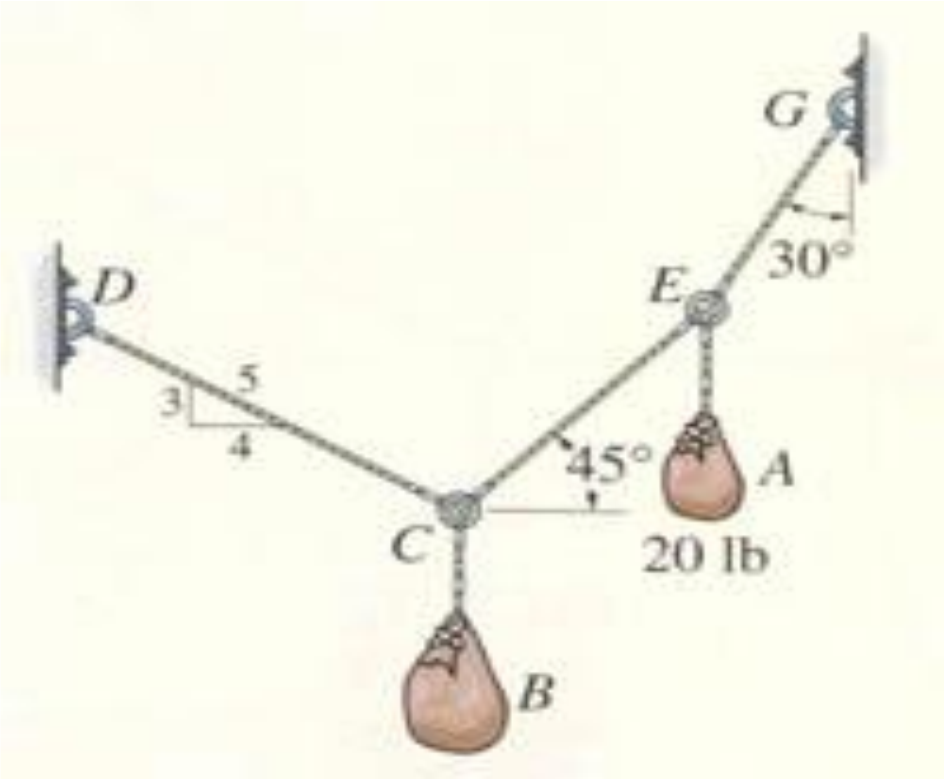
(cable supports more of W)

eventually, $N \to \emptyset$ just before ball lifts

$$N = W - T\sin 30° = W - \left(\frac{F}{\cos 30°}\right)\sin 30° \Rightarrow \boxed{N = (-\tan 30°)F + W}$$

$$N=0 \Rightarrow F = W/\tan 30° = 170\,N$$

EXAMPLE 2



Given: Sack A weighs 20 lb. and geometry is as shown.

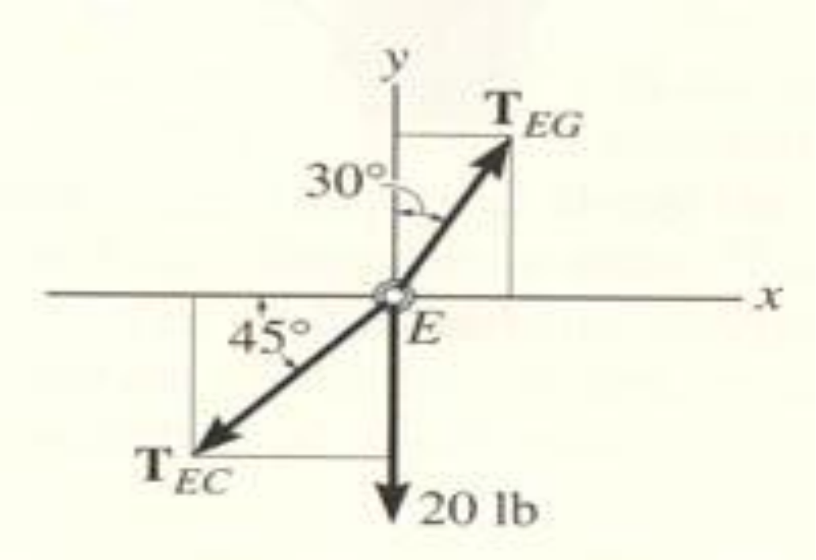
Find: Forces in the cables and weight of sack B.

<u>Plan</u>:

1. Draw a FBD for Point E.

2. Apply EofE at Point E to solve for the unknowns (T_{EG} & T_{EC}).

3. Repeat this process at C.



EXAMPLE 2
(continued)

A FBD at E should look like the one to the left. Note the assumed directions for the two cable tensions.

The scalar E-of-E are:

$+\rightarrow \quad \Sigma F_x = T_{EG} \sin 30^{\circ} - T_{EC} \cos 45^{\circ} = 0$

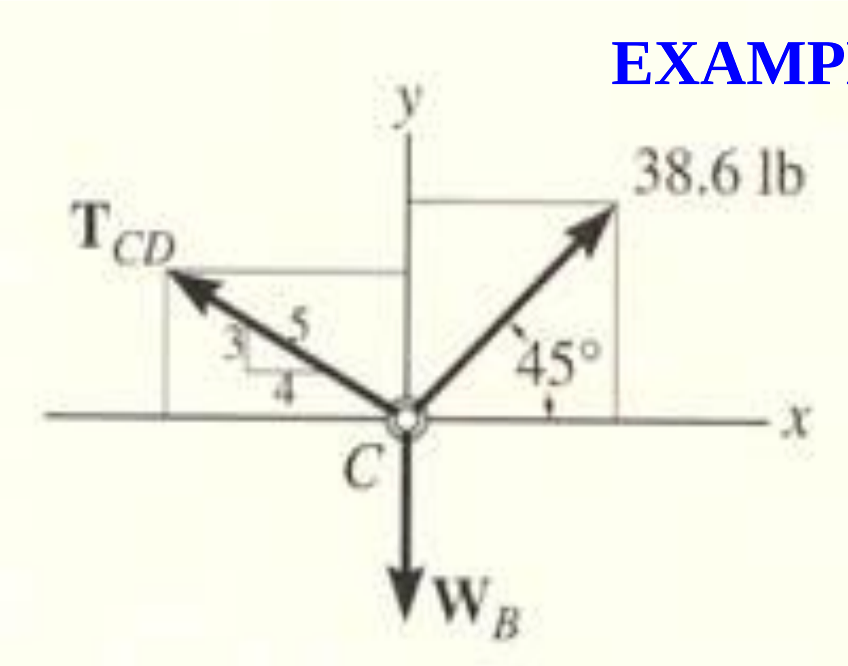
$+\uparrow \quad \Sigma F_y = T_{EG} \cos 30^{\circ} - T_{EC} \sin 45^{\circ} - 20 \text{ lbs} = 0$

Solving these two simultaneous equations for the two unknowns yields:

$T_{EC} = 38.6$ lb

$T_{EG} = 54.6$ lb

EXAMPLE 2 (continued)



Now move on to ring C. A FBD for C should look like the one to the left.

The scalar E-of-E are:

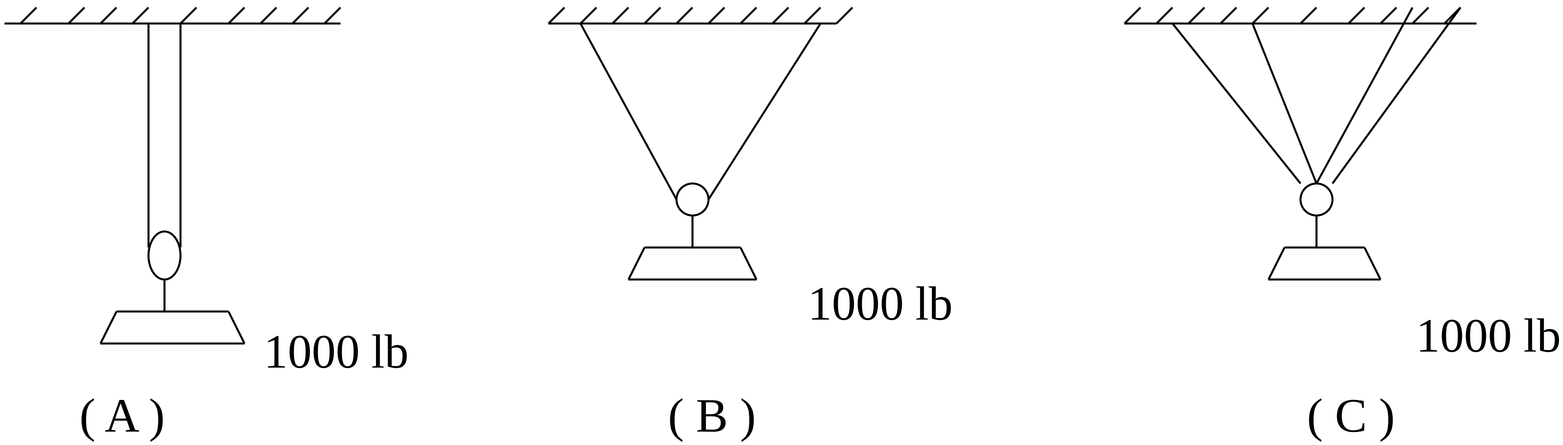
$$+\rightarrow \Sigma F_x = 38.64 \cos 45° - (4/5)\, T_{CD} = 0$$

$$+\uparrow \Sigma F_y = (3/5)\, T_{CD} + 38.64 \sin 45° - W_B = 0$$

Solving the first equation and then the second yields

$T_{CD} = 34.2$ lb and $W_B = 47.8$ lb .

CONCEPT QUESTIONS



1) Assuming you know the geometry of the ropes, you cannot determine the forces in the cables in which system above?

2) Why?

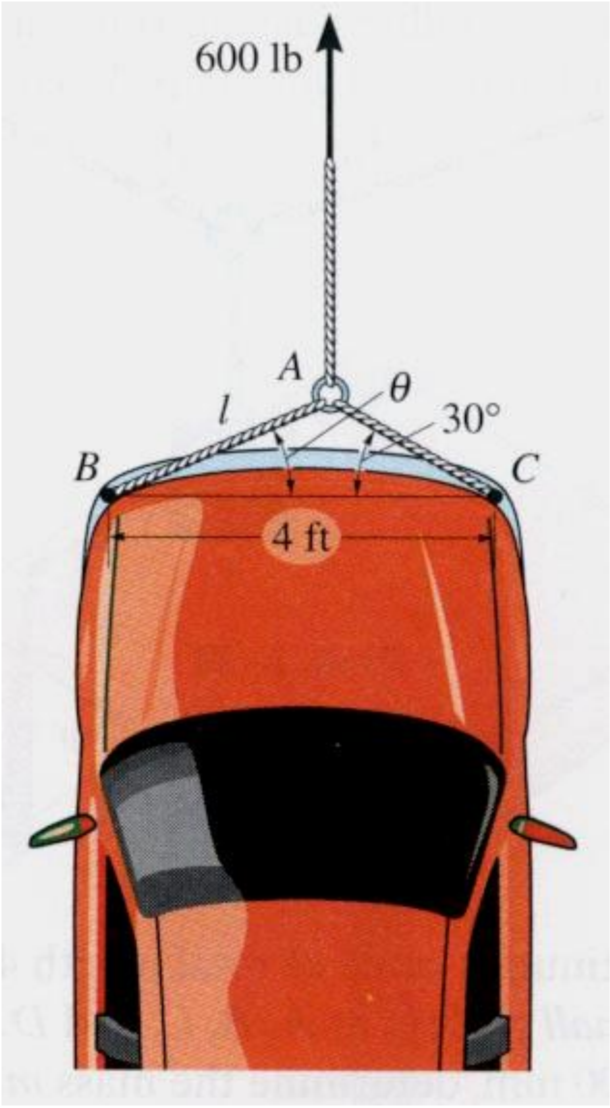
A) The weight is too heavy.

B) The cables are too thin.

C) There are more unknowns than equations.

D) There are too few cables for a 1000 lb weight.

GROUP PROBLEM SOLVING



Given: The car is towed at constant speed by the 600 lb force and the angle θ is 25°.

Find: The forces in the ropes AB and AC.

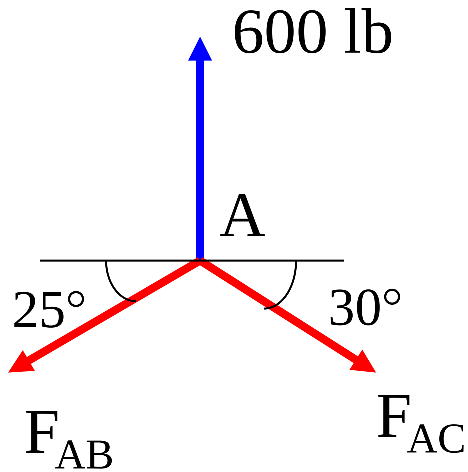
Plan:

1. Draw a FBD for point A.

2. Apply the E-of-E to solve for the forces in ropes AB and AC.

GROUP PROBLEM SOLVING

(continued)



FBD at point A

Applying the scalar E-of-E at A, we get;

$+ \rightarrow \sum F_x = F_{AC} \cos 30° – F_{AB} \cos 25° = 0$

$+ \rightarrow \sum F_y = -F_{AC} \sin 30° – F_{AB} \sin 25° + 600 = 0$

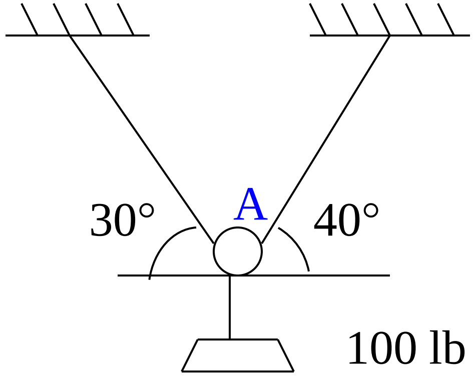
Solving the above equations, we get;

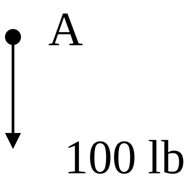
$F_{AB} = 634$ lb

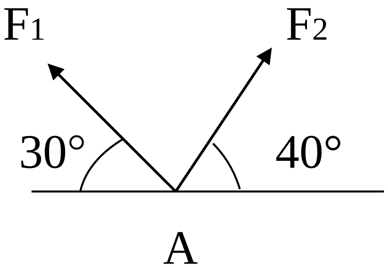
$F_{AC} = 664$ lb

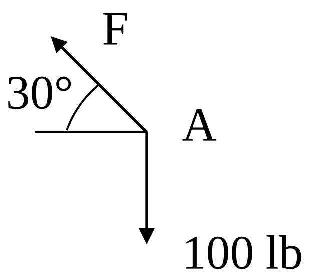
ATTENTION QUIZ

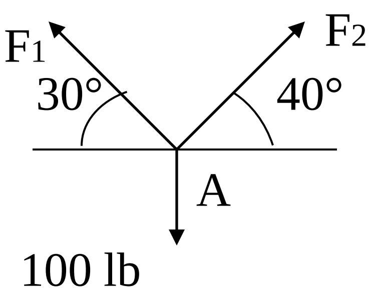
1. Select the correct FBD of particle A.



A) 

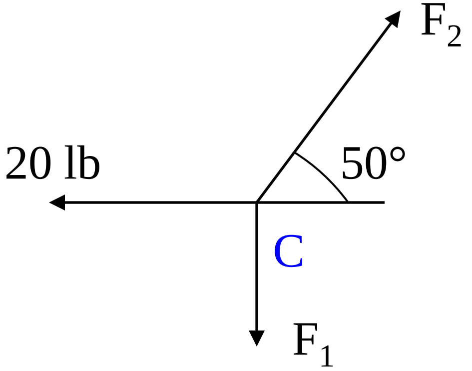
B) 

C) 

D) 

ATTENTION QUIZ

2. Using this FBD of Point C, the sum of forces in the x-direction (ΣF_X) is ___ . Use a sign convention of + $\rightarrow$.



A) $F_2 \sin 50° - 20 = 0$

B) $F_2 \cos 50° - 20 = 0$

C) $F_2 \sin 50° - F_1 = 0$

D) $F_2 \cos 50° + 20 = 0$

End of the Lecture

Let Learning Continue