POSITION VECTORS & FORCE VECTORS

Today's Objectives:

Students will be able to :

a) Represent a position vector in Cartesian coordinate form, from given geometry.

b) Represent a force vector directed along a line.

In-Class Activities:

- Check Homework
- Reading Quiz
- Applications / Relevance
- Write Position Vectors
- Write a Force Vector
- Concept Quiz
- Group Problem
- Attention Quiz

READING QUIZ

1. A position vector, $\boldsymbol{r}_{PQ}$, is obtained by

   A) Coordinates of Q minus coordinates of P

   B) Coordinates of P minus coordinates of Q

   C) Coordinates of Q minus coordinates of the origin

   D) Coordinates of the origin minus coordinates of P

2. A force of magnitude F, directed along a unit vector $\boldsymbol{U}$, is given by $\boldsymbol{F}$ = _____ .

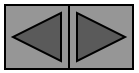
   A) F ($\boldsymbol{U}$)

   B) $\boldsymbol{U}$ / F

   C) F / $\boldsymbol{U}$

   D) F + $\boldsymbol{U}$

   E) F – $\boldsymbol{U}$



APPLICATIONS

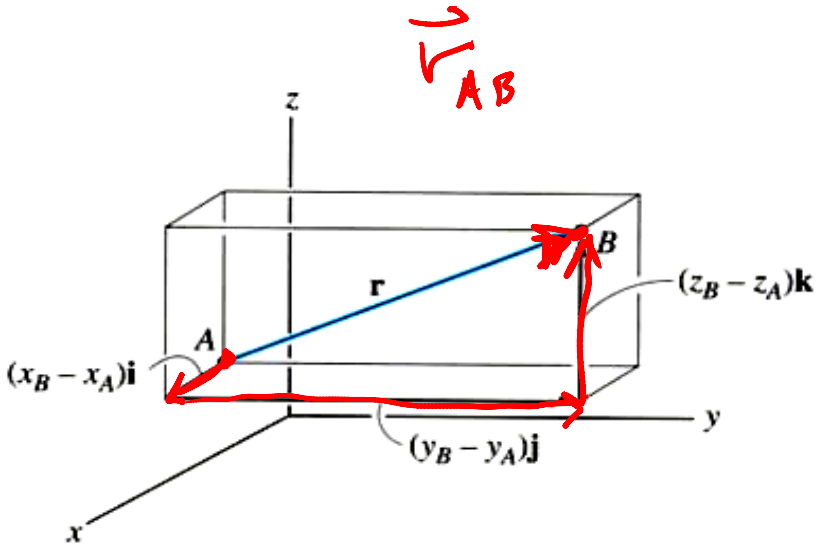
Wing strut

How can we represent the force along the wing strut in a 3-D Cartesian vector form?

POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.

Consider two points, A & B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.

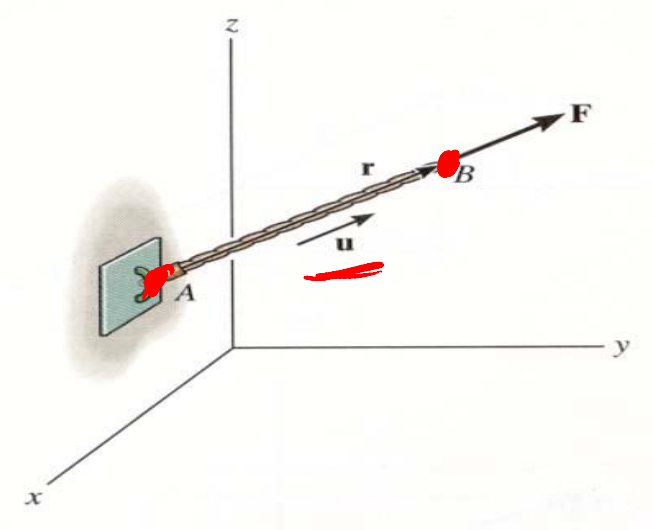


The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$$\boldsymbol{r}_{AB} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$$

Please note that B is the ending point and A is the starting point. So ALWAYS subtract the "tail" coordinates from the "tip" coordinates!

FORCE VECTOR DIRECTED ALONG A LINE
(Section 2.8)

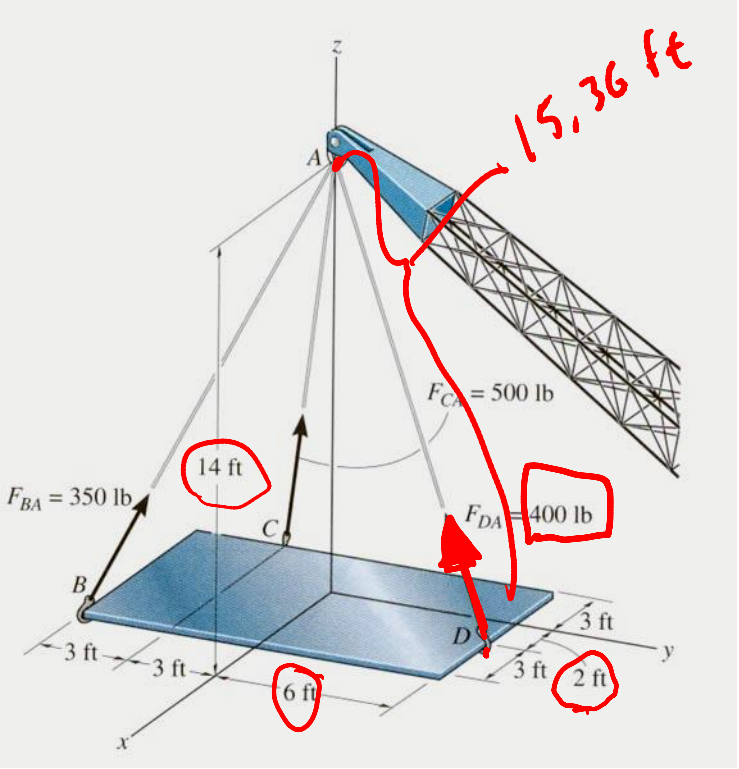


If a force is directed along a line, then we can represent the force vector in Cartesian Coordinates by using a unit vector and the force magnitude. So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\boldsymbol{u}_{AB} = (\boldsymbol{r}_{AB}/r_{AB})$.

c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = F\,\boldsymbol{u}_{AB}$.



15.36 ft

EXAMPLE **Given:** 400 lb force along the cable DA.

Find: The force $\boldsymbol{F_{DA}}$ in the Cartesian vector form.

1) Find $\vec{r}_{DA}$ $\quad A = \{0, 0, 14\}$ $\quad D = \{2, 6, 0\}$

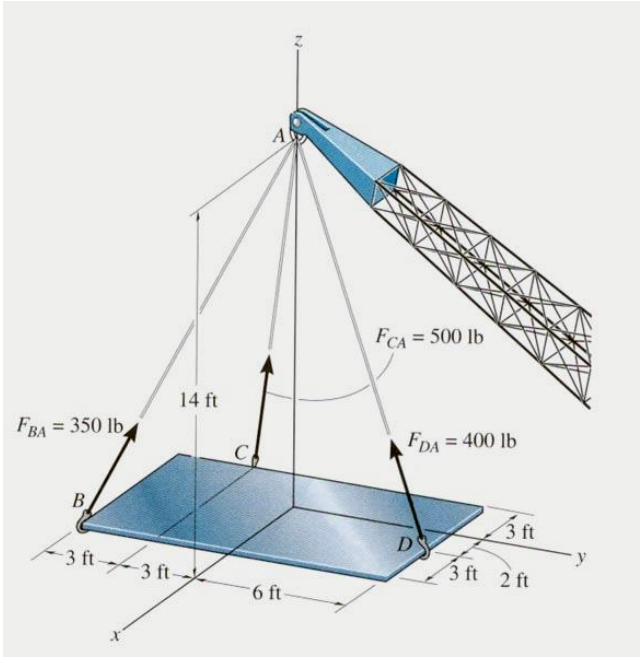
$\vec{r}_{DA} = A - D = -2\hat{i} - 6\hat{j} + 14\hat{k} = \vec{r}_{DA}$

2) Find $|r_{DA}| = \sqrt{(-2)^2 + (-6)^2 + (14)^2} = 15.36 \text{ ft}$

3) Find $\vec{u}_{DA} = \dfrac{\vec{r}_{DA}}{r_{DA}} = -\dfrac{2}{15.36}\hat{i} - \dfrac{6}{15.36}\hat{j} + \dfrac{14}{15.36}\hat{k}$

4) Find $\vec{F}_{DA} = 400\,\vec{u}_{DA} = 400\{(-2\hat{i} - 6\hat{j} + 14\hat{k})/15.36\} = (-52.1\hat{i} - 156\hat{j} + 365\hat{k})$ lb

EXAMPLE (continued)



The figure shows that when relating D to A, we will have to go -2 ft in the x-direction, -6 ft in the y-direction, and +14 ft in the z-direction. Hence,

$\boldsymbol{r}_{DA} = \{-2\,\boldsymbol{i} - 6\,\boldsymbol{j} + 14\,\boldsymbol{k}\}$ ft.

We can also find $\boldsymbol{r}_{DA}$ by subtracting the coordinates of D from the coordinates of A.

$r_{DA} = (2^2 + 6^2 + 14^2)^{0.5} = 15.36$ ft

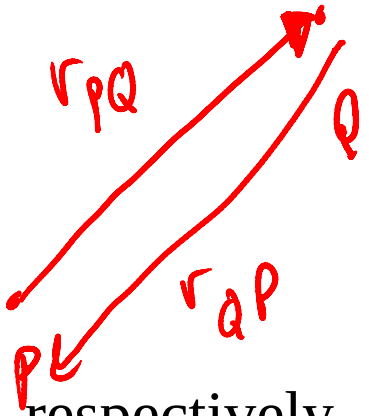
$\boldsymbol{u}_{DA} = \boldsymbol{r}_{DA}/r_{DA}$ and $\boldsymbol{F}_{DA} = 400\,\boldsymbol{u}_{DA}$ lb

$\boldsymbol{F}_{DA} = 400\{(-2\,\boldsymbol{i} - 6\,\boldsymbol{j} + 14\,\boldsymbol{k})/15.36\}$ lb

$= \{-52.1\,\boldsymbol{i} - 156\,\boldsymbol{j} + 365\,\boldsymbol{k}\}$ lb

CONCEPT QUIZ

1. **P** and **Q** are two points in a 3-D space. How are the position vectors $\boldsymbol{r}_{PQ}$ and $\boldsymbol{r}_{QP}$ related?



A) $\boldsymbol{r}_{PQ} = \boldsymbol{r}_{QP}$ B) $\boldsymbol{r}_{PQ} = -\boldsymbol{r}_{QP}$

C) $\boldsymbol{r}_{PQ} = 1/\boldsymbol{r}_{QP}$ D) $\boldsymbol{r}_{PQ} = 2\,\boldsymbol{r}_{QP}$

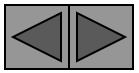
2. If $\boldsymbol{F}$ and $\boldsymbol{r}$ are force vector and position vectors, respectively, in SI units, what are the units of the expression (r * ($\boldsymbol{F}$ / F)) ?

dimensionless

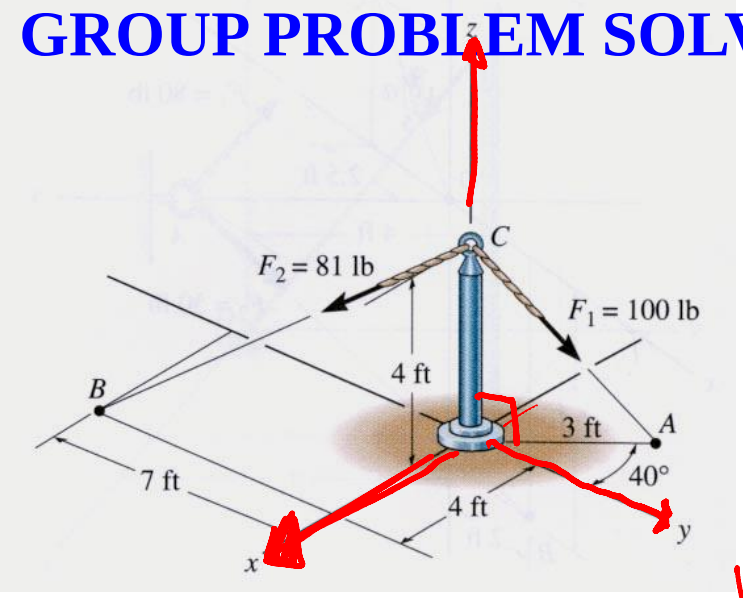
A) Newton B) Dimensionless

C) Meter D) Newton - Meter

E) The expression is algebraically illegal.



GROUP PROBLEM SOLVING



Given: Two forces are acting on a pipe as shown in the figure.

Find: The magnitude and the coordinate direction angles of the resultant force.

1) Find $\vec{F}_1$

A: $(-3\sin 40, 3\cos 40, 0)$

C: $(0, 0, 4)$

$\vec{r}_{CA} = A - C = -3\sin 40\,\hat{i} + 3\cos 40\,\hat{j} - 4\,\hat{k}$

$|r_{CA}| = 5$

$\vec{F}_1 = \frac{100}{5}\vec{r}_{CA} = -38.57\,\hat{i} + 45.96\,\hat{j} - 80\,\hat{k}$

2) Find $\vec{F}_2$

$\vec{F}_2 = 81 \cdot \frac{\vec{r}_{CB}}{r_{CB}}$

$= 81\left(4\,\hat{i} - 7\,\hat{j} - 4\,\hat{k}\right)/9$

$= 36\,\hat{i} - 63\,\hat{j} - 36\,\hat{k}$

3) $\vec{F}_R = \vec{F}_1 + \vec{F}_2 = -2.57\,\hat{i} - 17.04\,\hat{j} - 116\,\hat{k}$

$F_R = \sqrt{(\ )^2 + (\ )^2 + (\ )^2} = 117$

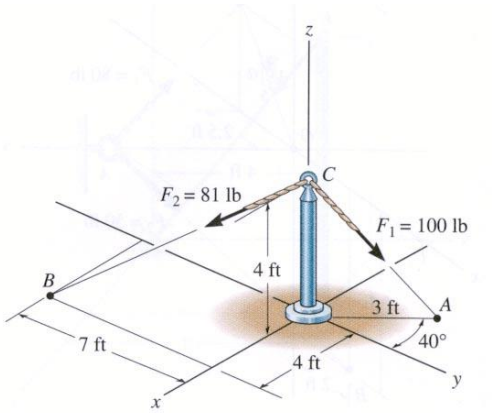
$\alpha = \cos^{-1}\left(\frac{-2.57}{117}\right) = 91.3°$

$\beta = \cos^{-1}\left(\frac{-17.04}{117}\right) = 98.4°$

$\gamma = \cos^{-1}\left(\frac{-116}{117}\right) = 172°$

GROUP PROBLEM SOLVING

(continued)



$$\boldsymbol{F}_{CA} = 100 \text{ lb}\{\boldsymbol{r}_{CA}/r_{CA}\}$$

$$\boldsymbol{F}_{CA} = 100 \text{ lb}(-3 \sin 40° \boldsymbol{i} + 3 \cos 40° \boldsymbol{j} - 4 \boldsymbol{k})/5$$

$$\boldsymbol{F}_{CA} = \{-38.57 \boldsymbol{i} + 45.96 \boldsymbol{j} - 80 \boldsymbol{k}\} \text{ lb}$$

$$\boldsymbol{F}_{CB} = 81 \text{ lb}\{\boldsymbol{r}_{CB}/r_{CB}\}$$

$$\boldsymbol{F}_{CB} = 81 \text{ lb}(4 \boldsymbol{i} - 7 \boldsymbol{j} - 4 \boldsymbol{k})/9$$

$$\boldsymbol{F}_{CB} = \{36 \boldsymbol{i} - 63 \boldsymbol{j} - 36 \boldsymbol{k}\} \text{ lb}$$

$$\boldsymbol{F}_R = \boldsymbol{F}_{CA} + \boldsymbol{F}_{CB} = \{-2.57 \boldsymbol{i} - 17.04 \boldsymbol{j} - 116 \boldsymbol{k}\} \text{ lb}$$

$$F_R = (2.57^2 + 17.04^2 + 116^2) = 117.3 \text{ lb} = 117 \text{ lb}$$

$$\alpha = \cos^{-1}(-2.57/117.3) = 91.3°, \quad \beta = \cos^{-1}(-17.04/117.3) = 98.4°$$

$$\gamma = \cos^{-1}(-116/117.3) = 172°$$

ATTENTION QUIZ

1. Two points in 3 – D space have coordinates of P (1, 2, 3) and Q (4, 5, 6) meters. The position vector $\boldsymbol{r}_{QP}$ is given by

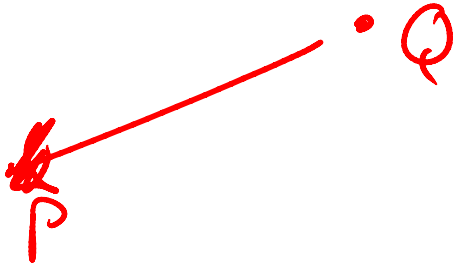
A) $\{3\,\boldsymbol{i} + 3\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

B) $\{-3\,\boldsymbol{i} - 3\,\boldsymbol{j} - 3\,\boldsymbol{k}\}$ m

C) $\{5\,\boldsymbol{i} + 7\,\boldsymbol{j} + 9\,\boldsymbol{k}\}$ m

D) $\{-3\,\boldsymbol{i} + 3\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

E) $\{4\,\boldsymbol{i} + 5\,\boldsymbol{j} + 6\,\boldsymbol{k}\}$ m



2. Force vector, $\boldsymbol{F}$, directed along a line PQ is given by

A) $(\boldsymbol{F}/F)\,\boldsymbol{r}_{PQ}$

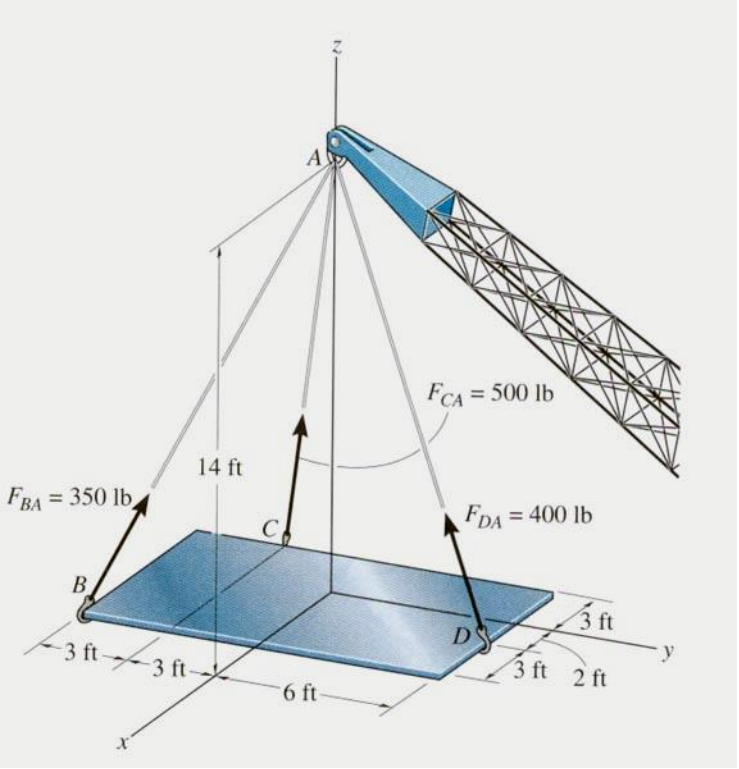
B) $\boldsymbol{r}_{PQ}/r_{PQ}$

C) $F(\boldsymbol{r}_{PQ}/r_{PQ})$

D) $F(r_{PQ}/\boldsymbol{r}_{PQ})$

End of the Lecture

Let Learning Continue



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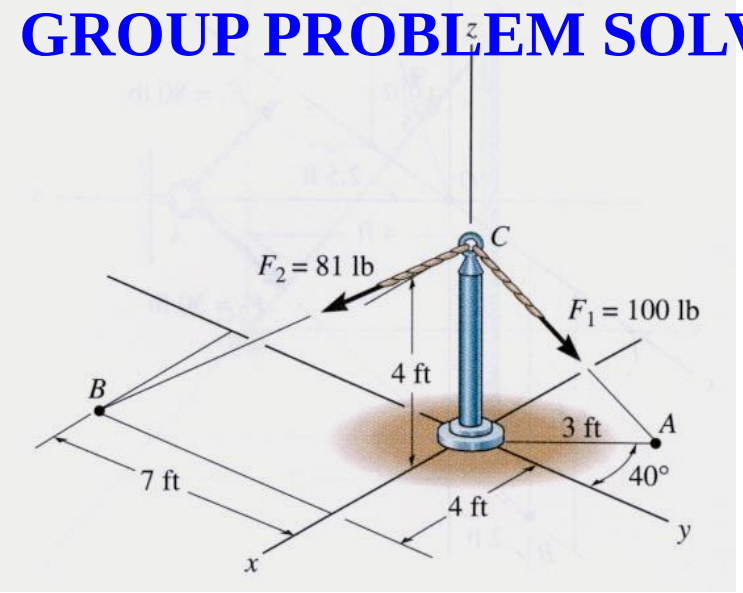

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