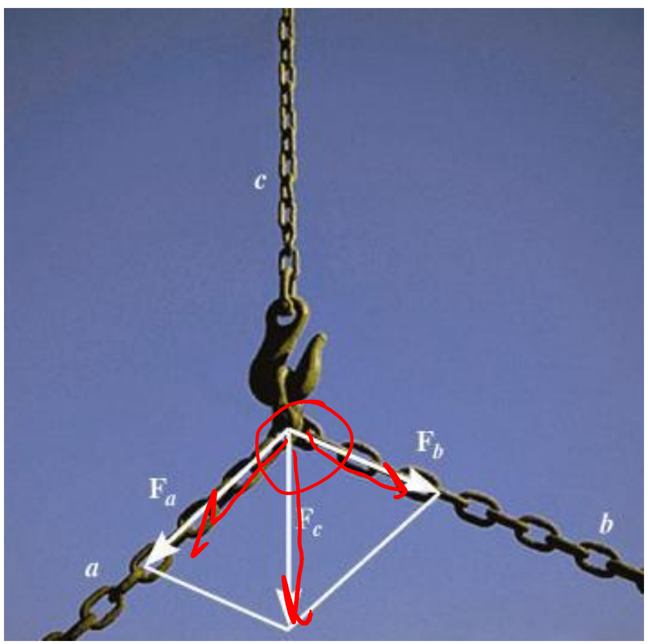
FORCE VECTORS, VECTOR OPERATIONS & ADDITION COPLANAR FORCES

<u>Today's Objective</u>:

Students will be able to :

a) Resolve a 2-D vector into components.

b) Add 2-D vectors using Trig Laws and/or Cartesian vector notations.



<u>In-Class activities</u>:

- Check Homework
- Reading Quiz
- Application of Adding Forces
- Parallelogram Law
- Resolution of a Vector Using Cartesian Vector Notation (CVN)
- Addition Using CVN
- Attention Quiz

READING QUIZ

1. Which one of the following is a scalar quantity?

A) Force B) Position C) Mass D) Velocity

2. For vector addition you have to use ______ law.

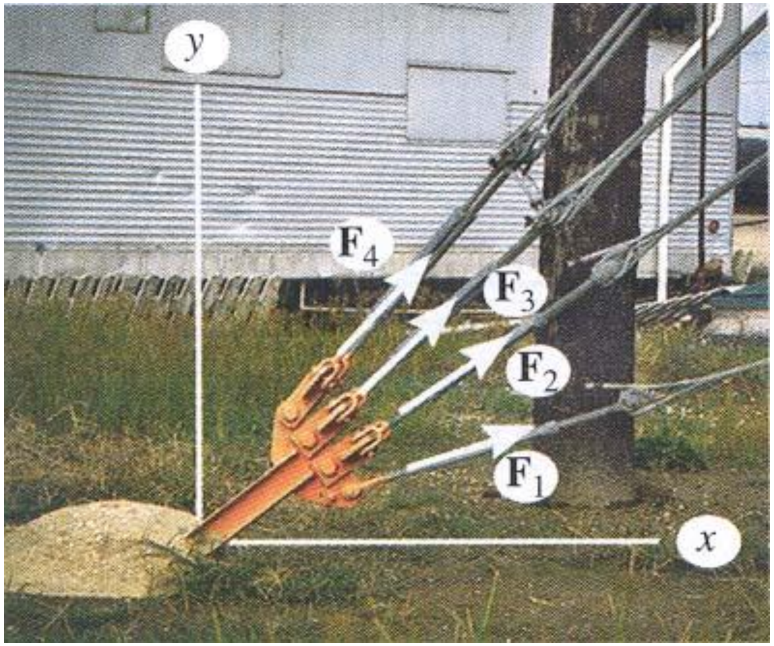
A) Newton's Second

B) the arithmetic

C) Pascal's

D) the parallelogram

APPLICATION OF VECTOR ADDITION



There are four concurrent cable forces acting on the bracket.

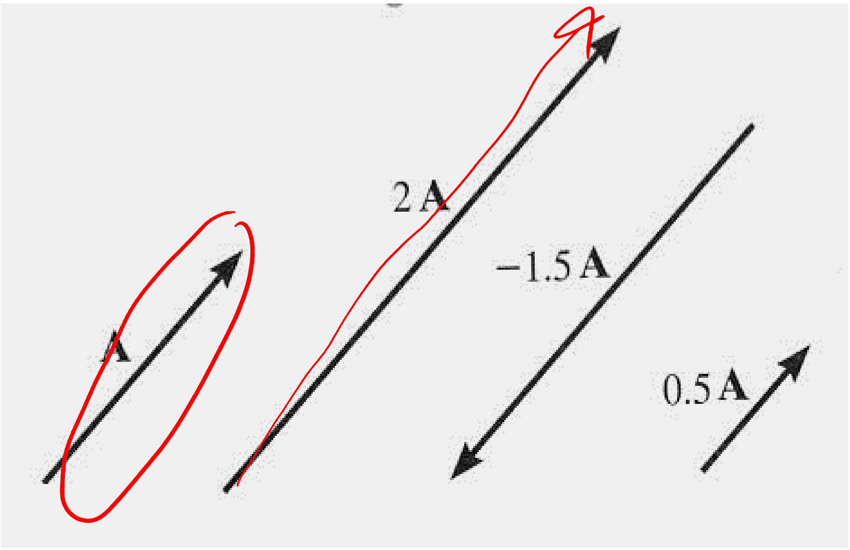
How do you determine the resultant force acting on the bracket ?

SCALARS AND VECTORS (Section 2.1)

	Scalars	***Vectors***
Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a "carrot"

In the PowerPoint presentation vector quantity is represented ***Like this*** (in **bold**, *italics*, and yellow).

VECTOR OPERATIONS
(Section 2.2)



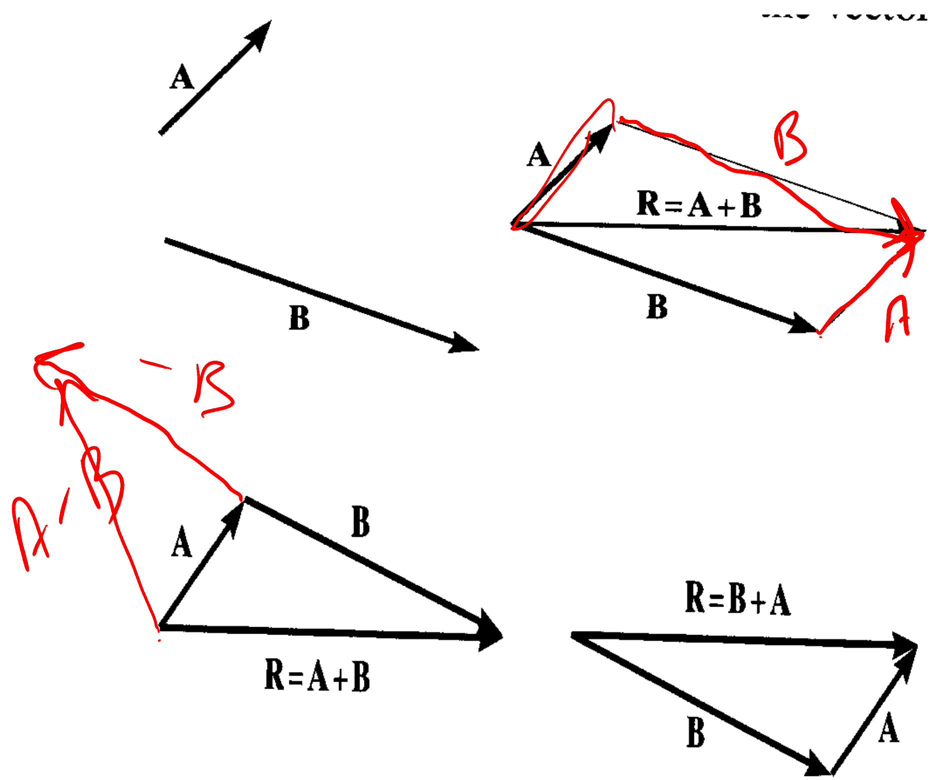
Scalar Multiplication

and Division

VECTOR ADDITION USING EITHER THE PARALLELOGRAM LAW OR TRIANGLE

Parallelogram Law:

Triangle method
(always 'tip to tail'):



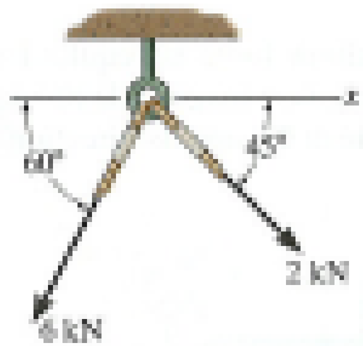
How do you subtract a vector?

How can you add more than two concurrent vectors graphically ?

Example

Example

F2-1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



F2-1

Given: 2 forces acting on screw

Find: Resultant force mag and direction

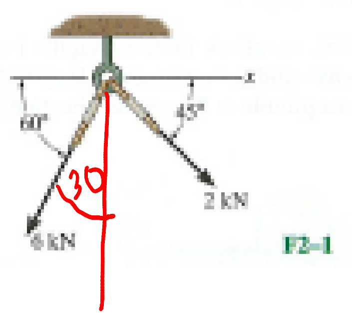
Plan:

1) Draw forces (not ropes)
2) Use Parallelogram Law for the resultant
3) Use Trig to solve for the angle

Example (continued)

Force vectors

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



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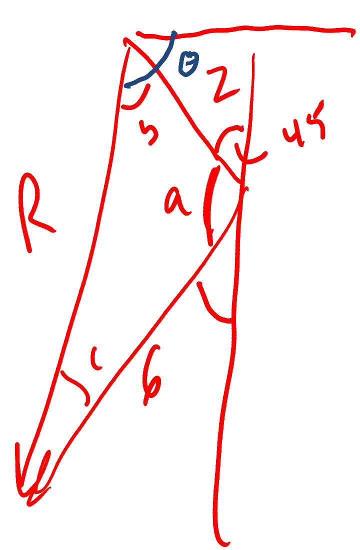
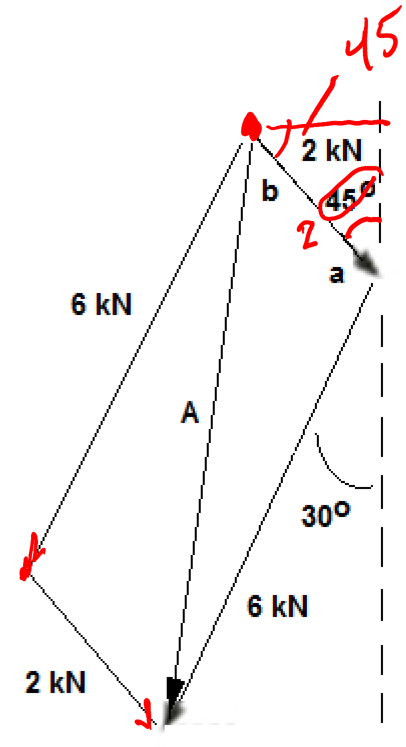




Law of cosines:

$A^2 = 2^2 + 6^2 - 2*2*6*\cos(a)$

Law of supplementary angles:

$45 + a + 30 = 180, \quad a = 105$

So: $A^2 = 4 + 36 - 24\cos(105) = 46.21$

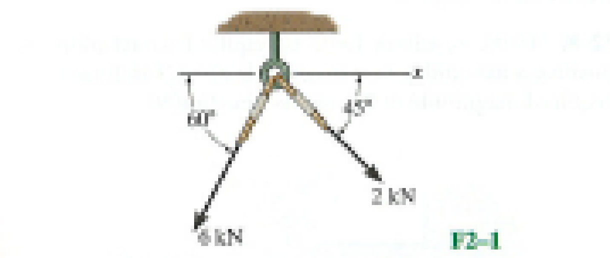
$A = \mathrm{sqrt}(46.21) = \underline{6.798\ \mathrm{kN}}$ (mag of Resultant)

6.80

Example (continued)

Force vectors

F2-1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



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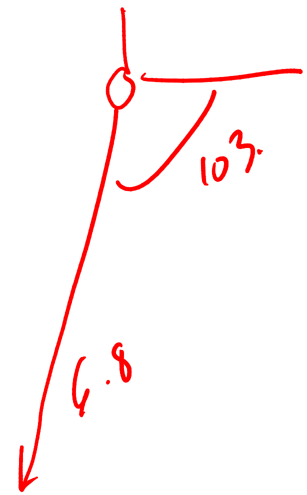
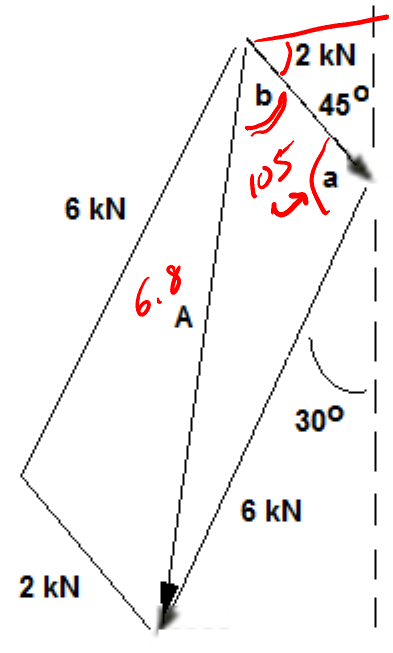




Law of sines:

$\sin(105)/A = \sin(b)/6$

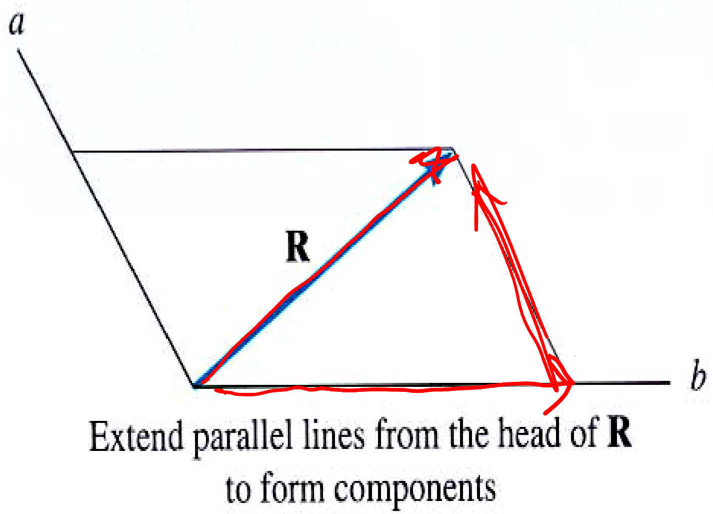
$b = \sin^{-1}(6\sin(105)/A) = \underline{58.49}\ °$

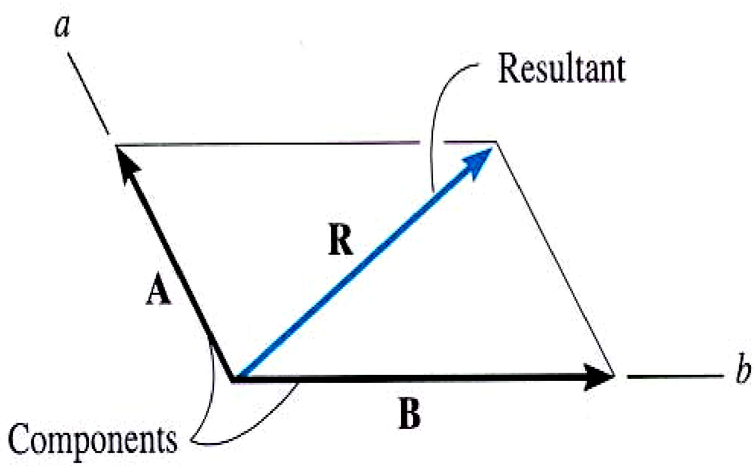
But the angle relative to x-axis is $b + 45 = 103.49$

angle = 103 ° (angle of resultant)

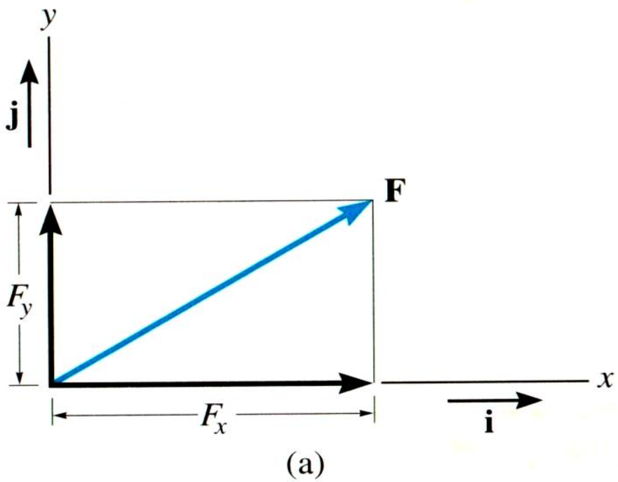
RESOLUTION OF A VECTOR

"Resolution" of a vector is breaking up a vector into components. It is kind of like using the parallelogram law in reverse.





CARTESIAN VECTOR NOTATION (Section 2.4)

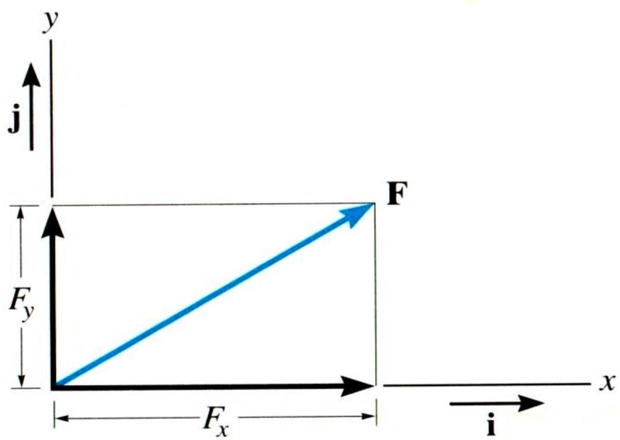


- We 'resolve' vectors into components using the x and y axes system.
- Each component of the vector is shown as a magnitude and a direction.

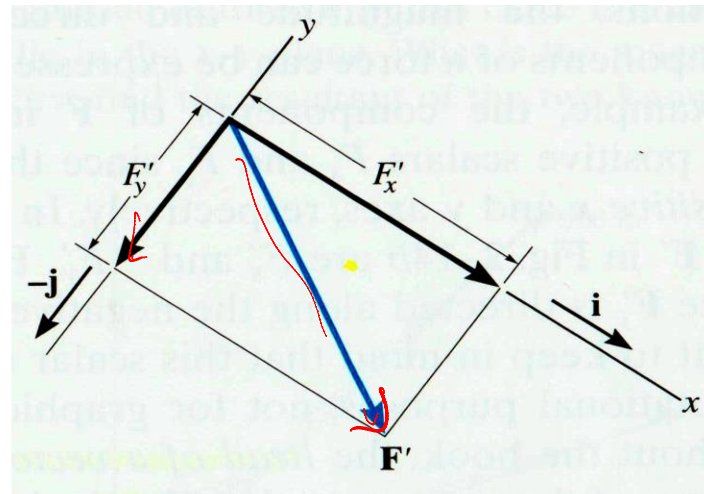
- The directions are based on the x and y axes. We use the "unit vectors" ***i*** and ***j*** to designate the x and y axes.

For example,

$\boldsymbol{F} = F_x\, \boldsymbol{i} + F_y\, \boldsymbol{j}$ ***or*** $\boldsymbol{F'} = F'_x\, \boldsymbol{i} + F'_y\, \boldsymbol{j}$



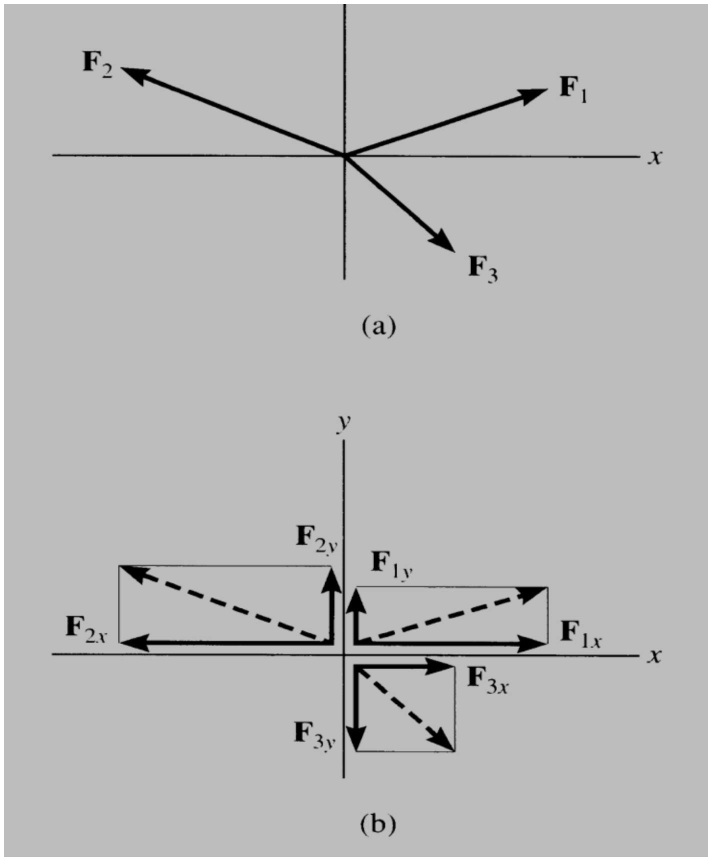
(a)



(b)

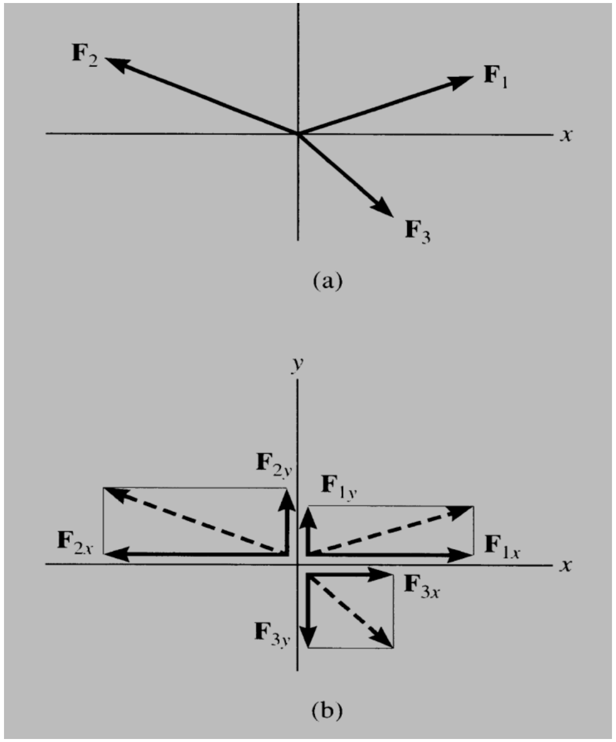
The x and y axes are always perpendicular to each other. Together,they can be directed at any inclination.

ADDITION OF SEVERAL VECTORS
Cartesian Method is easiest



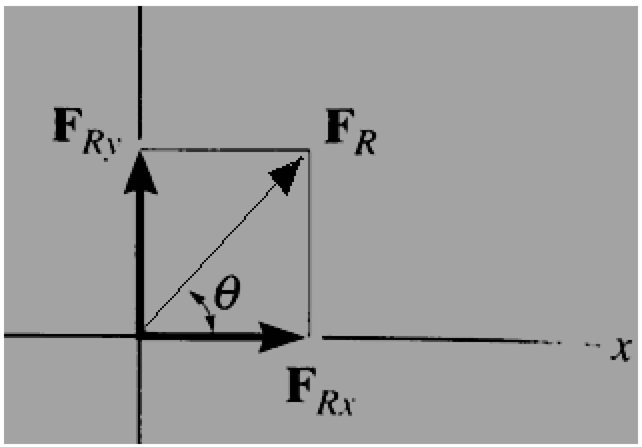
- Step 1 is to resolve each force into its Cartesian components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.

Example of this process,



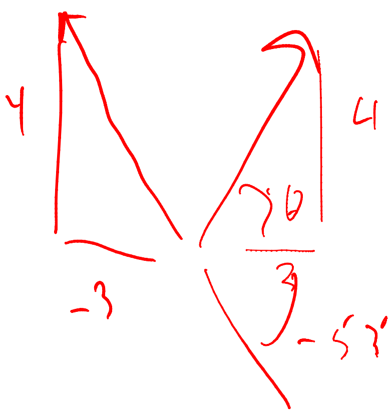
$$
\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\
&= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\
&= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\
&= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}
\end{aligned}
$$

You can also represent a 2-D vector with a magnitude and angle.



$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

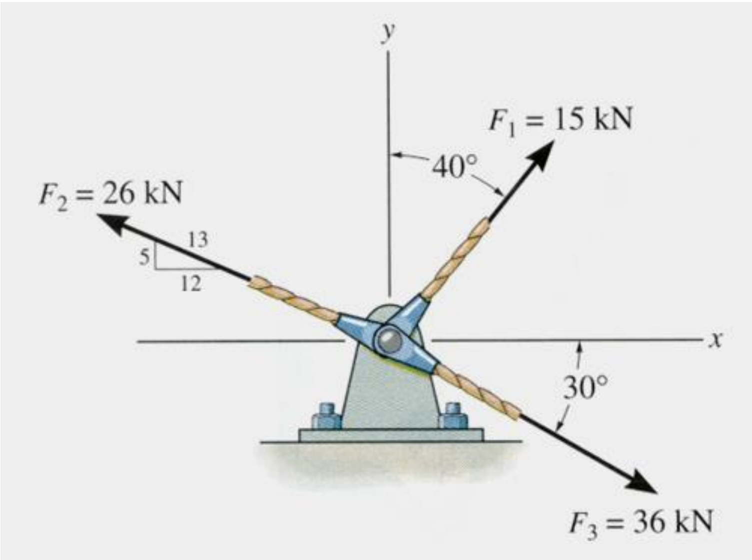
$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$



$\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$\tan^{-1}\left(\frac{4}{-3}\right)$

EXAMPLE



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force.

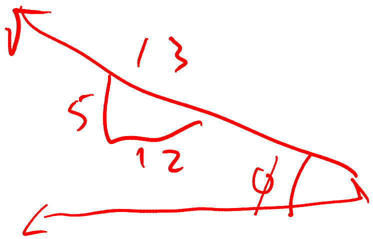
<u>Plan:</u>

a) Resolve the forces in their x-y components.

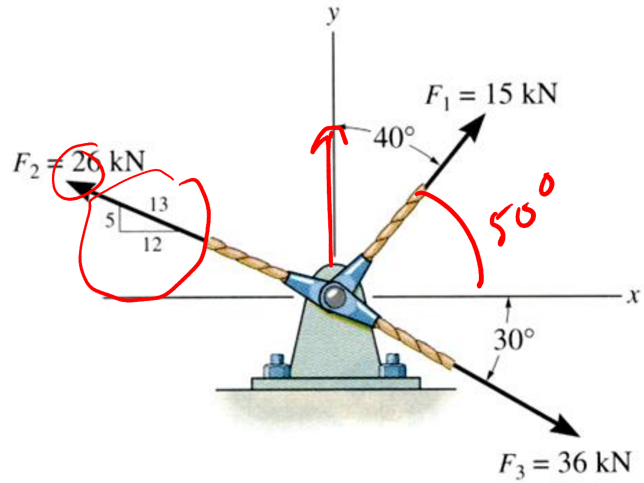
b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

EXAMPLE (continued)



$$\frac{26 \cdot 12}{13}$$



$F_1 = \{\ 15 \sin 40°\ \mathbf{i} + 15 \cos 40°\ \mathbf{j}\ \}$ kN

$= \{\ 9.642\ \mathbf{i} + 11.49\ \mathbf{j}\ \}$ kN

$F_2 = \{\ -(12/13)26\ \mathbf{i} + (5/13)26\ \mathbf{j}\ \}$ kN

$= \{\ -24\ \mathbf{i} + 10\ \mathbf{j}\ \}$ kN

$F_3 = \{\ 36 \cos 30°\ \mathbf{i}\ - 36 \sin 30°\ \mathbf{j}\ \}$ kN

$= \{\ 31.18\ \mathbf{i} - 18\ \mathbf{j}\ \}$ kN

$F_3 = 36\cos(-30)\,\hat{i} + 36\sin(-30)\,\hat{j}$

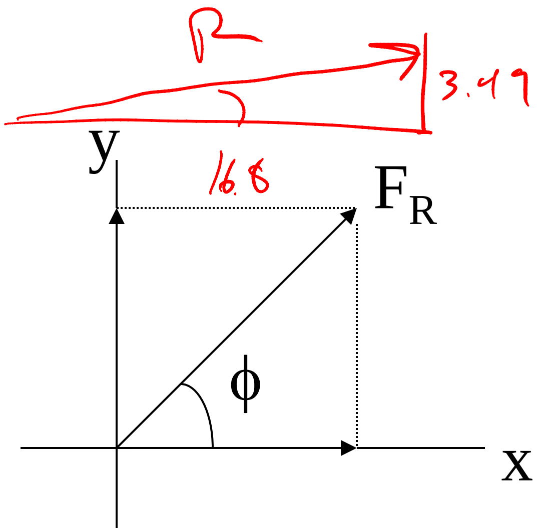
$31.17\ \hat{i} - 18\,j$

EXAMPLE
(continued)

Summing up all the *i* and *j* components respectively, we get,

$\boldsymbol{F_R} = \{\, (9.642 - 24 + 31.18)\, \boldsymbol{i} + (11.49 + 10 - 18)\, \boldsymbol{j} \,\}$ kN

$= \{\, 16.82\, \boldsymbol{i} + 3.49\, \boldsymbol{j} \,\}$ kN



$F_R = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2$ kN

$\phi = \tan^{-1}(3.49/16.82) = 11.7°$

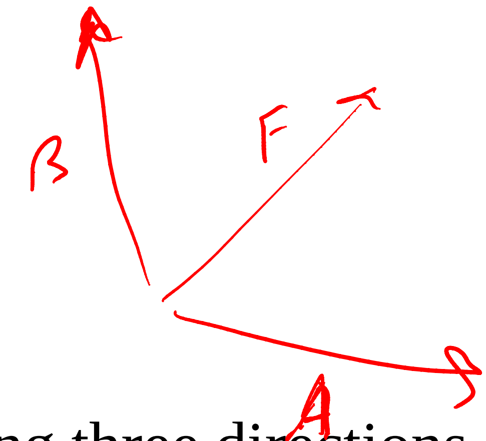
CONCEPT QUIZ

1. Can you resolve a 2-D vector along two directions, which are not at 90° to each other?

   A) Yes, but not uniquely.

   B) No.

   C) Yes, uniquely.



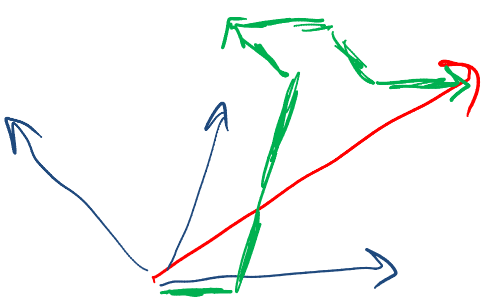
2. Can you resolve a 2-D vector along three directions (say at 0, 60, and 120°)?

   A) Yes, but not uniquely.

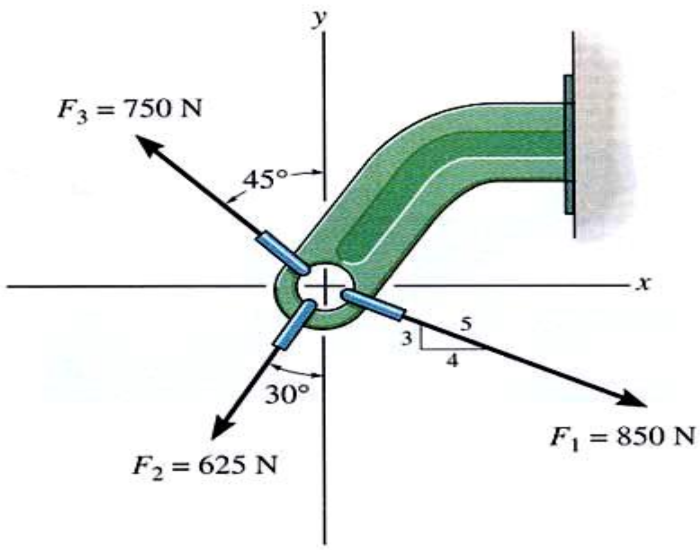
   B) No.

   C) Yes, uniquely.

$$\vec{F} = F_a \vec{A} + F_b \vec{B}$$



GROUP PROBLEM SOLVING



Given: Three concurrent forces acting on a bracket

Find: The magnitude and angle of the resultant force.

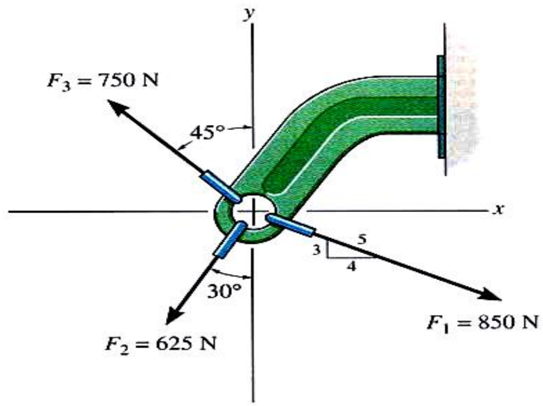
Plan:

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

GROUP PROBLEM SOLVING (continued)



$\mathbf{F_1}$ = { (4/5) 850 $\mathbf{i}$ - (3/5) 850 $\mathbf{j}$ } N

= { 680 $\mathbf{i}$ - 510 $\mathbf{j}$ } N

$\mathbf{F_2}$ = { -625 sin(30°) $\mathbf{i}$ - 625 cos(30°) $\mathbf{j}$ } N

= { -312.5 $\mathbf{i}$ - 541.3 $\mathbf{j}$ } N

$\mathbf{F_3}$ = { -750 sin(45°) $\mathbf{i}$ + 750 cos(45°) $\mathbf{j}$ } N

{ -530.3 $\mathbf{i}$ + 530.3 $\mathbf{j}$ } N

GROUP PROBLEM SOLVING (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

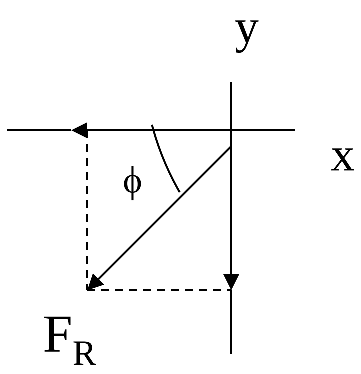
$\boldsymbol{F_R}$ = { (680 – 312.5 – 530.3) ***i*** + (-510 – 541.3 + 530.3) ***j*** }N

= { - 162.8 ***i*** - 521 ***j*** } N

$F_R = ((162.8)^2 + (521)^2)^{½} = 546$ N

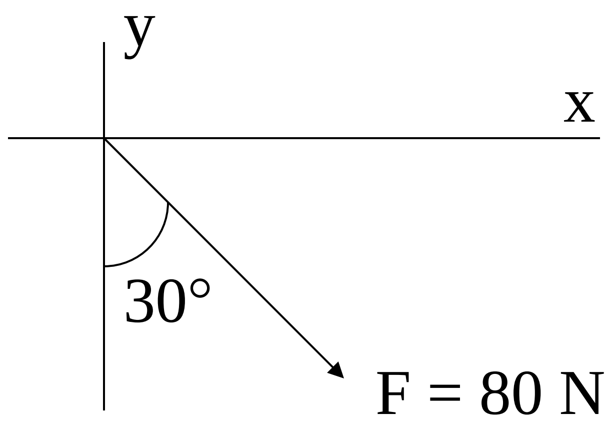
$\phi = \tan^{-1}(521/162.8) = 72.64°$ or

From Positive x axis $\theta = 180 + 72.64 = 253°$



ATTENTION QUIZ

1. Resolve $\boldsymbol{F}$ along x and y axes and write it in vector form. $\boldsymbol{F}$ = { ____________ } N



A) 80 cos (30°) $\boldsymbol{i}$ - 80 sin (30°) $\boldsymbol{j}$

B) 80 sin (30°) $\boldsymbol{i}$ + 80 cos (30°) $\boldsymbol{j}$

C) 80 sin (30°) $\boldsymbol{i}$ - 80 cos (30°) $\boldsymbol{j}$

D) 80 cos (30°) $\boldsymbol{i}$ + 80 sin (30°) $\boldsymbol{j}$

2. Determine the magnitude of the resultant ($\boldsymbol{F_1}$ + $\boldsymbol{F_2}$) force in N when $\boldsymbol{F_1}$ = { 10 $\boldsymbol{i}$ + 20 $\boldsymbol{j}$ } N and $\boldsymbol{F_2}$ = { 20 $\boldsymbol{i}$ + 20 $\boldsymbol{j}$ } N .

A) 30 N

B) 40 N

C) 50 N

D) 60 N

E) 70 N

End of the Lecture

Let Learning Continue