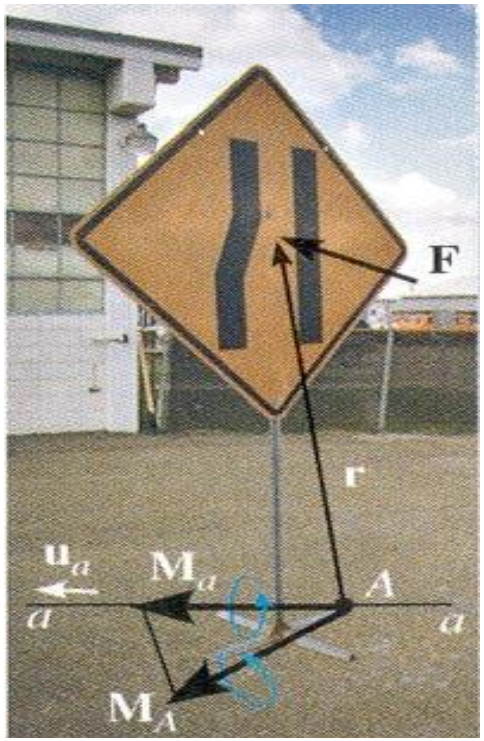
MOMENT ABOUT AN AXIS (Section 4.5)

Today's Objectives:

Students will be able to determine the moment of a force about an axis using

a) scalar analysis, and

b) vector analysis.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Scalar Analysis
- Vector Analysis
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. When determining the moment of a force about a specified axis, the axis must be along _____.

A) the x axis B) the y axis C) the z axis

D) any line in 3-D space E) any line in the x-y plane

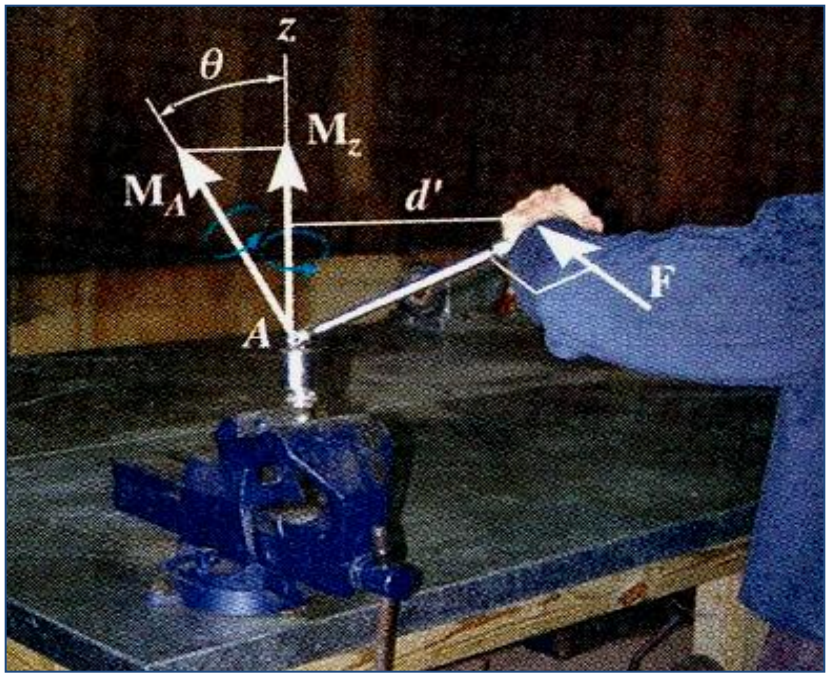
2. The triple scalar product $\boldsymbol{u \cdot (r \times F)}$ results in

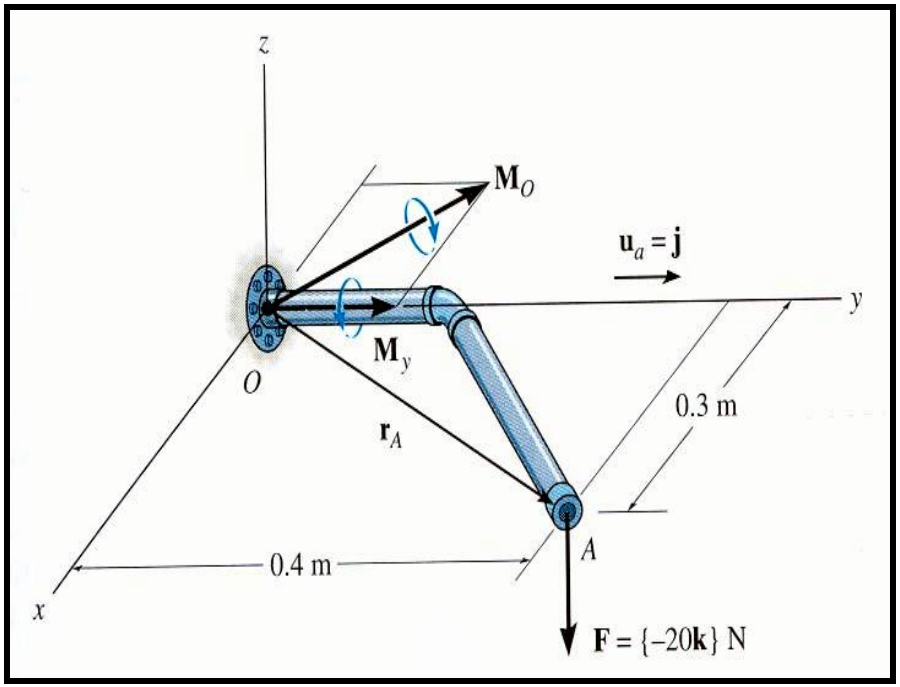
A) a scalar quantity (+ or -). B) a vector quantity.

C) zero. D) a unit vector.

E) an imaginary number.

APPLICATIONS





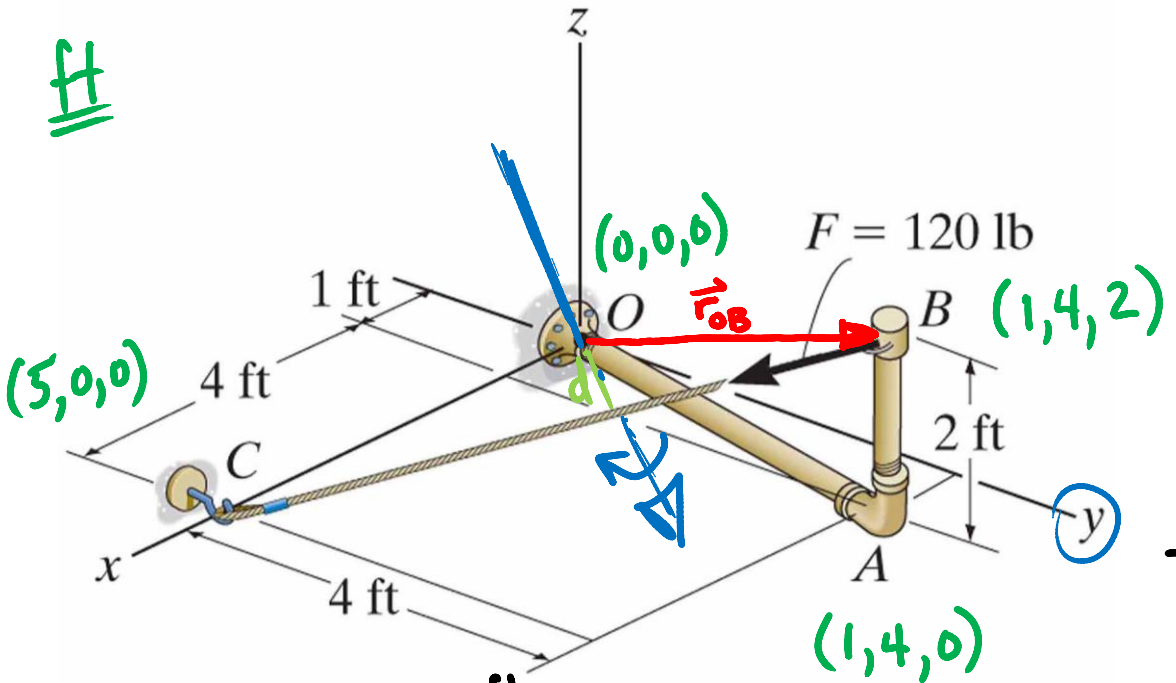
With the force ***F***, a person is creating the moment M_A. What portion of M_A is used in turning the socket?

The force ***F*** is creating the moment M_O. How much of M_O acts to unscrew the pipe?

From Last Lecture →

Example 2a: Find moment of F about point O

$\vec{M}_O$



ft

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$

$$\vec{r}_{OB} = \left(\hat{i} + 4\hat{j} + 2\hat{k}\right) \text{ ft}$$

$$\vec{F} = F\,\vec{u}_{BC}$$

$$\vec{u}_{BC} = \frac{\vec{r}_{BC}}{r_{BC}} = \frac{(C-B)}{r_{BC}}$$

$$= \frac{(5-1)\hat{i} + (0-4)\hat{j} + (0-2)\hat{k}}{\sqrt{4^2+4^2+2^2}}$$

$$\vec{u}_{BC} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

$$\vec{r}_{OB} \times \vec{F} = \left(\hat{i} + 4\hat{j} + 2\hat{k}\right)^{ft} \times (120\ lb)\left(\tfrac{2}{3}\hat{i} - \tfrac{2}{3}\hat{j} - \tfrac{1}{3}\hat{k}\right)$$

$$= \begin{matrix} \\ ft \\ lb \end{matrix} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 80 & -80 & -40 \end{vmatrix}$$

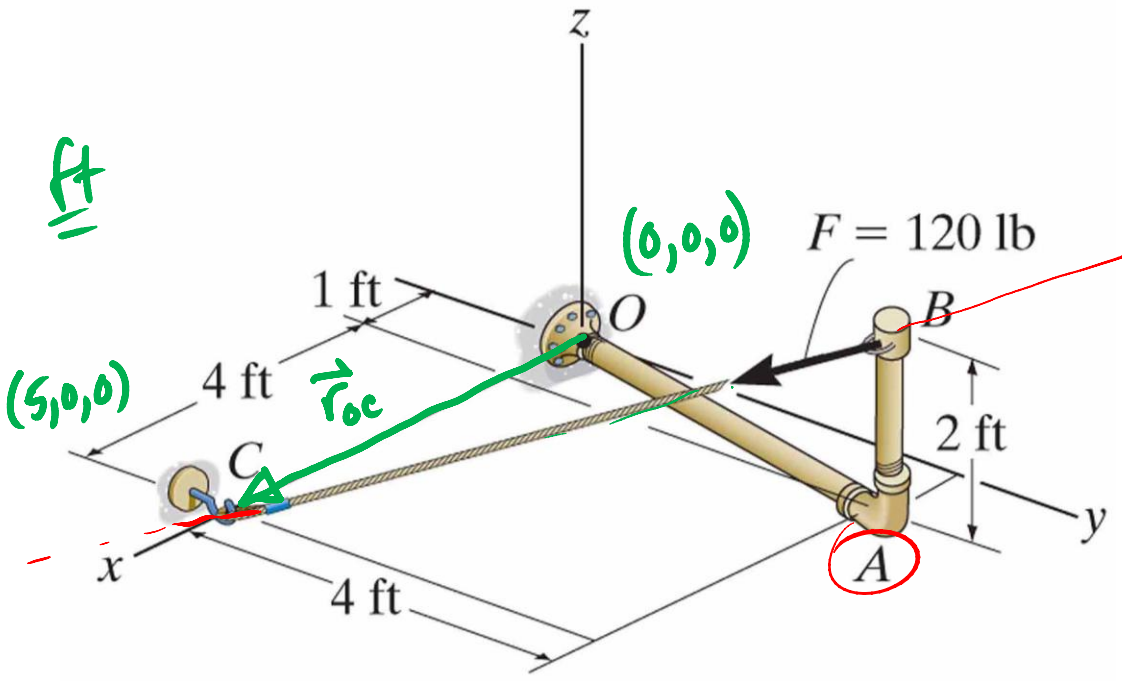
$$= (-160+160)\hat{i} + (160+40)\hat{j} + (-80-320)\hat{k}$$

$$\vec{M}_O = (200\hat{j} - 400\hat{k})\ lb \cdot ft$$

$M_O = 447$ lb-ft

We also discovered we could choose a different r vector

Example 2a': Find moment of F about point O



ft

$$\vec{M}_O = \vec{r}_{OC} \times \vec{F}$$

$$\vec{F} = (80\hat{i} - 80\hat{j} - 40\hat{k})\ \text{lb}$$

$$\vec{r}_{OC} = 5\hat{i}\ \text{ft}$$

$$= 5\hat{i} \times (80\hat{i} - 80\hat{j} - 40\hat{k})\ \text{ft lb}$$

$$= (5)(-80)(\hat{i}\times\hat{j}) + (5)(-40)(\hat{i}\times\hat{k})$$

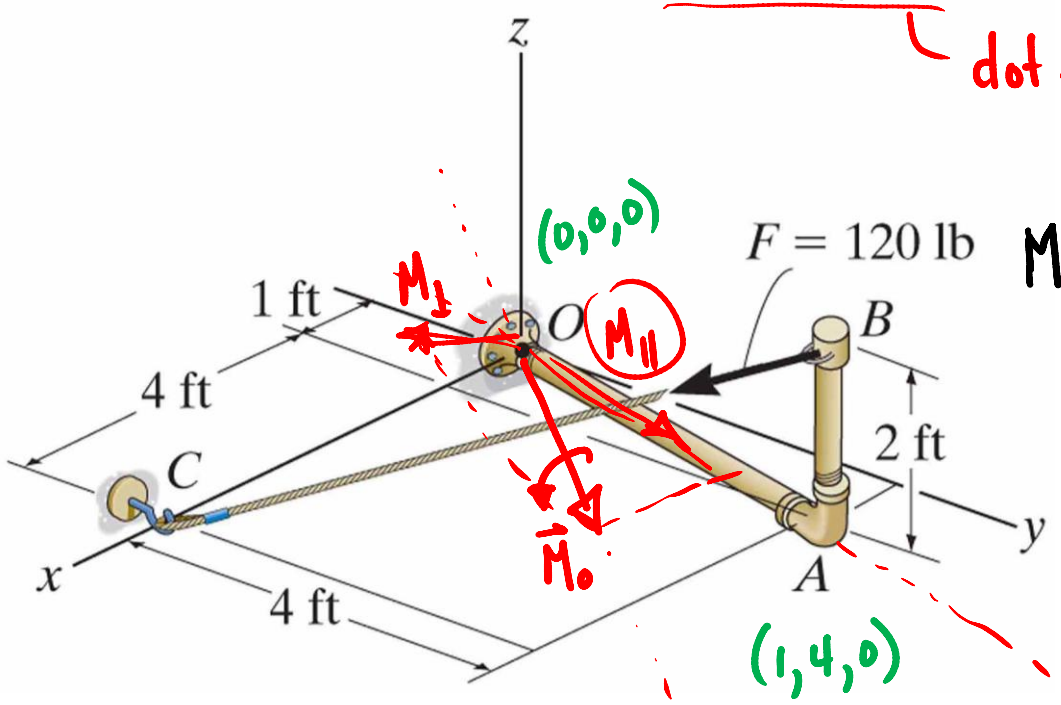
$\hat{i}\times\hat{j} = \hat{k}$, $\hat{i}\times\hat{k} = -\hat{j}$

$$\vec{M}_O = (200\hat{j} - 400\hat{k})\ \text{lb ft}$$

Today →

Example 2c: Find moment of F about OA axis (Projection of M_O along line of OA)

dot product w/ $\vec{u}_{OA}$



$$M_{OA} = \vec{u}_{OA} \cdot \vec{M}_O$$

$$\vec{u}_{OA} = \frac{\vec{r}_{OA}}{r_{OA}} = \frac{\vec{A} - \vec{O}}{r_{OA}}$$

$$= \frac{\hat{i} + 4\hat{j}}{\sqrt{1^2 + 4^2}}$$

$$\vec{u}_{OA} = 0.243\hat{i} + 0.970\hat{j}$$

$$M_{OA} = (0.243\hat{i} + 0.970\hat{j}) \cdot (200\hat{j} - 400\hat{k})\ \text{lb ft}$$

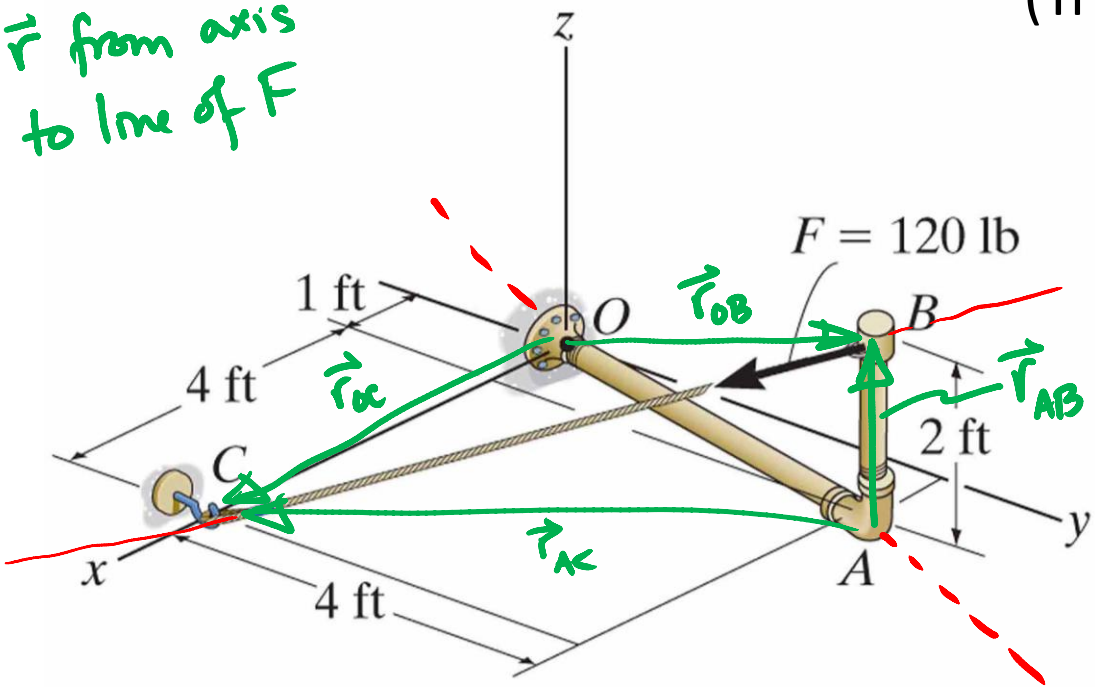
$$= (0.243)(0) + 0.970(200) + (0)(-400)$$

$$M_{OA} = 194\ \text{lb ft}$$

$$\vec{M}_{OA} = M_{OA}\vec{u}_{OA} = (47.1\hat{i} + 188\hat{j})\ \text{lb ft}$$

Example 2c': Find moment of F about OA axis (Triple-Product approach)

$\vec{r}$ from axis to line of F



$$M_{OA} = \vec{u}_{OA} \cdot \vec{M}_O$$

$$M_{OA} = \vec{u}_{OA} \cdot (\vec{r}_{OC} \times \vec{F})$$

$$= \det \begin{vmatrix} u_{OAx} & u_{OAy} & u_{OAz} \\ r_{OCx} & r_{OCy} & r_{OCz} \\ F_x & F_y & F_z \end{vmatrix}$$

$$= \begin{vmatrix} 0.243 & 0.970 & 0 \\ 5 & 0 & 0 \\ 80 & -80 & -40 \end{vmatrix} \begin{matrix} 0.243 & 0.970 \\ 5 & 0 \\ 80 & -80 \end{matrix} \quad \text{ft} \quad \text{lb}$$

$$= 0 + 0 + 0 - 0 - 0 - (0.970)(5)(-40)$$

$$M_{OA} = 194 \text{ lb ft}$$

Summary: Vector Formulation of Moment about an Axis

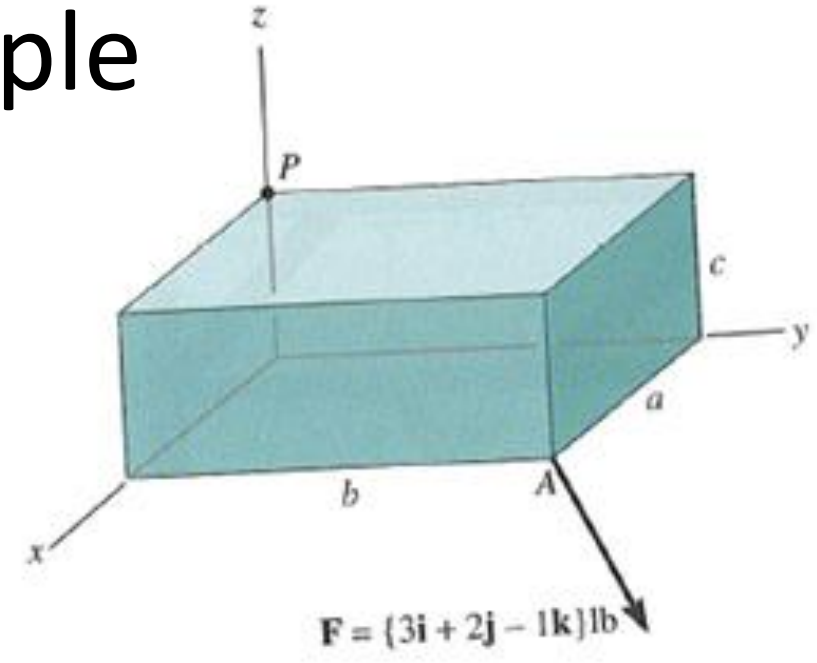
- Given or determine F in Cartesian Coordinates
- Find u_{axis}
- Find an r
  - Must go from desired axis to line of action of F
  - There can be several possible r vectors
- Find Magnitude M about axis = $u_{axis} \bullet (r \times F)$
  - Using Triple Product
  - Or, if geometry is simple, use vector dot product
  - Also consider Scalar Analysis (next)

Scalar Analysis
(Moment about an Axis) is often easier…

- When the axis of interest is either
  - x, y, or z axes
- In this case,
  - decompose the force into it's i,j,k parts
  - find the shortest distance of each part to axis
  - Compute moment of each force part and sum

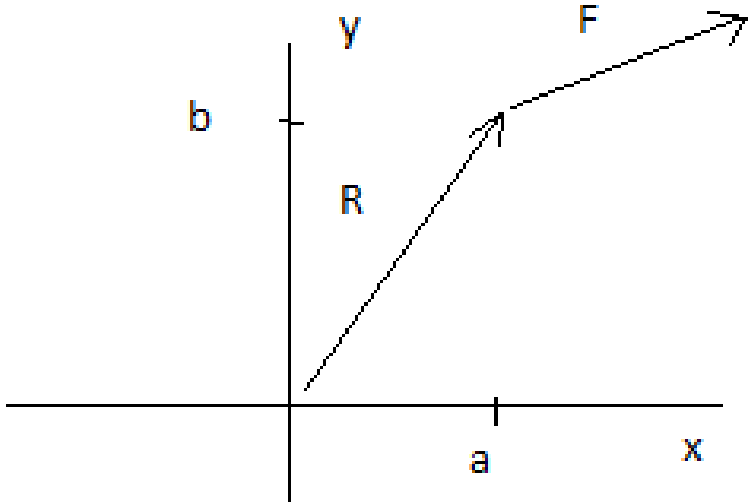
Scalar Example

- IF a = 3 and b = 4 feet,
  Find the moment of force F
  About the z axis



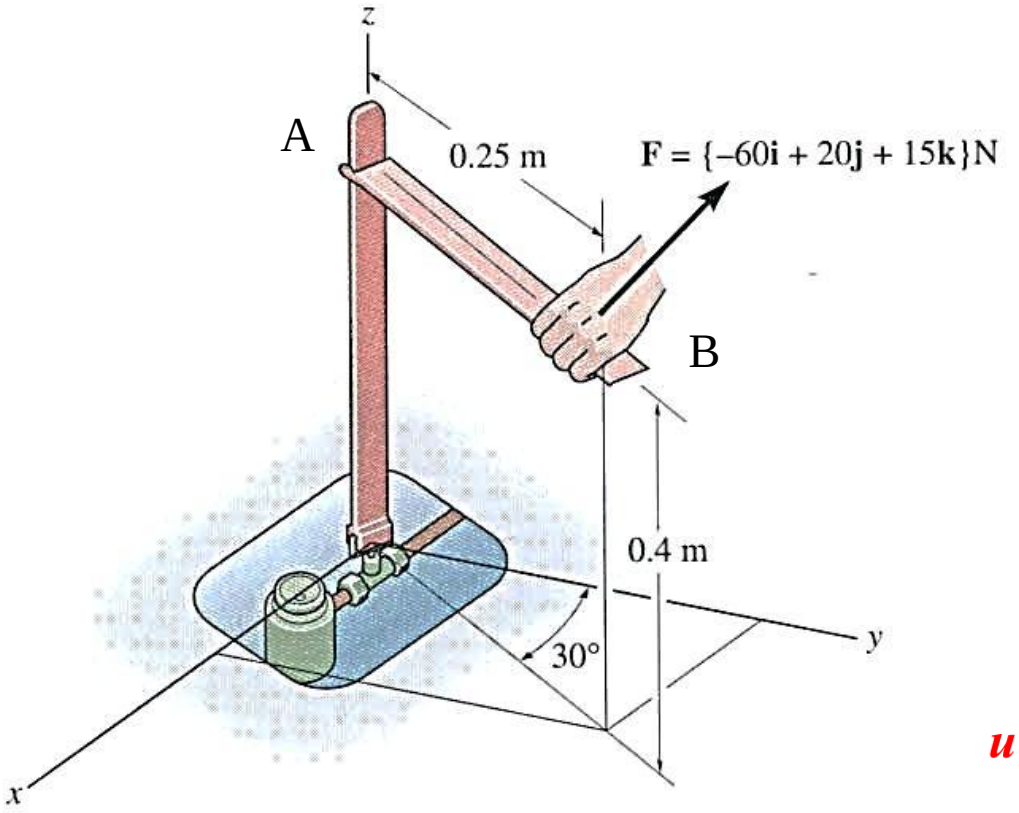
$$Mz = - b\,Fx + a\,Fy = -4*3 + 3*2$$
$$= -6 \text{ lb-ft}$$

"Top View" (z-axis leaving page)



Note that Z component of F
Cannot create a moment around
Z-axis

EXAMPLE 3



Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

$\boldsymbol{u} = 1\,\boldsymbol{k}$

$\boldsymbol{r}_{AB} = \{0.25 \sin 30°\,\boldsymbol{i} + 0.25 \cos 30°\,\boldsymbol{j}\}$ m

$= \{0.125\,\boldsymbol{i} + 0.2165\,\boldsymbol{j}\}$ m

$\boldsymbol{F} = \{-60\,\boldsymbol{i} + 20\,\boldsymbol{j} + 15\,\boldsymbol{k}\}$ N

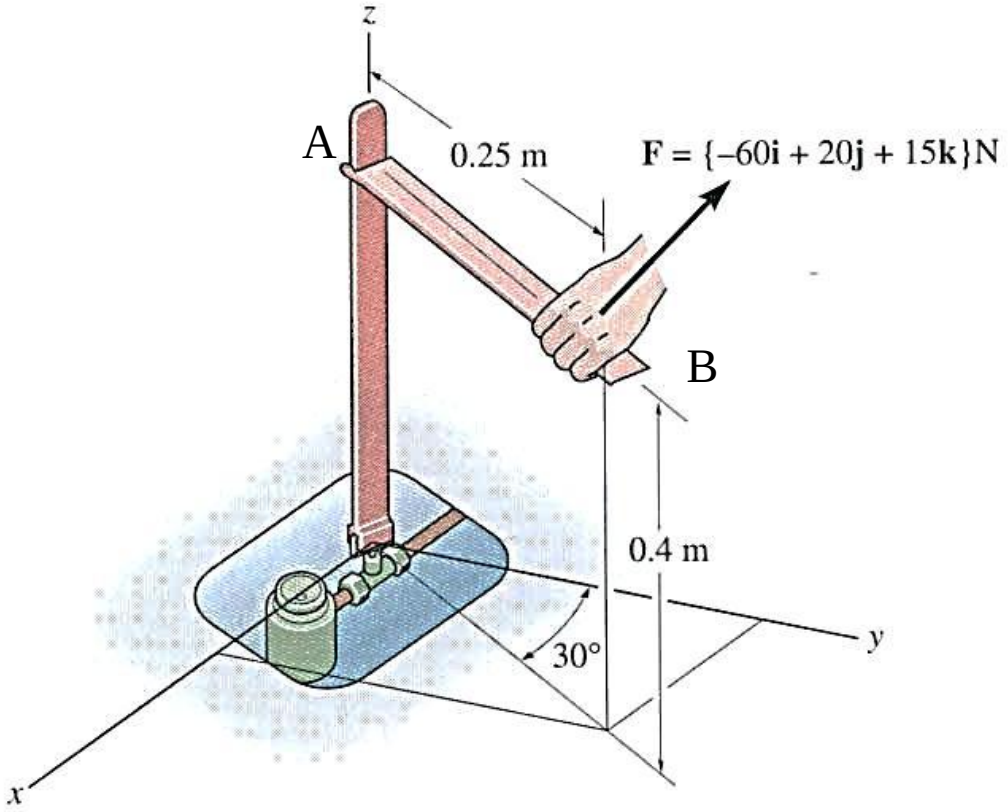
$M_z = \boldsymbol{u} \bullet (\boldsymbol{r}_{AB} \times \boldsymbol{F})$

$$M_z = \begin{vmatrix} 0 & 0 & 1 \\ 0.125 & 0.2165 & 0 \\ -60 & 20 & 15 \end{vmatrix}$$

$= 1\{0.125(20) - 0.2165(-60)\}$ N·m

$= 15.5$ N·m

EXAMPLE 3'

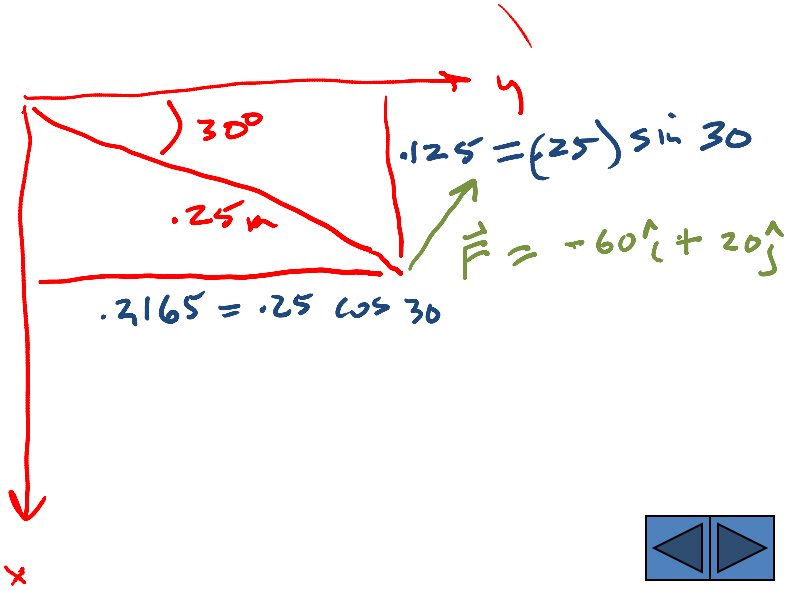


Given: A force is applied to the tool to open a gas valve.

Find: The magnitude of the moment of this force about the z axis of the value.

Use Scalar Analysis

$$M_{Oz} = F_x r_y + F_y r_x$$
$$= (20)(.125) + 60(.2165)$$
$$= 15.5 \text{ N·m}$$



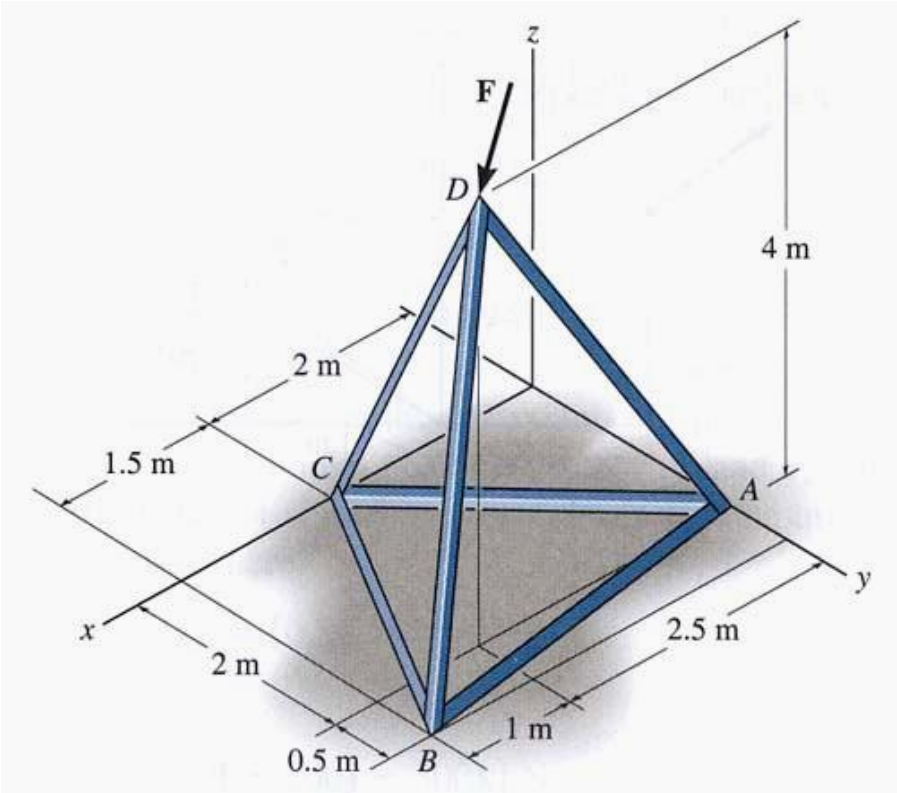
CONCEPT QUIZ

2. The force ***F*** is acting along DC. Using the triple product to determine the moment of ***F*** about the bar BA, you could use any of the following position vectors except _____.

A) $\boldsymbol{r_{BC}}$ B) $\boldsymbol{r_{AD}}$

C) $\boldsymbol{r_{AC}}$ D) $\boldsymbol{r_{DB}}$

E) $\boldsymbol{r_{BD}}$

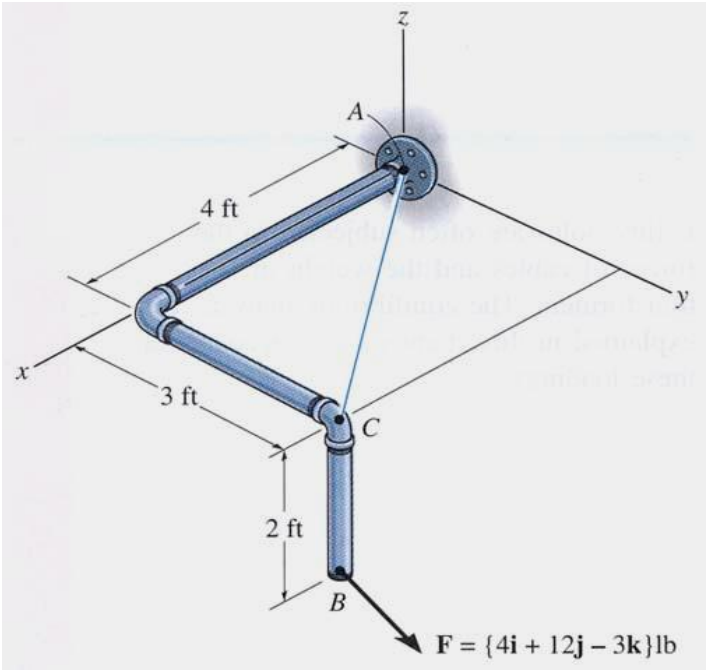


ATTENTION QUIZ

1. For finding the moment of the force ***F*** about the x-axis, the position vector in the triple scalar product should be ___ .

A) $\boldsymbol{r}_{AC}$ B) $\boldsymbol{r}_{BA}$

C) $\boldsymbol{r}_{AB}$ D) $\boldsymbol{r}_{BC}$



2. If $\boldsymbol{r} = \{1\,\boldsymbol{i} + 2\,\boldsymbol{j}\}$ m and $\boldsymbol{F} = \{10\,\boldsymbol{i} + 20\,\boldsymbol{j} + 30\,\boldsymbol{k}\}$ N, then the moment of ***F*** about the y-axis is ____ N·m.

A) 10 B) -30

C) -40 D) None of the above.

End of the Lecture

Let Learning Continue