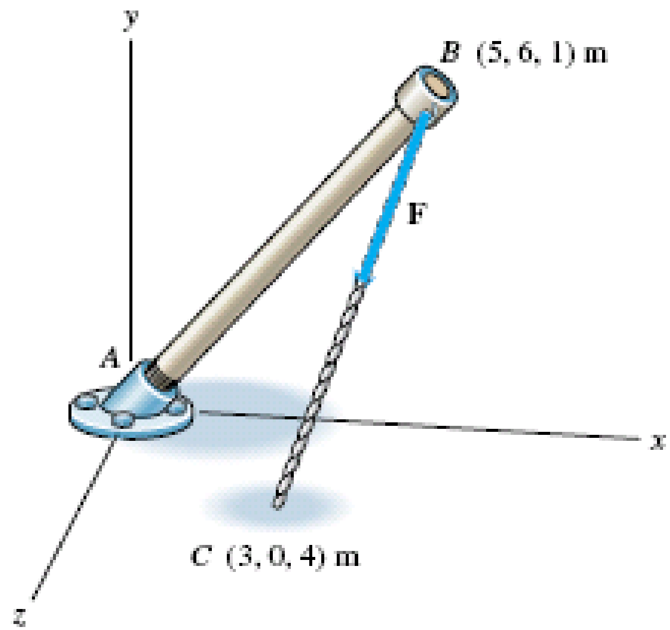
Review (Pos Vector)



The tension in cable BC is of magnitude F = 400 N.

a) Derive the unit vector for F

b) Express F as a Cartesian vector

a) Find $\vec{r}_{BC} = C - B$

$$= (3-5)\hat{i} + (0-6)\hat{j} + (4-1)\hat{k}$$

$$\vec{r}_{BC} = \{-2\hat{i} - 6\hat{j} + 3\hat{k}\}\ m$$

$$\vec{u}_{BC} = \vec{u}_F = \frac{\vec{r}_{BC}}{r_{BC}} = \frac{\{-2\hat{i} - 6\hat{j} + 3\hat{k}\}\ \cancel{m}}{\sqrt{2^2 + 6^2 + 3^2}\ \cancel{m}}$$

$$\vec{u}_F = -\frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}$$

b) $\vec{F} = F\vec{u}_F = 400N\left(-\frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}\right)$

$$= (-114\hat{i} - 343\hat{j} + 171\hat{k})\ N$$

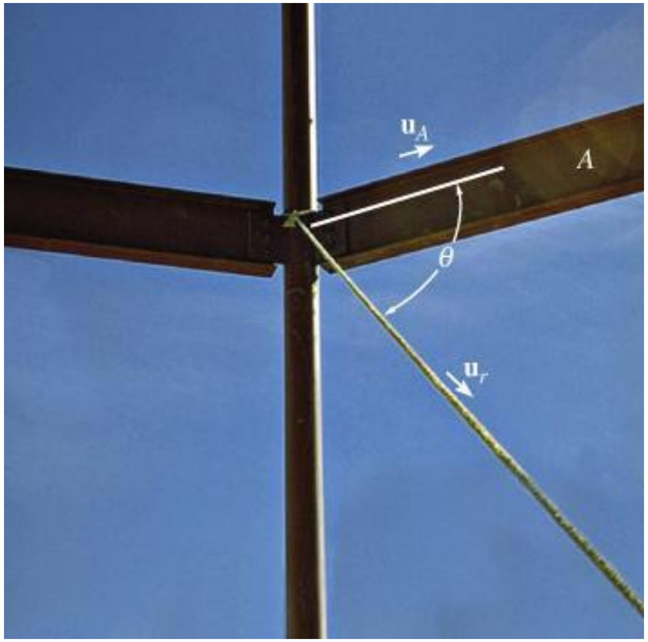
DOT PRODUCT

<u>Today's Objective</u>:

Students will be able to use the dot product to:

a) determine an angle between two vectors, and,

b) determine the projection of a vector along a specified line.

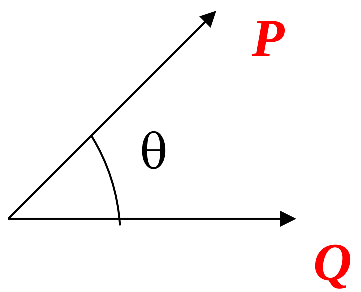


<u>In-Class Activities</u>:

- Check Homework
- Reading Quiz
- Applications / Relevance
- Dot product - Definition
- Angle Determination
- Determining the Projection
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. The dot product of two vectors ***P*** and ***Q*** is defined as



A) P Q cos θ B) P Q sin θ

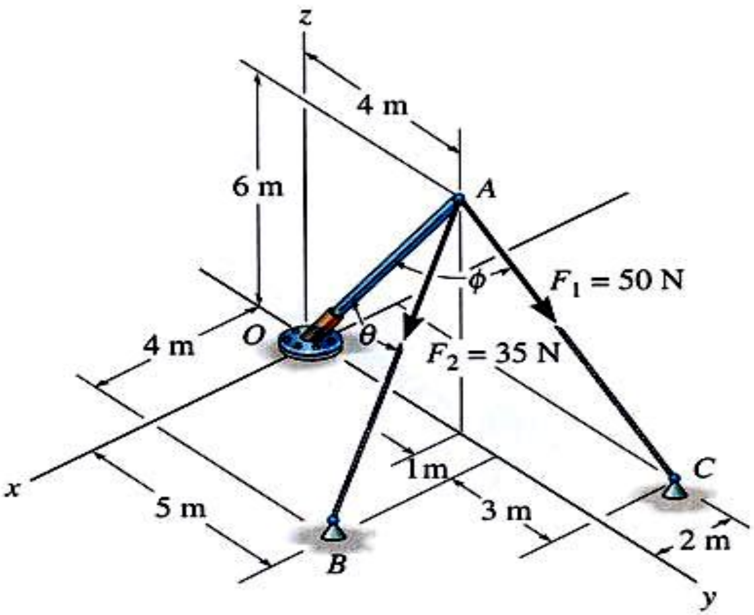
C) P Q tan θ D) P Q sec θ

2. The dot product of two vectors results in a ___________ quantity.

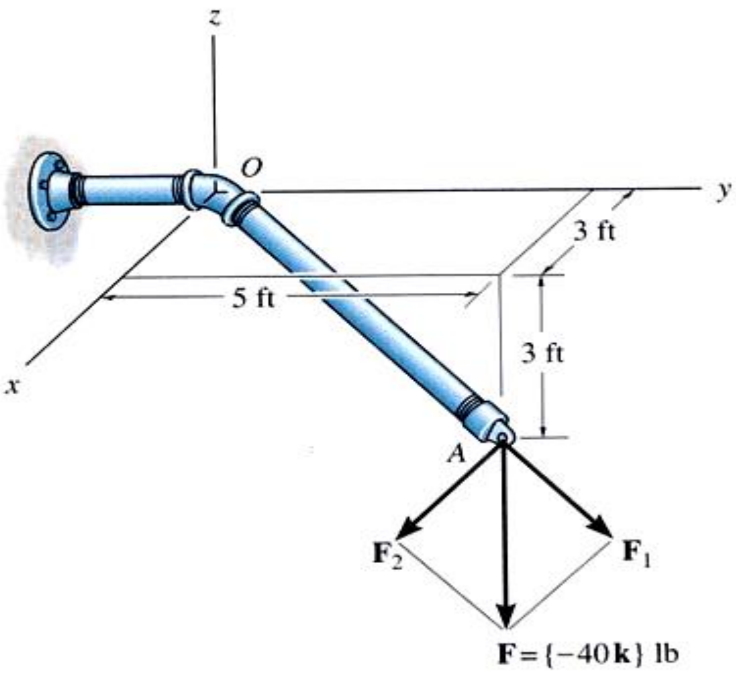
A) scalar B) vector

C) complex D) zero

APPLICATIONS

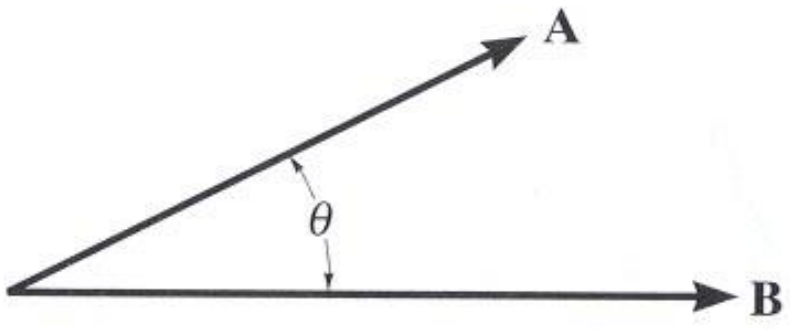


For this geometry, can you determine angles between the pole and the cables?



For force ***F*** at Point A, what component of it (F_1) acts along the pipe OA? What component (F_2) acts perpendicular to the pipe?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A•B*** = A B cos θ.

Angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DOT PRODUCT DEFINITON
(continued)

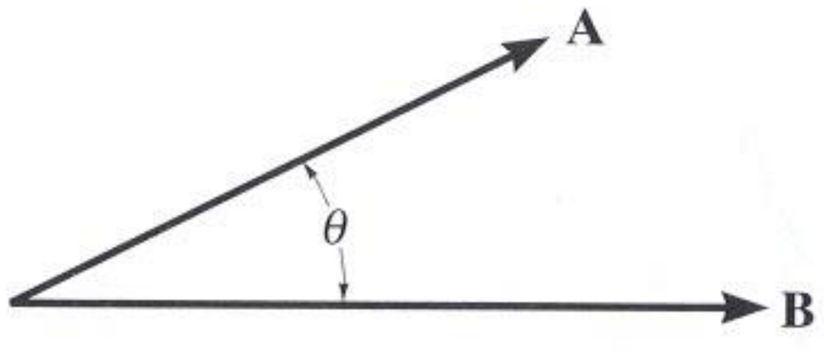
Examples: $\mathbf{i} \bullet \mathbf{j} = 0$

$\mathbf{i} \bullet \mathbf{i} = 1$

$$\mathbf{A} \bullet \mathbf{B} = (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}) \bullet (B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For the given two vectors in the Cartesian form, one can find the angle by

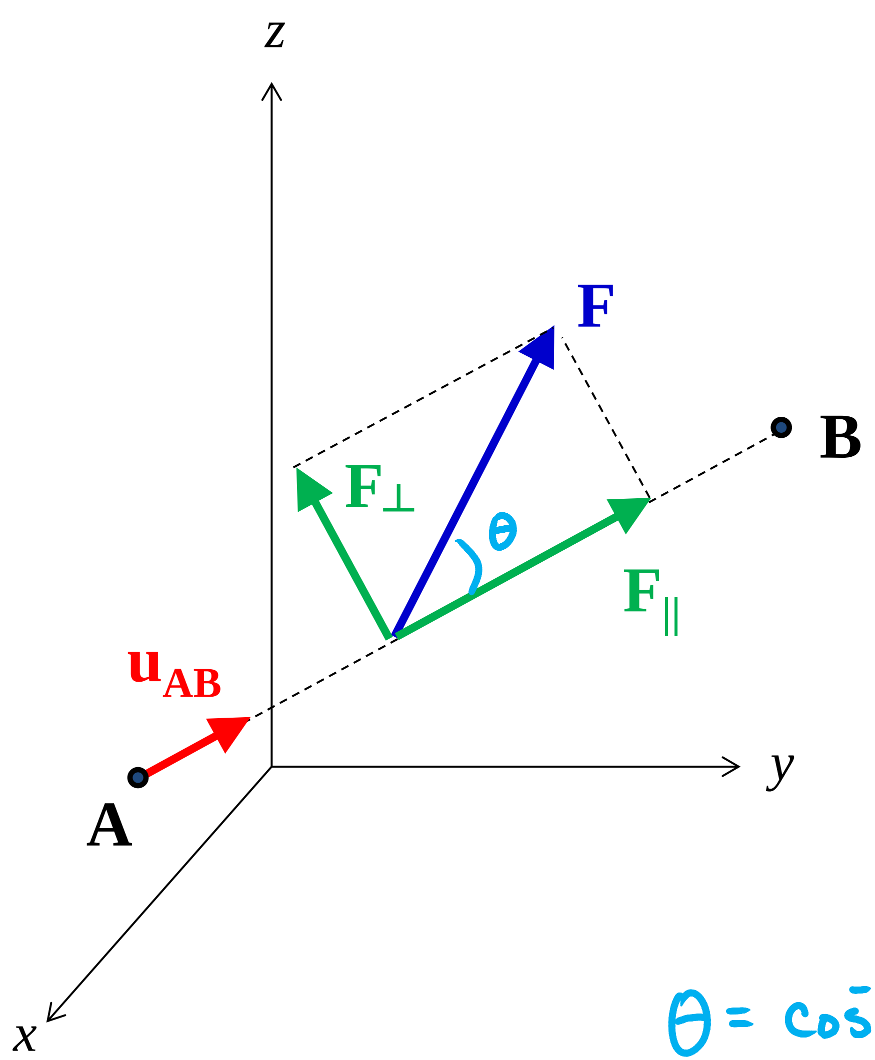
a) Finding the dot product, $\mathbf{A} \bullet \mathbf{B} = (A_x B_x + A_y B_y + A_z B_z)$,

b) Finding the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Using the definition of dot product and solving for θ, i.e.,

$\theta = \cos^{-1} [(\mathbf{A} \bullet \mathbf{B})/(A\,B)]$, where $0^{\circ} \le \theta \le 180^{\circ}$.

Projection of a Vector along a line



$$\vec{F} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

$$F_{\parallel} = F \cos\theta$$

$$= \vec{F} \cdot \vec{u}_{AB}$$

$$= F_x u_{ABx} + F_y u_{ABy} + F_z u_{ABz}$$

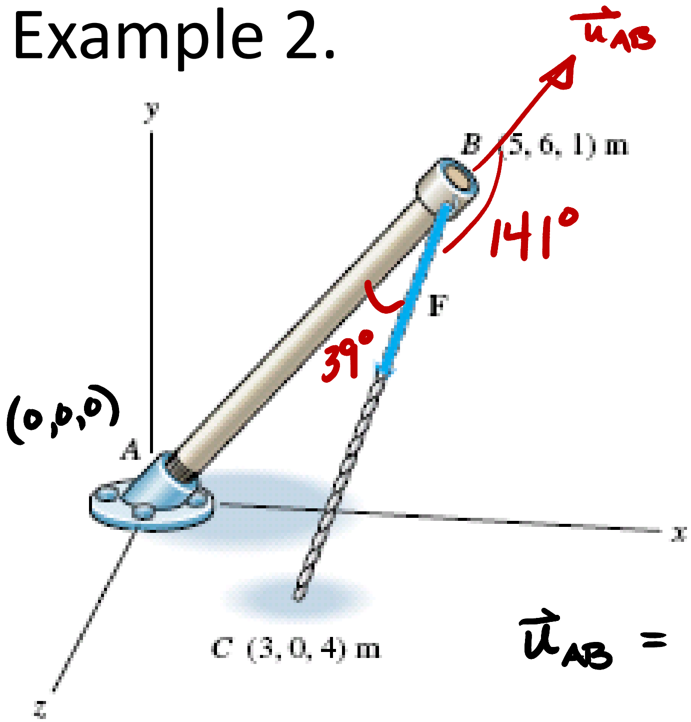
scalar magnitude

$$\vec{F}_{\parallel} = F_{\parallel} \vec{u}_{AB}$$

$$\vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel}$$

$$\theta = \cos^{-1}\left(\frac{\vec{F} \cdot \vec{u}_{AB}}{F}\right) = \cos^{-1}\left(\vec{u}_F \cdot \vec{u}_{AB}\right)$$

Example 2.



c) Find the component of F parallel to AB

d) Perpendicular to AB

e) Find the angle between AB and F

c) $F_{\parallel} = \vec{F} \cdot \vec{u}_{AB}$

$$\vec{F} = 400\text{N}\left(-\frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}\right)$$

$$\vec{u}_{AB} = \frac{5\hat{i} + 6\hat{j} + 1\hat{k}}{\sqrt{5^2+6^2+1^2}} = 0.635\hat{i} + 0.762\hat{j} + 0.127\hat{k}$$

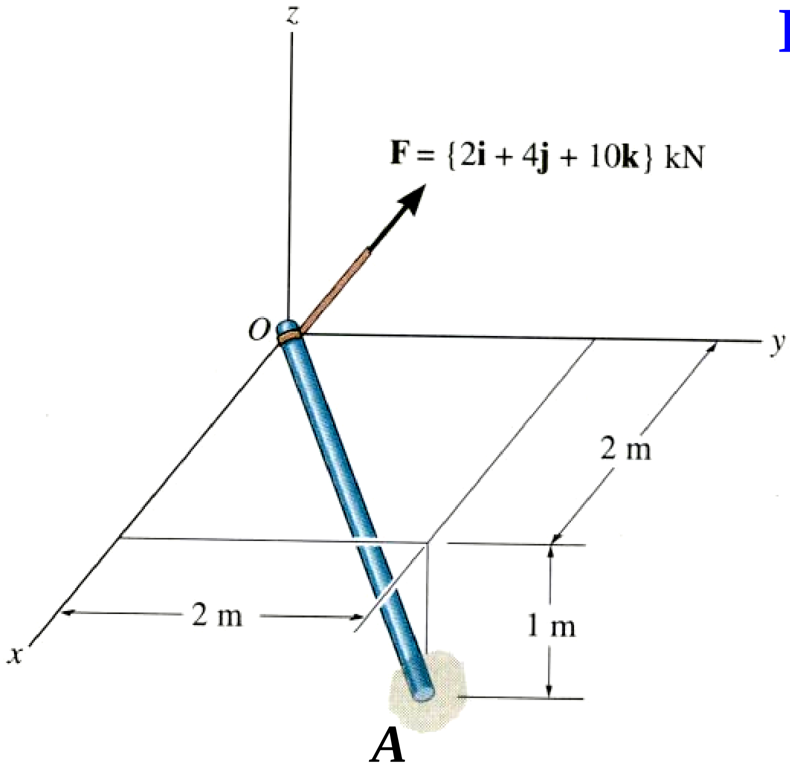
$$F_{\parallel} = \vec{F} \cdot \vec{u}_{AB} = 400\text{N}\left[\left(-\frac{2}{7}\right)(0.635) + \left(-\frac{6}{7}\right)(0.762) + \left(\frac{3}{7}\right)(0.127)\right]$$

$F_{\parallel} = -312\text{ N}$ (−) ⇒ opp. $\vec{u}_{AB}$ $\quad \vec{F}_{\parallel} = F_{\parallel}\vec{u}_{AB} = -198\hat{i} - 238\hat{j} - 40\hat{k}$ N

d) $\vec{F}_{\perp} = \vec{F} - \vec{F}_{\parallel} = (-114\hat{i} - 343\hat{j} + 171\hat{k}) - (\quad) = 84\hat{i} - 105\hat{j} + 211\hat{k}$ N

e) $\theta = \cos^{-1}(\vec{u}_F \cdot \vec{u}_{AB}) = \cos^{-1}\left(\frac{\vec{F} \cdot \vec{u}_{AB}}{F}\right) = \cos^{-1}\left(\frac{-312}{400}\right) = 141°$ (or 39°)

EXAMPLE



Given: The force acting on the pole

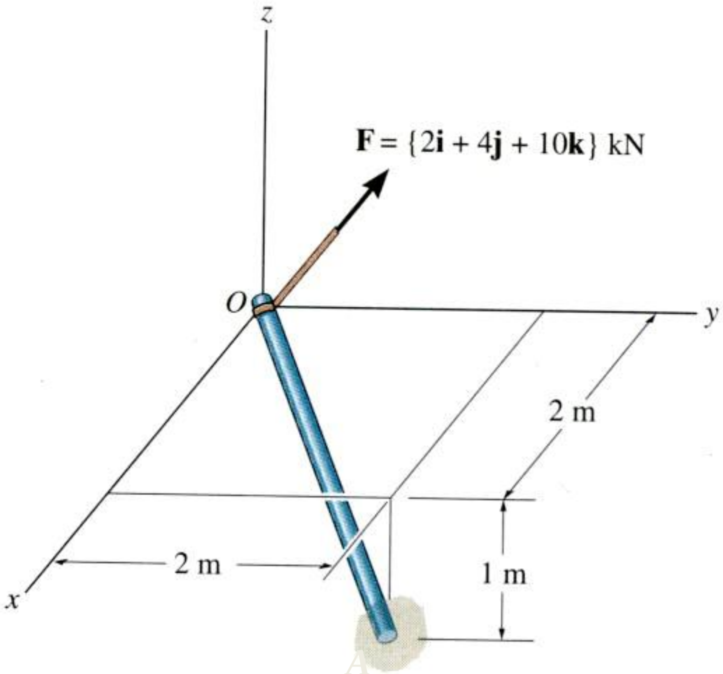
Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole OA.

<u>Plan</u>:

1. Get $\boldsymbol{r_{OA}}$

2. $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{OA}})/(F\ r_{OA})\}$

3. $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{OA}}$ or $F \cos \theta$



EXAMPLE
(continued)

$\boldsymbol{r_{OA}} = \{2\,\boldsymbol{i} + 2\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$r_{OA} = (2^2 + 2^2 + 1^2)^{1/2} = 3$ m

$\boldsymbol{F} = \{2\,\boldsymbol{i} + 4\,\boldsymbol{j} + 10\,\boldsymbol{k}\}$kN

$F = (2^2 + 4^2 + 10^2)^{1/2} = 10.95$ kN

$\boldsymbol{F} \bullet \boldsymbol{r_{OA}} = (2)(2) + (4)(2) + (10)(-1) = 2$ kN·m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{OA}})/(F\, r_{OA})\}$

$\theta = \cos^{-1}\{2/(10.95 * 3)\} = 86.5°$

$\boldsymbol{u_{OA}} = \boldsymbol{r_{OA}}/r_{OA} = \{(2/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} - (1/3)\,\boldsymbol{k}\}$

$F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{OA}} = (2)(2/3) + (4)(2/3) + (10)(-1/3) = 0.667$ kN

Or $F_{OA} = F \cos\theta = 10.95 \cos(86.51°) = 0.667$ kN

CONCEPT QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be _______________ to each other.

   A) parallel (pointing in the same direction)

   B) parallel (pointing in the opposite direction)

   C) perpendicular

   D) cannot be determined.

2. If a dot product of two non-zero vectors equals -1, then the vectors must be _________ to each other.

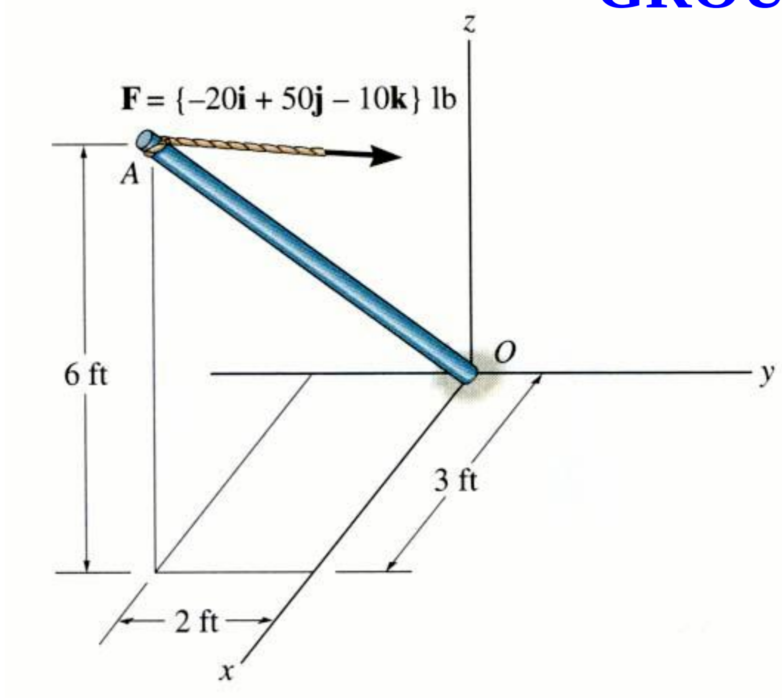
   A) parallel (pointing in the same direction)

   B) parallel (pointing in the opposite direction)

   C) perpendicular

   D) cannot be determined.

GROUP PROBLEM SOLVING



Given: The force acting on the pole.

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole AO.

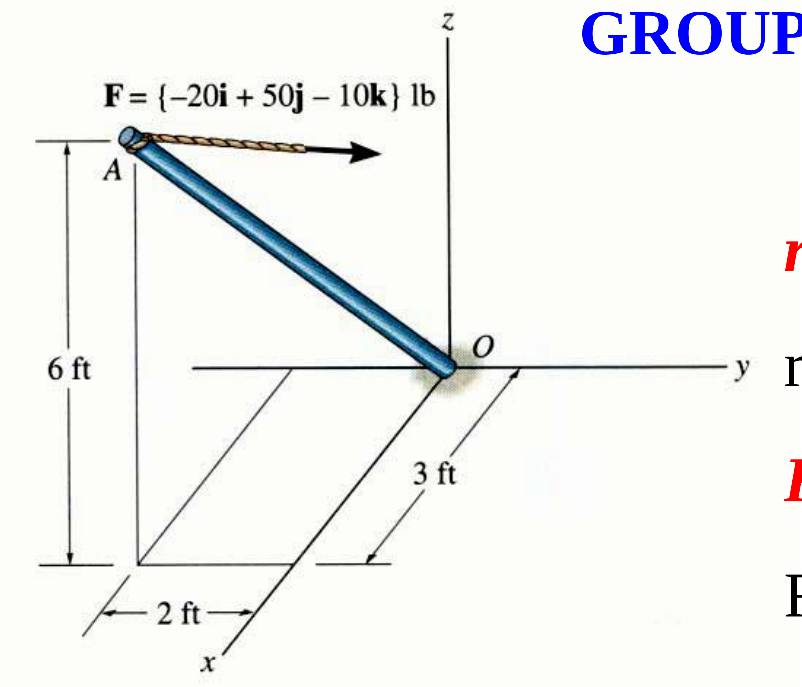
<u>Plan</u>:

1. Get $\boldsymbol{r_{AO}}$

2. $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\ \ r_{AO})\}$

3. $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}}$ or $F \cos\theta$

GROUP PROBLEM SOLVING (continued)



$\boldsymbol{r_{AO}} = \{-3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ ft.

$r_{AO} = (3^2 + 2^2 + 6^2)^{1/2} = 7$ ft.

$\boldsymbol{F} = \{-20\,\boldsymbol{i} + 50\,\boldsymbol{j} - 10\,\boldsymbol{k}\}$lb

$F = (20^2 + 50^2 + 10^2)^{1/2} = 54.77$ lb

$\boldsymbol{F} \bullet \boldsymbol{r_{AO}} = (-20)(-3) + (50)(2) + (-10)(-6) = 220$ lb·ft

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\, r_{AO})\}$

$\theta = \cos^{-1}\{220/(54.77 \times 7)\} = 55.0°$

$\boldsymbol{u_{AO}} = \boldsymbol{r_{AO}}/r_{AO} = \{(-3/7)\,\boldsymbol{i} + (2/7)\,\boldsymbol{j} - (6/7)\,\boldsymbol{k}\}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{u_{AO}} = (-20)(-3/7) + (50)(2/7) + (-10)(-6/7) = 31.4$ lb

Or $F_{AO} = F \cos\theta = 54.77 \cos(55.0°) = 31.4$ lb

ATTENTION QUIZ

1. The Dot product can be used to find all of the following except _____ .

   A) sum of two vectors

   B) angle between two vectors

   C) component of a vector parallel to another line

   D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors ***P*** and ***Q***.

   $\boldsymbol{P} = \{5\,\boldsymbol{i} + 2\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

   $\boldsymbol{Q} = \{-2\,\boldsymbol{i} + 5\,\boldsymbol{j} + 4\,\boldsymbol{k}\}$ m

   A) -12 m   B) 12 m   C) 12 m^2

   D) -12 m^2 E) 10 m^2

End of the Lecture

Let Learning Continue