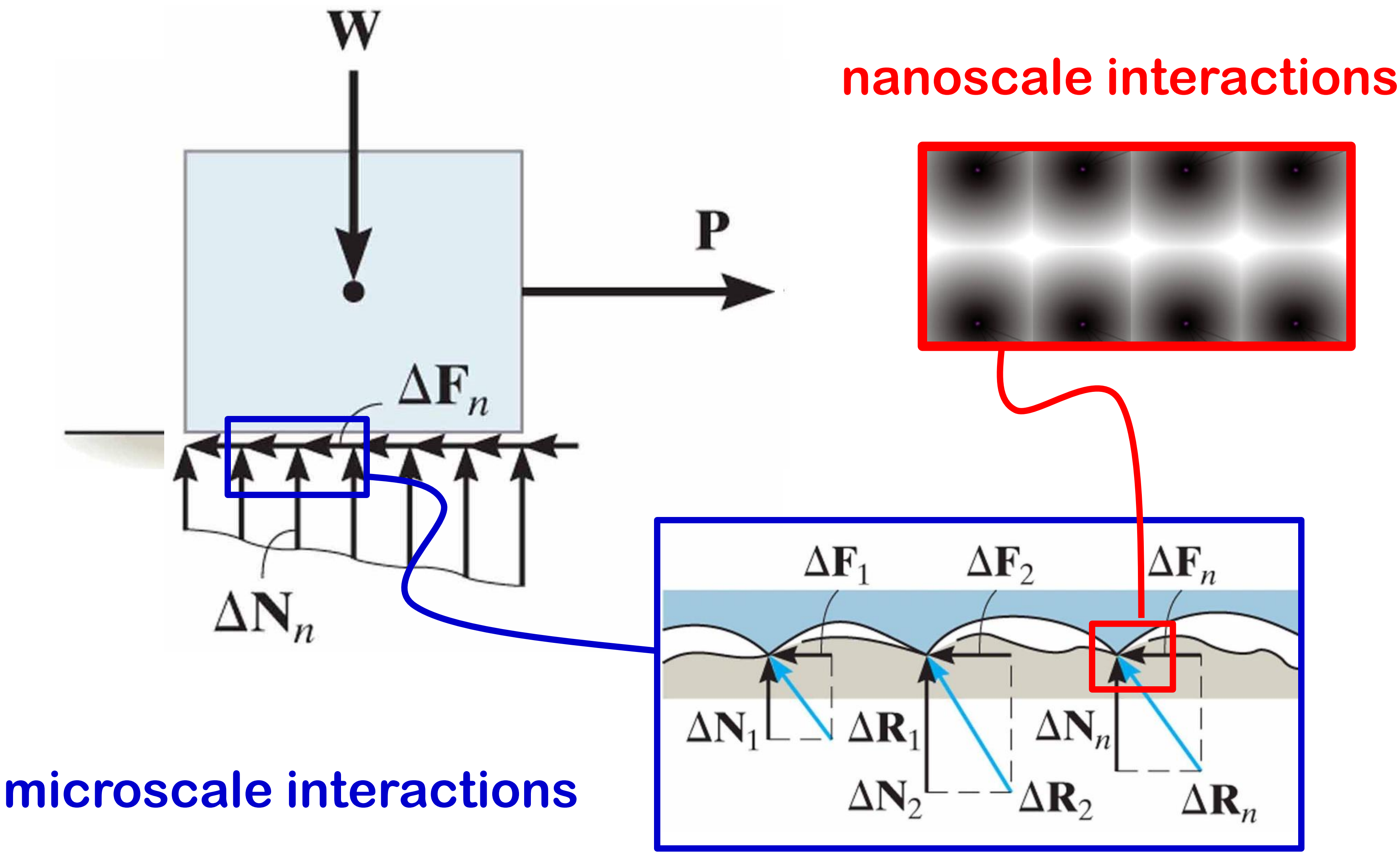
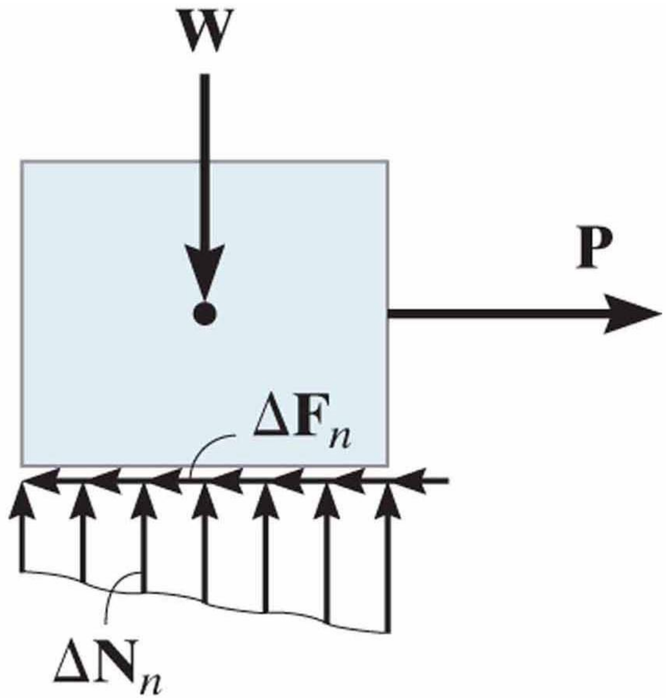
Friction (Ch 8)

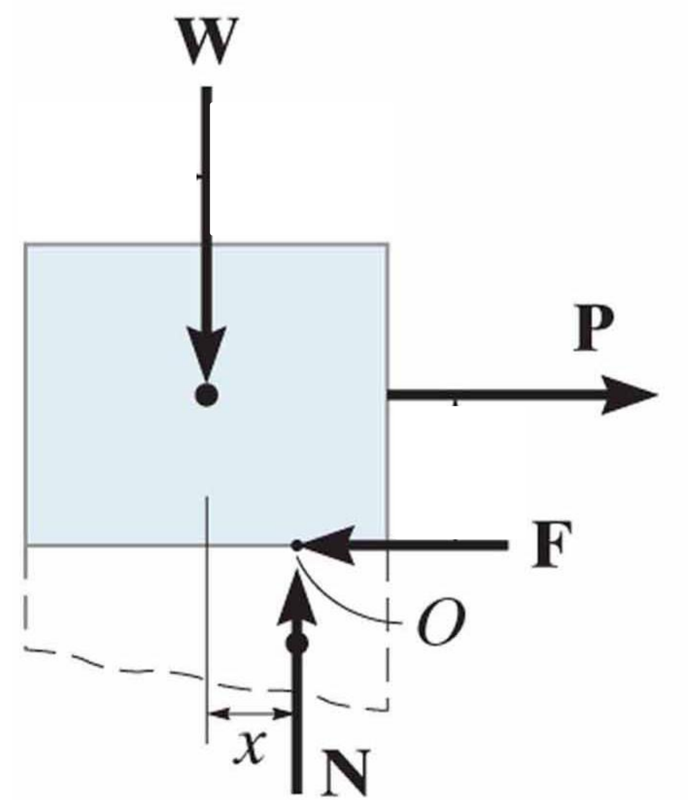
- Basics of Dry Friction
- Types of Friction Analyses
- Selected Applications:
    - Wedges
    - Belts
    - Bearings

Basics of Dry Friction



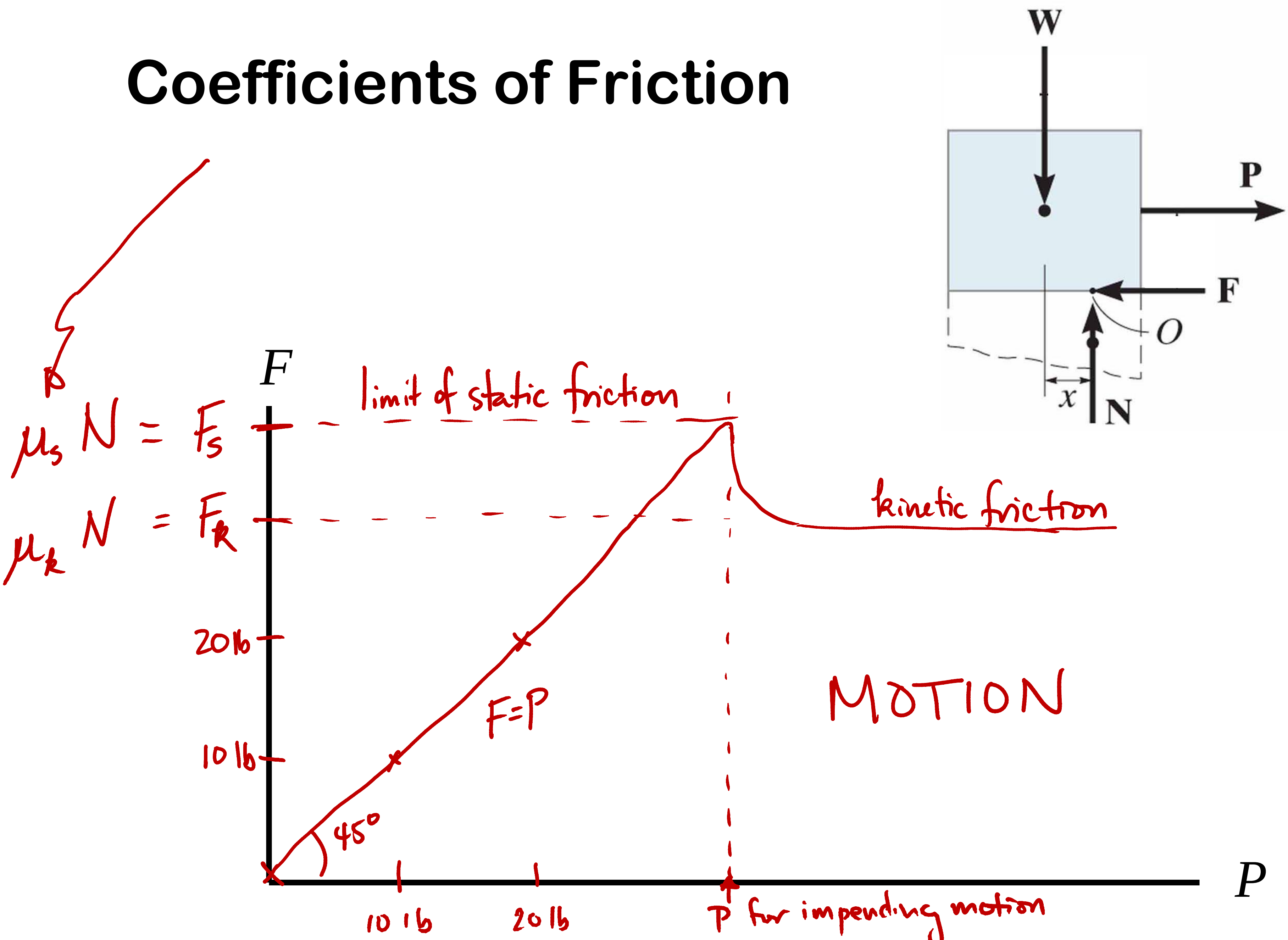
Basics of Dry Friction



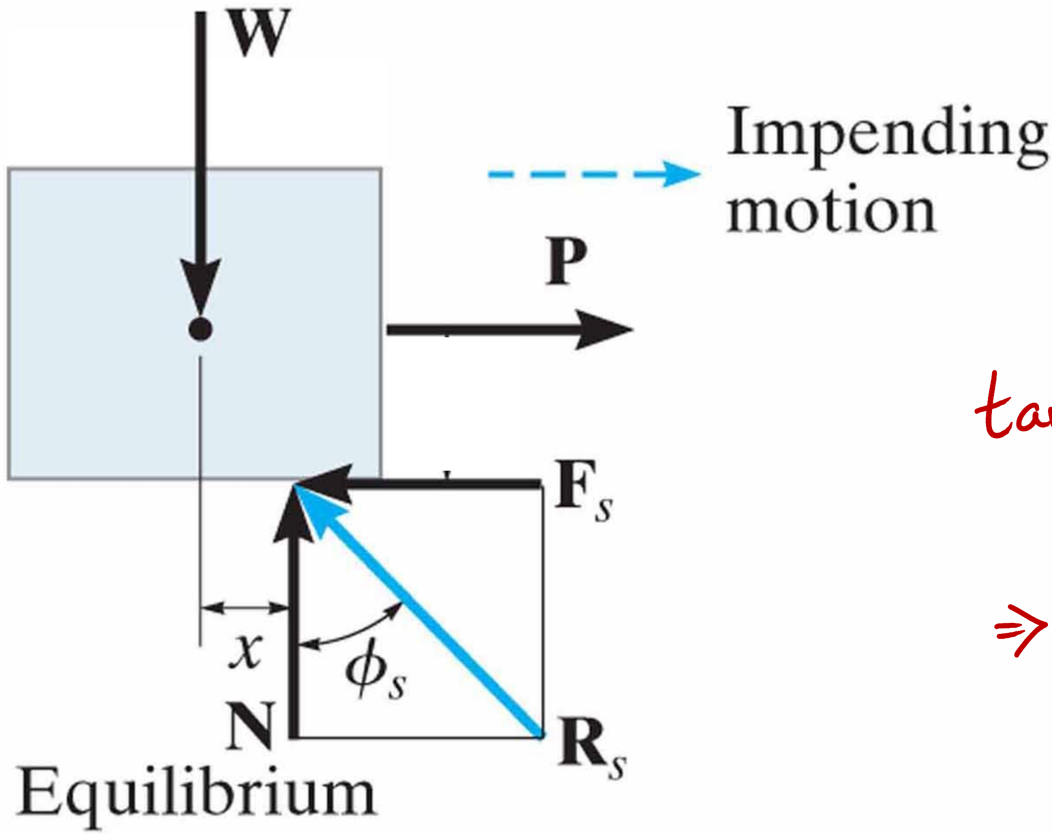


Resultant Normal
and Frictional Forces

Coefficients of Friction



Angle of Friction (ϕ_s)



$F_s = \mu_s N$

$$\tan\phi_s = \frac{F_s}{N} = \mu_s$$

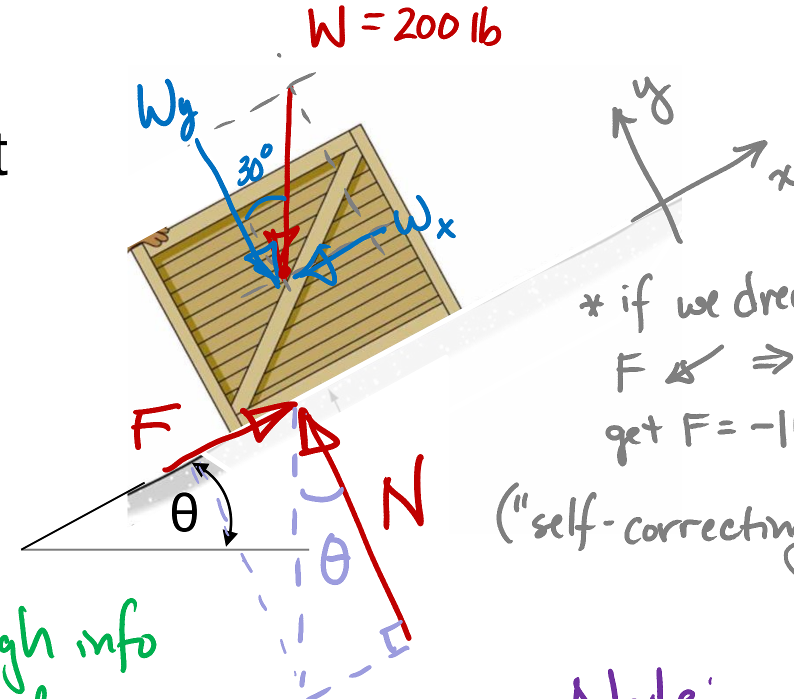
$$\Rightarrow \quad \phi_s = \tan^{-1}\mu_s$$

Types of Friction Analyses

- Equilibrium – True or False
  - Slip?
  - Tip?
- Impending Motion
  - at a Single Point
  - at All Points
  - at Some Points

Equilibrium – Slip?

Will the 200-lb crate slide down the θ=30° incline, if the coefficient of static friction is 0.35? YES



* if we drew F ↙ ⇒ get F = −100 ("self-correcting")

$$\Sigma F_y = 0 = N - W_y$$

$$\Rightarrow N = 200\cos 30° = 173 \text{ lb}$$

$$\Sigma F_x = 0 = F - W_x$$

$$\Rightarrow F = 200\sin 30° = 100 \text{ lb}$$

enough info to solve for F, N

$$F \le \mu_s N \ ?$$

$$100 \le 0.35(173) \ ?$$

No! Not @ Equil.

test limit $F_{max} = \mu_s N$

Note:

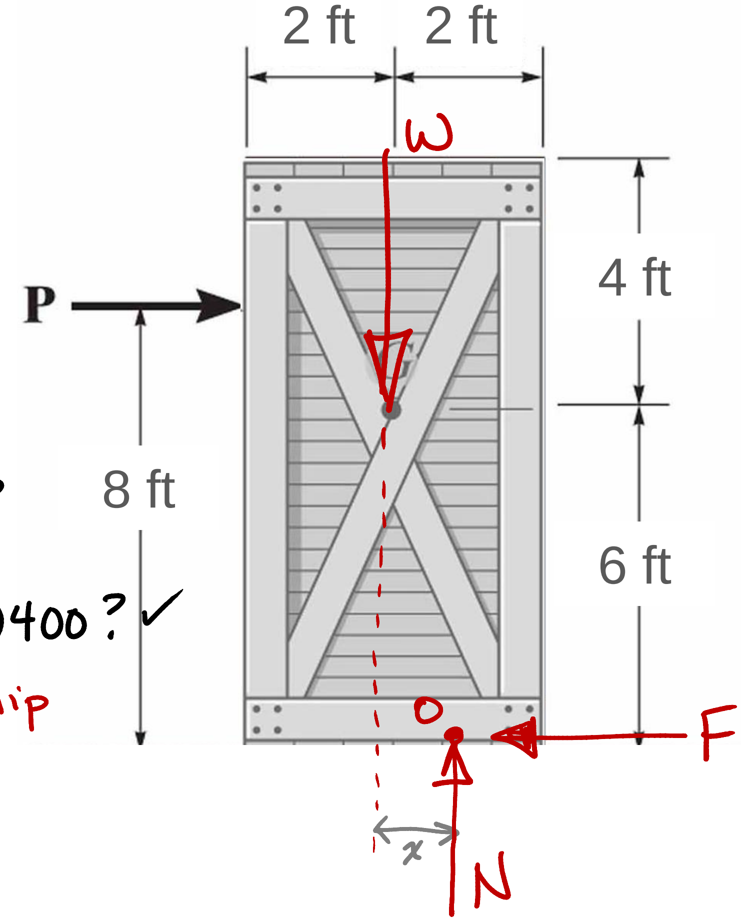
$$F \le \mu_s N \Rightarrow \frac{F}{N} \le \mu_s$$

$$\tan\theta \le \tan\phi$$

$$\Rightarrow \theta \le \phi$$ for Eq. (angle of repose)

Equilibrium – Tip?

Will the 400-lb crate slip, tip, or remain stable if P = 125 lb? The coefficient of static friction is 0.35.



$$\left.\begin{array}{l}\Sigma F_x = 0 \Rightarrow F = P = 125 \text{ lb} \\ \Sigma F_y = 0 \Rightarrow N = W = 400 \text{ lb}\end{array}\right\} \quad \begin{array}{l} F \le \mu_s N \,? \\ 125 \le (0.35)400 \,? \checkmark \end{array}$$

$\Rightarrow$ Does not slip

P & F form couple $P(8\text{ft})$

W & N form opposing couple Wx

$$\Sigma M_O = Wx - P(8\text{ft}) = 0 \text{ if } x = \frac{125(8)}{400} = 2.5 \text{ ft}$$ Not Possible

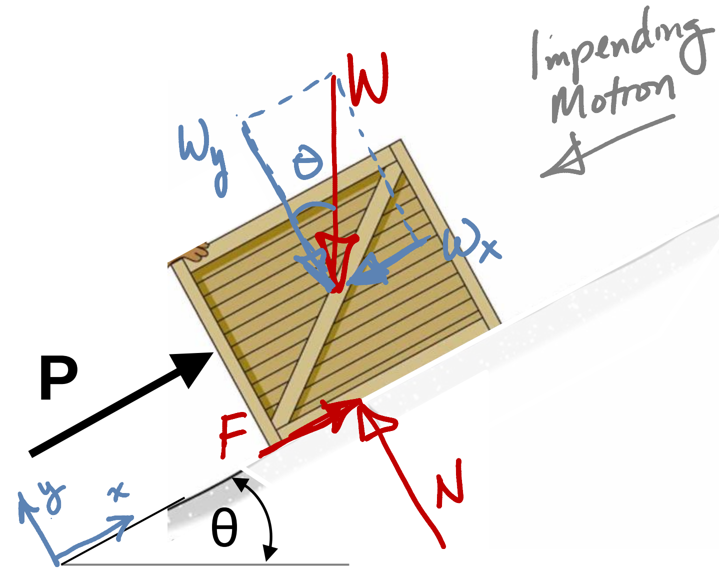
$$\max x = 2 \text{ ft} \Rightarrow \Sigma M_O = (400)(2) - (125)(8) = -200 \text{ lbft}$$ Net M

$\Rightarrow$ Crate Tips

Impending Motion at One Point: A

$\Rightarrow$ use equil. eqns + $F_s = \mu_s N$

What is the minimum force P to prevent the 200-lb crate from sliding down the θ=30° incline, if the coefficient of static friction is 0.25?



$$\Sigma F_y = 0 = N - W_y = N - 200\,\text{lb} \cos 30°$$

$$\Rightarrow N = 173\,\text{lb}$$

$$\Sigma F_x = 0 = P + F - W_x$$

@ limit of friction $F = \mu_s N = 0.25(173\,\text{lb})$

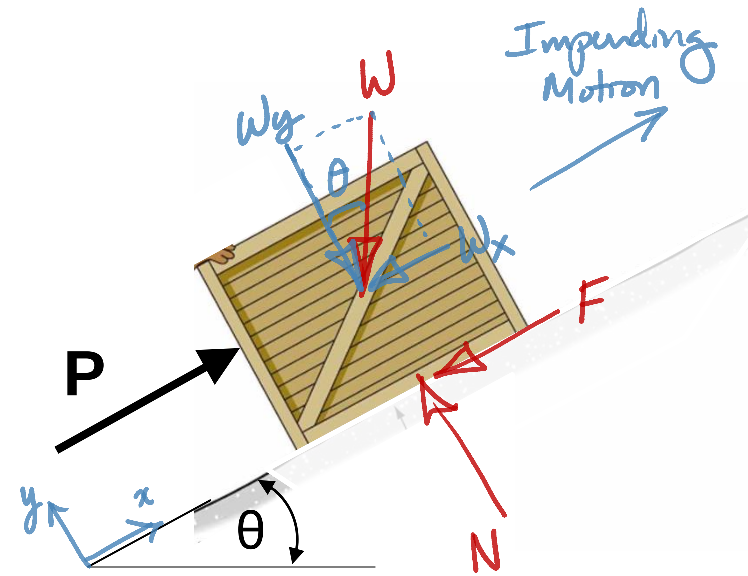
$$\Rightarrow P = 200\,\text{lb} \sin 30° - 0.25(173\,\text{lb})$$

$$\Rightarrow P = 56.7\,\text{lb}$$

Note: larger P will result in smaller F

Impending Motion at One Point: B

What minimum force P is needed to start the 200-lb crate sliding up the θ=30° incline, if the coefficient of static friction is 0.25?



$$\Sigma F_y = 0 = N - W_y = N - 200\text{lb}\cos 30°$$

$$\Rightarrow N = 173\text{ lb}$$

$$\Sigma F_x = 0 = P - F - W_x \qquad F = \mu_s N = 43.3\text{lb}$$

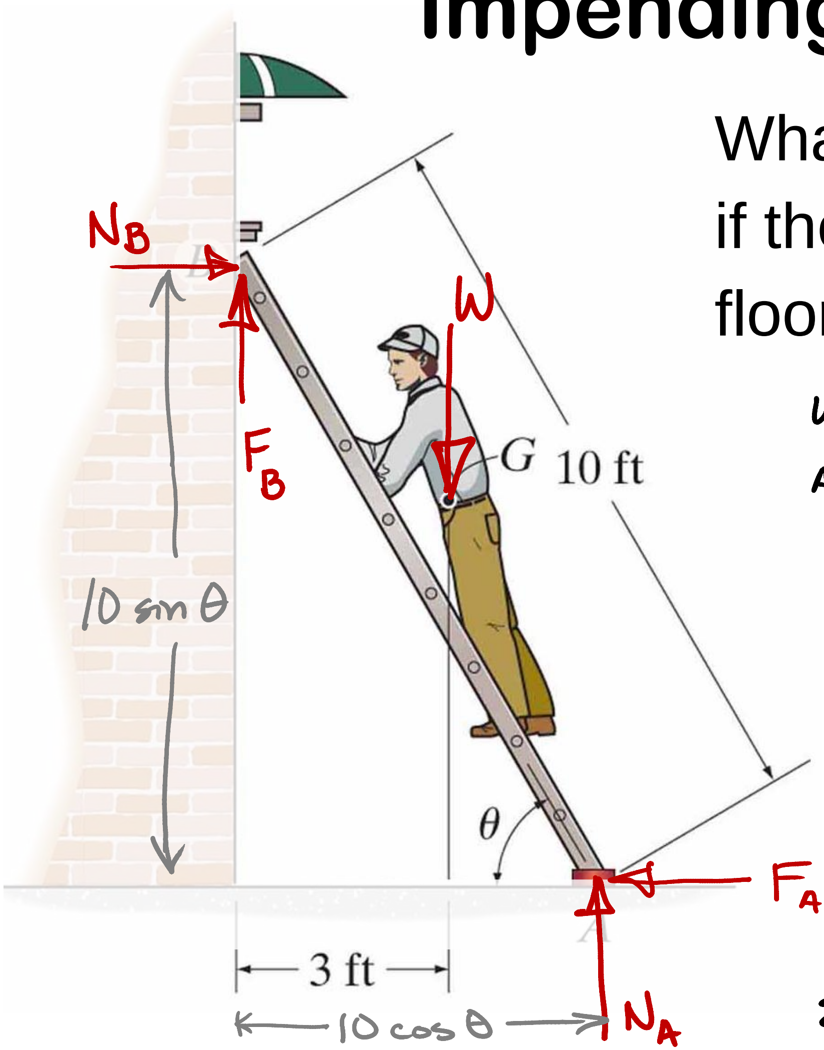
$$\Rightarrow P = 200\text{lb}\sin 30° + 43.3\text{ lb}$$

$$\Rightarrow P = 143\text{ lb}$$

Note: only difference from previous problem is sense of F... ⇒ diff. P

Not self-correcting!

Impending Motion at All Points



What is the maximum angle for stability, if the coefficient of static friction with the floor is 0.3 and with the wall 0.2?

When ladder moves, slips @ A & B

At limit $\Rightarrow$ $F_A = 0.3 N_A$ & $F_B = 0.2 N_B$

Equil. Eqns.

$$\Sigma F_x = 0 = N_B - F_A \Rightarrow N_B = 0.3 N_A$$

$$\Sigma F_y = 0 = N_A + F_B - W = N_A + 0.2 N_B - W$$

$$= N_A + 0.2(0.3 N_A) - W \Rightarrow W = 1.06 N_A$$

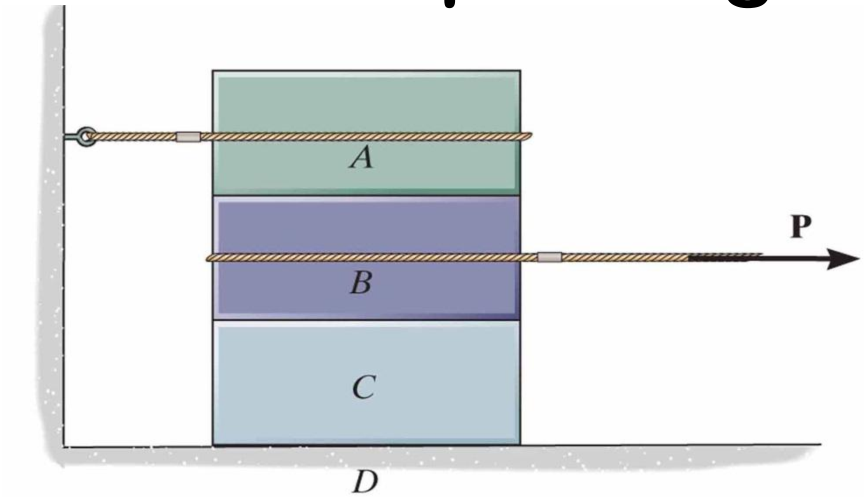
$$\Sigma M_B = 0 = N_A(10\cos\theta) - F_A(10\sin\theta) - W(3)$$

$$0 = \cancel{N_A}(10\cos\theta) - 0.3\cancel{N_A}(10\sin\theta) - 3.18\cancel{N_A}$$

$$\Rightarrow 10\cos\theta - 3\sin\theta = 3.18 \Rightarrow \theta = 55.6°$$

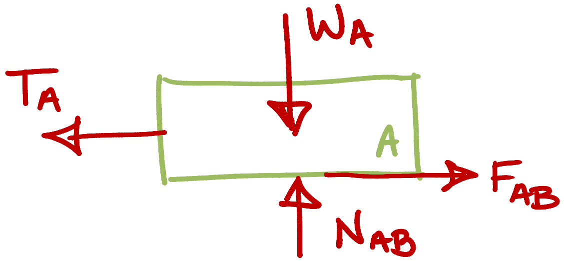
Equil Eqns + $F = \mu N$
at ALL points of contact

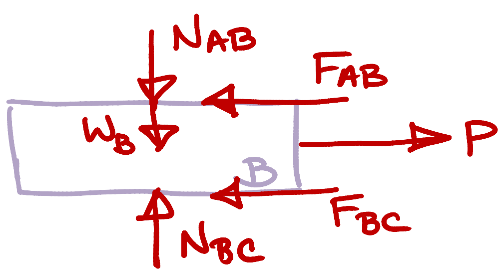
Impending Motion at Some Points



What force P is needed for motion, and will block C slide? Each block weighs 20N, and the coefficient of friction at each surface is 0.25.

Case 1: B slips at both surfaces





Block A:

$$\Sigma F_y = 0 = N_{AB} - W_A \Rightarrow N_{AB} = 20N$$

Block B:

$$\Sigma F_y = 0 = N_{BC} - N_{AB} - W_B \Rightarrow N_{BC} = 40N$$

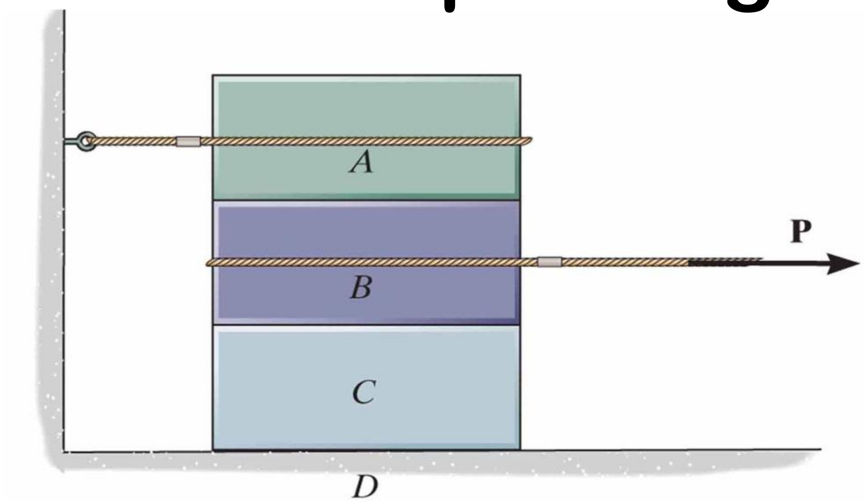
$$\Sigma F_x = 0 = P - F_{AB} - F_{BC}$$

$P = 15N$

Friction Eqns:

$$F_{AB} = \mu N_{AB} = 5N \qquad F_{BC} = \mu N_{BC} = 10N$$

Impending Motion at Some Points



* Consider situation with $P = 0$

$\Rightarrow$ blocks in equilibrium.

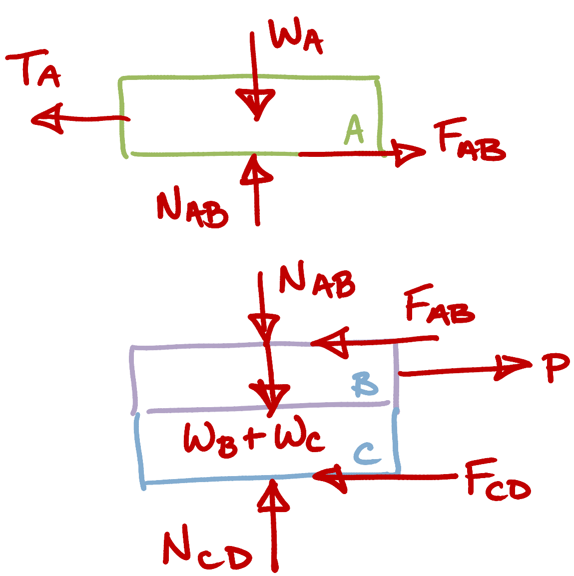
Increase P until "something happens"

Case with lowest P will occur first.

Block B slips when $P = 15\,N$.

C does NOT slide.

Case 2: Blocks B & C slide together



Block A

$$\Sigma F_y = 0 = N_{AB} - W_A \Rightarrow N_{AB} = 20N$$

Blocks B/C

$$\Sigma F_y = 0 = N_{CD} - N_{AB} - W_B - W_C \Rightarrow N_{CD} = 60N$$

$$\Sigma F_x = 0 = P - F_{AB} - F_{CD}$$

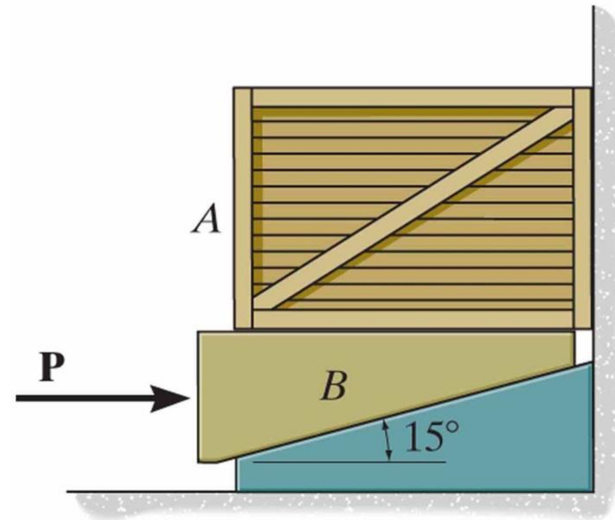
$P = 20N$

Friction Eqns

$$F_{AB} = \mu N_{AB} = 5N \qquad F_{CD} = \mu N_{CD} = 15N$$

Wedges

- Impending Motion at All Points
- Mechanical Advantage = W_A / P
- $P < 0$ indicates self-locking (for B ←)

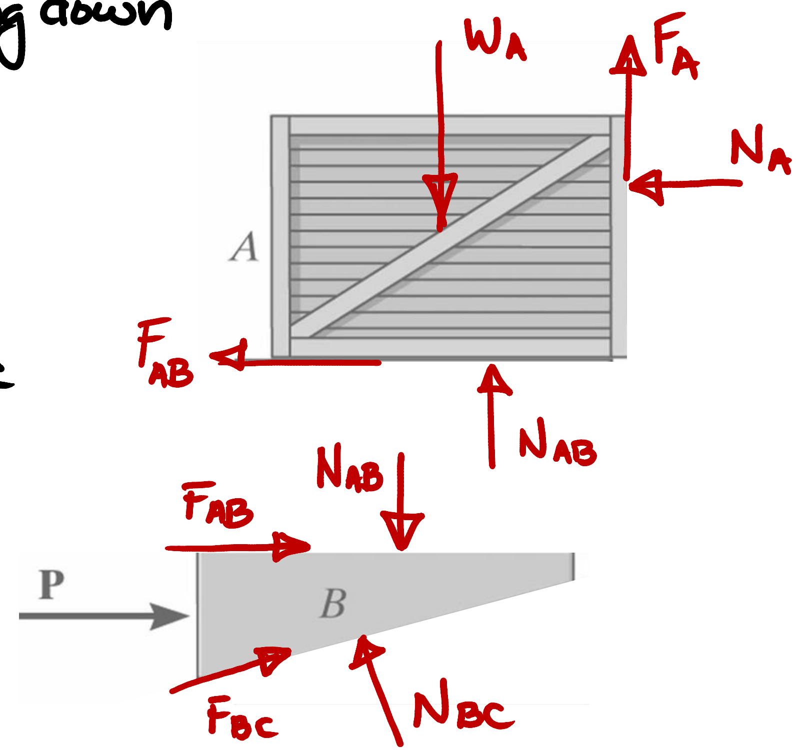


Consider min. P to keep crate from sliding down

for A, write: $\Sigma F_x = 0$, $\Sigma F_y = 0$

for B, write: $\Sigma F_x = 0$, $\Sigma F_y = 0$

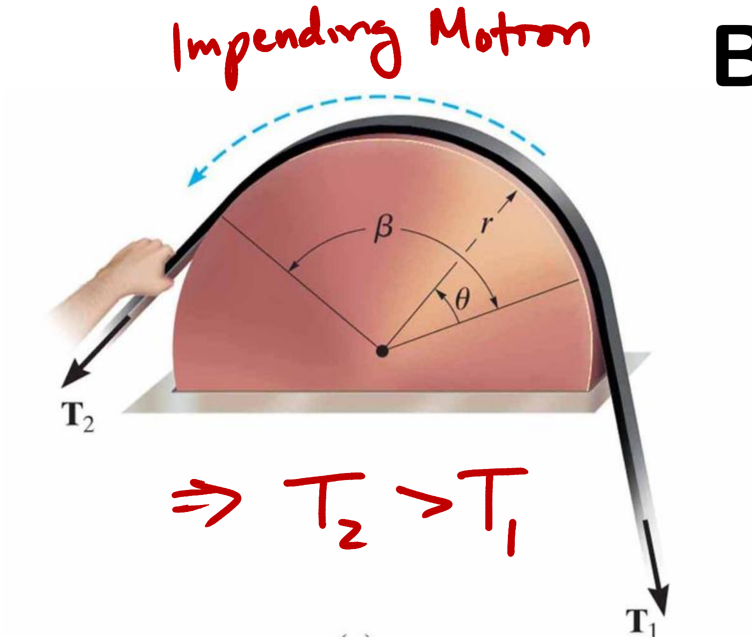
Impending Motion ⇒ $F_A = \mu N_A$, $F_{AB} = \mu N_{AB}$, $F_{BC} = \mu N_{BC}$

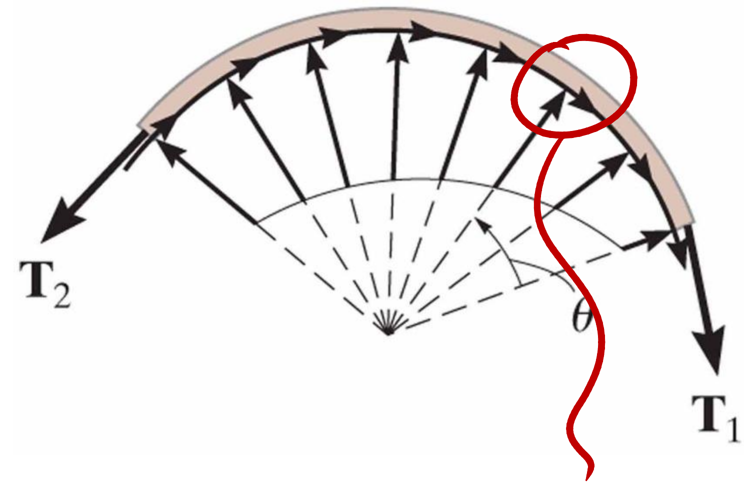


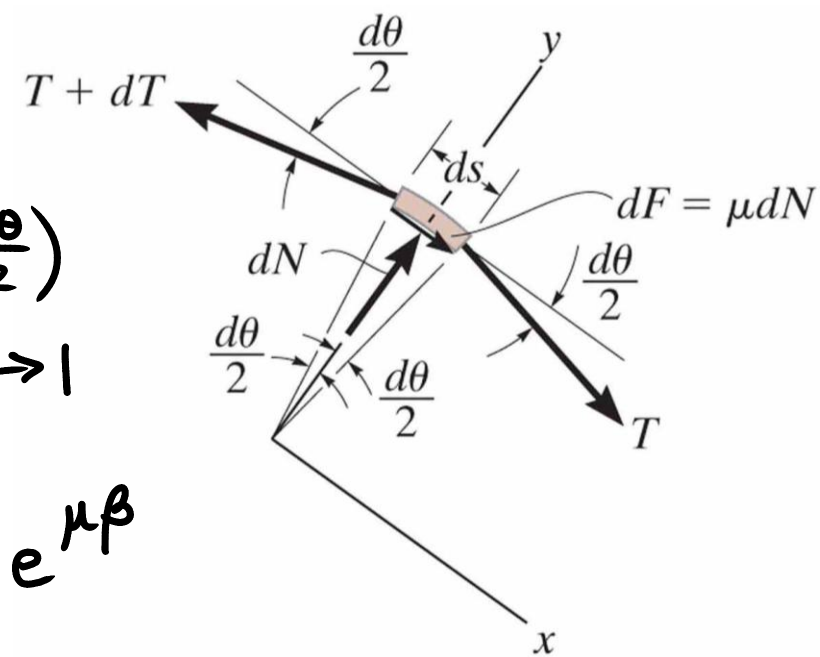
7 eqns for 7 unknowns if W given

$P, F_A, N_A, F_{AB}, N_{AB}, F_{BC}, N_{BC}$

Belt Friction







$$\sum F_x = 0 = T\cos\left(\frac{d\theta}{2}\right) + dF - (T+dT)\cos\left(\frac{d\theta}{2}\right)$$

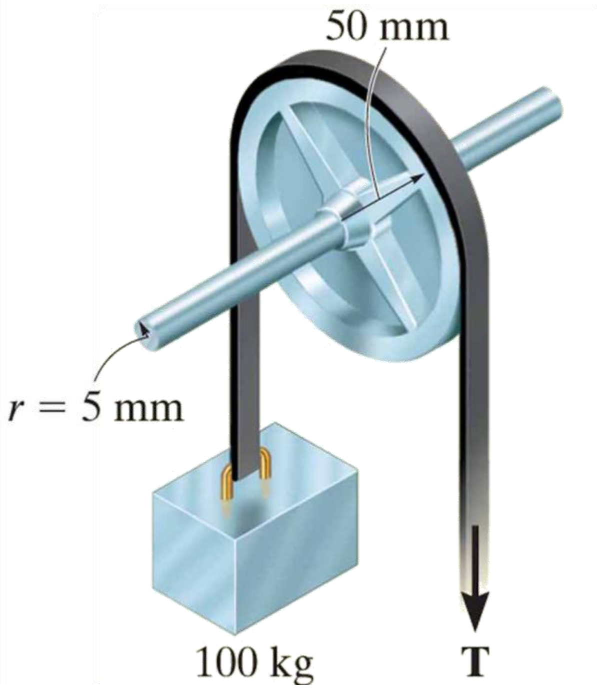
$$\sum F_y = 0 = dN - T\sin\left(\frac{d\theta}{2}\right) - (T+dT)\sin\left(\frac{d\theta}{2}\right)$$

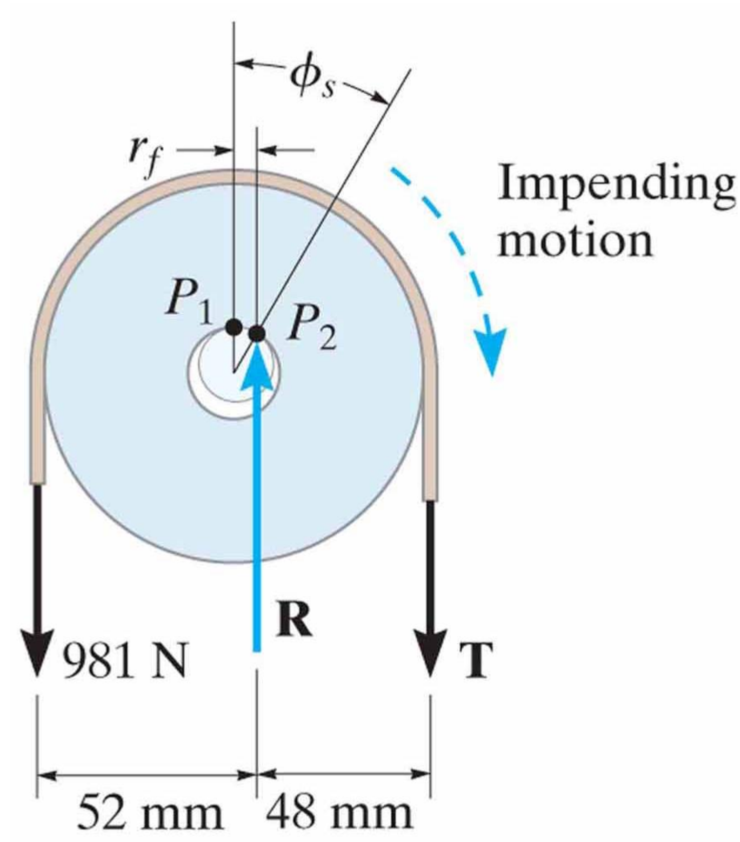
in $\lim d\theta \to 0 \Rightarrow \sin\left(\frac{d\theta}{2}\right) \to \frac{d\theta}{2}$ & $\cos\left(\frac{d\theta}{2}\right) \to 1$

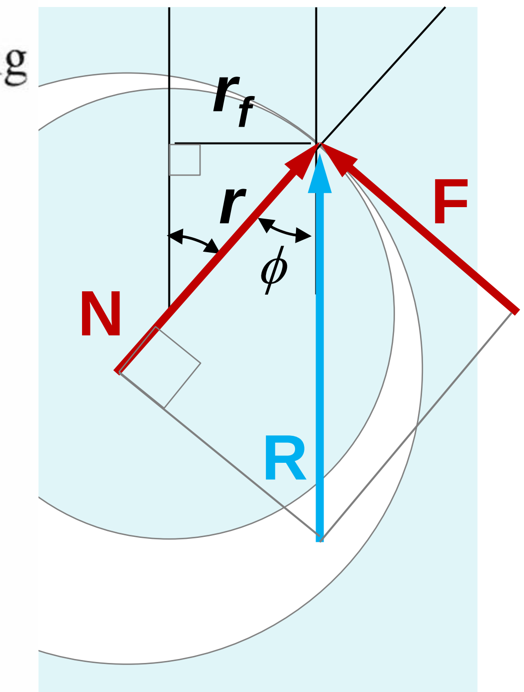
... (see text)

$$\Rightarrow \frac{dT}{T} = \mu\, d\theta \xRightarrow{\text{integrate}} T_2 = T_1 e^{\mu\beta}$$

Pulley (Bearing) Friction





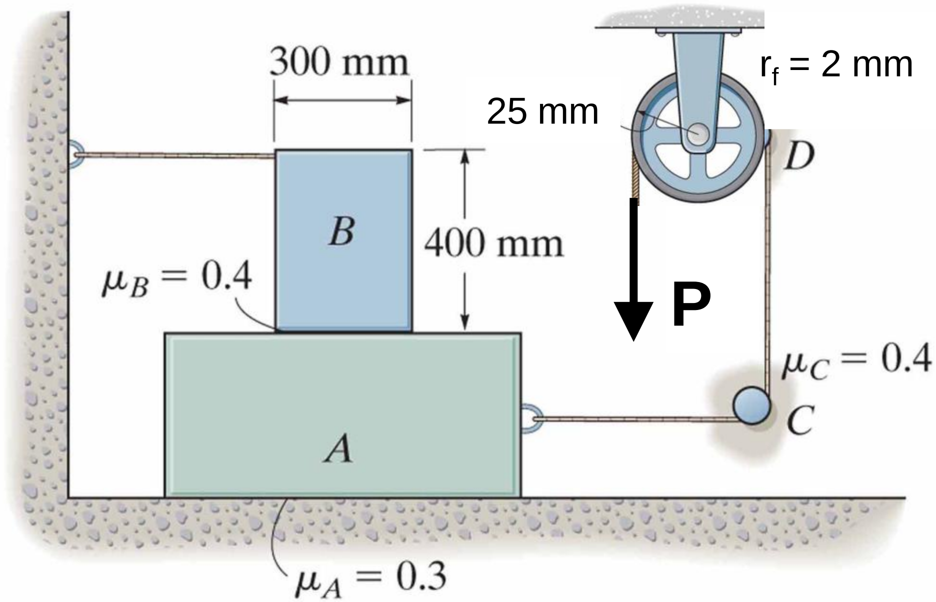


$$\Sigma M_{P_2} = 0 = 981\,N\,(r_p + r_f) - T\,(r_p - r_f)$$

$$\Rightarrow \quad T = 981\,N\left(\frac{r_p + r_f}{r_p - r_f}\right)$$

$$\tan\phi = \frac{F}{N} = \mu_s$$

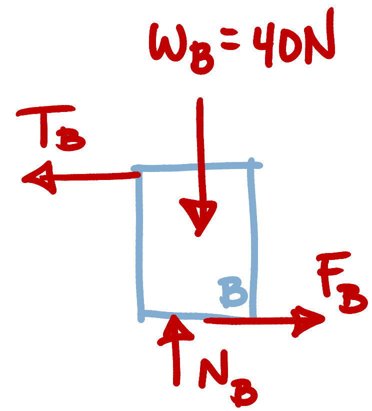
$$r_f = r\sin\phi$$



Example:

Determine the minimum force P needed to move block A (W_A = 100N), and whether block B (W_B = 40N) slips or tips. 2 cases

Case 1: Slip between A & B $\Rightarrow F_B = \mu N_B$



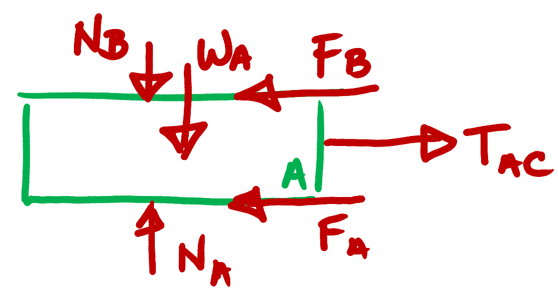
B $\Sigma F_y = 0 \Rightarrow N_B = W_B = 40\,N$

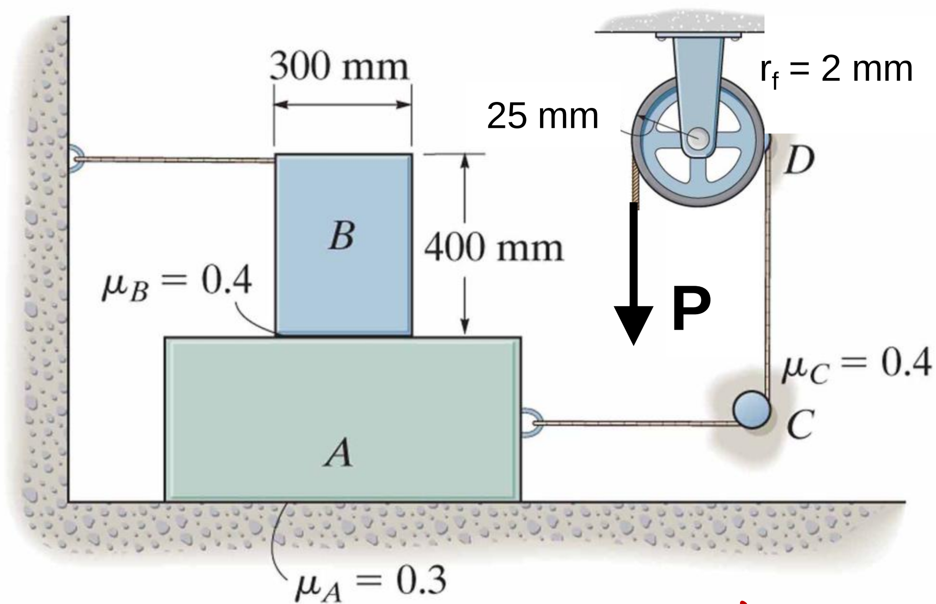
$F_B = \mu_B N_B = 16\,N$

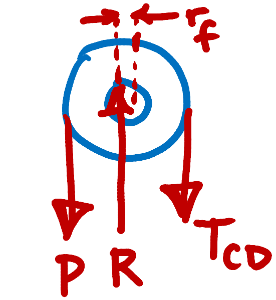
A $\Sigma F_y = 0 \Rightarrow N_A = N_B + W_A = 140\,N$

$F_A = \mu_A N_A = 42\,N$

$\Sigma F_x = 0 \Rightarrow T_{AC} = F_A + F_B = 58N$







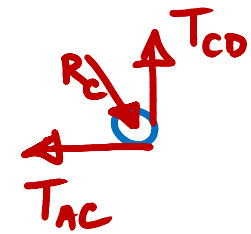
(C) $T_{CD} > T_{AC}$

$$T_{CD} = T_{AC}\, e^{\mu\beta}$$

$$= 58\,N\, e^{(0.4)(\pi/2)}$$

$$\Rightarrow T_{CD} = 108.7\,N$$

(D) $\Sigma M_R = 0 \Rightarrow$

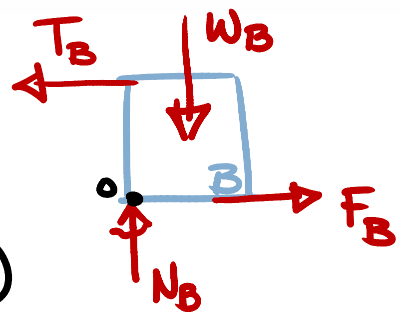


$$P(r_p - r_f) = T_{CD}(r_p + r_f)$$

$$\Rightarrow P = (108.7)\left(\frac{25+2}{25-2}\right)$$

$$\Rightarrow P = 128\,N$$

Case 2: B tips



(B) $\Sigma M_O = 0$

$$= T_B(400) - W_B(150)$$

$$\Rightarrow T_B = 15\,N$$

$$\Sigma F_x = 0 \Rightarrow F_B = 15\,N$$

(A) Previous eqns $\Rightarrow T_{AC} = 57\,N \overset{(C)}{\Rightarrow} T_{CD} = 106.8\,N \overset{(D)}{\Rightarrow}$ $\underline{P = 125\,N}$

Case 2 has smaller P
$\Rightarrow$ happens first
$\Rightarrow$ B tips