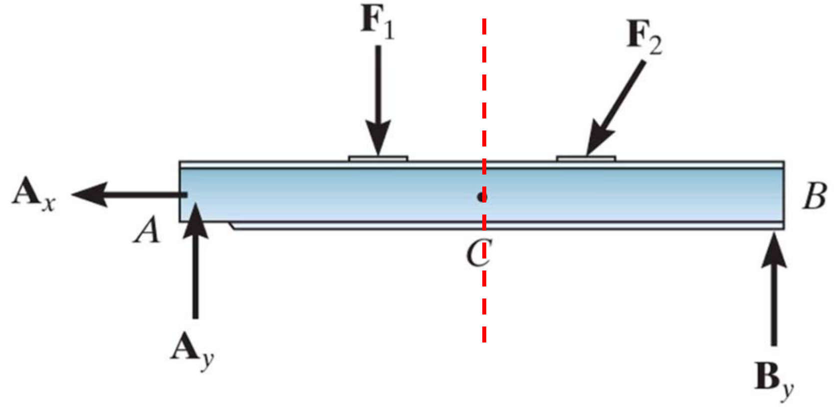
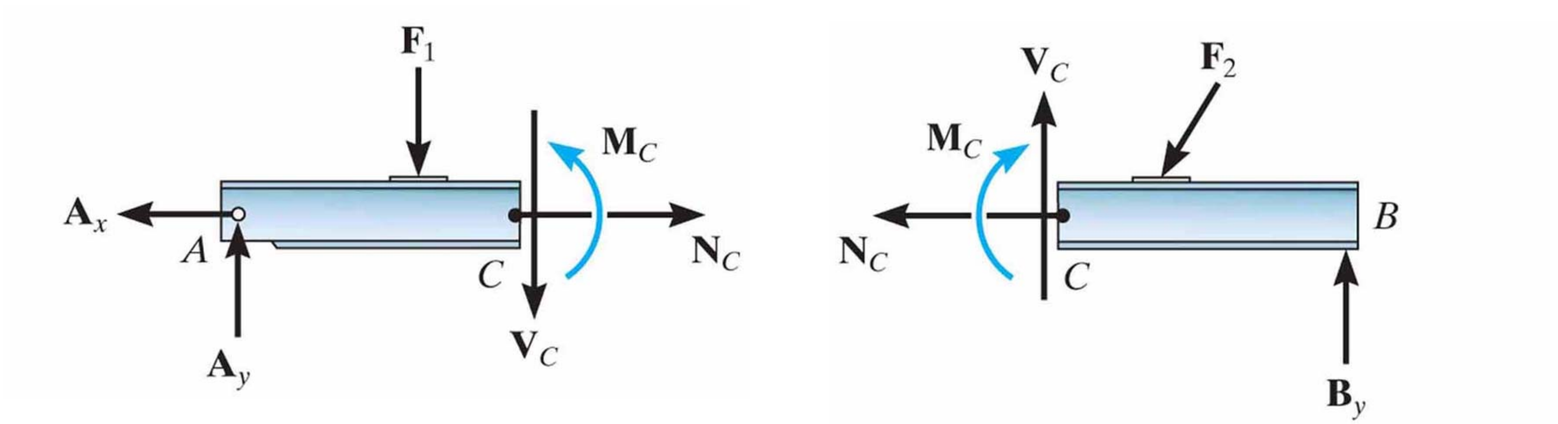
Internal Forces (Ch 7)



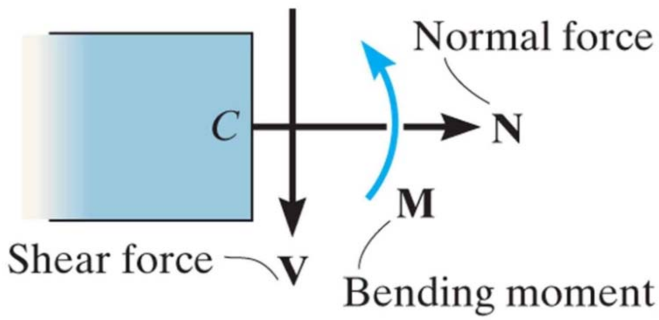


Section individual member at desired point

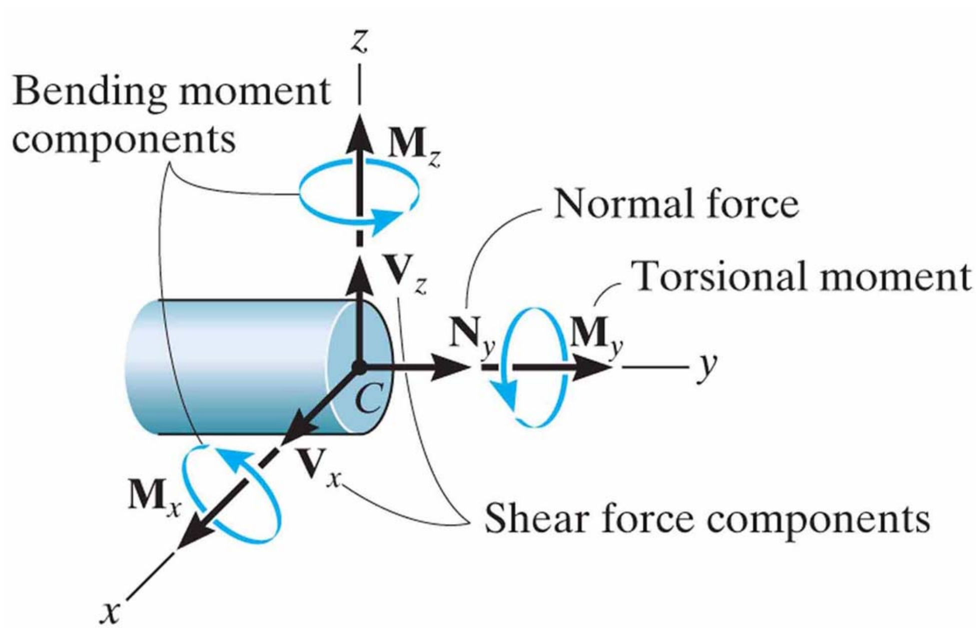
Types of Internal Forces

For 2D systems

(coplanar Fs, z-symmetry)



For 3D systems

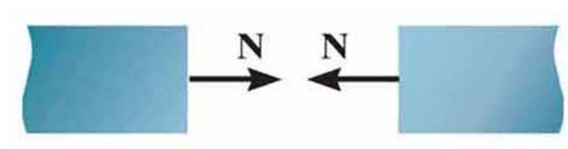


Sign Conventions

Axial Force, N:

Positive – tension

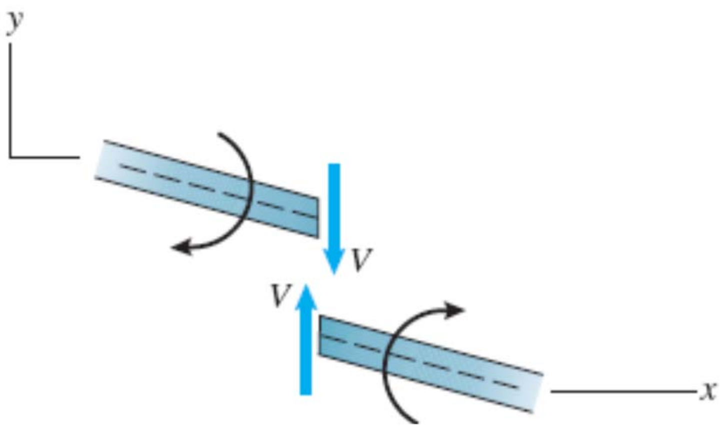
Negative – compression

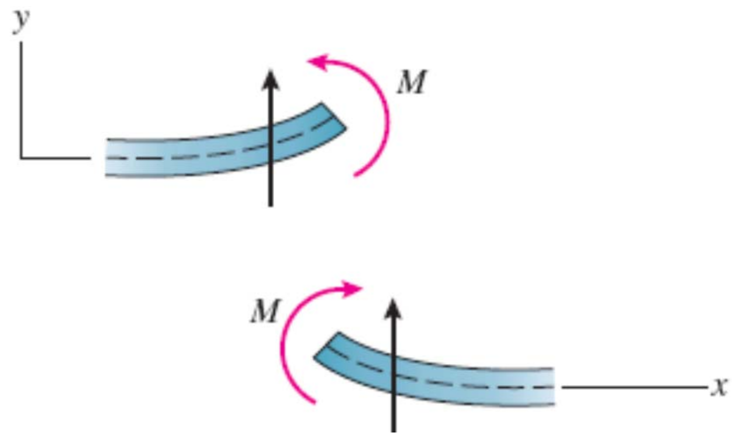


Shear Force, V:

Positive – causes beam axis to rotate clockwise

Negative – causes beam axis to rotate counterclockwise



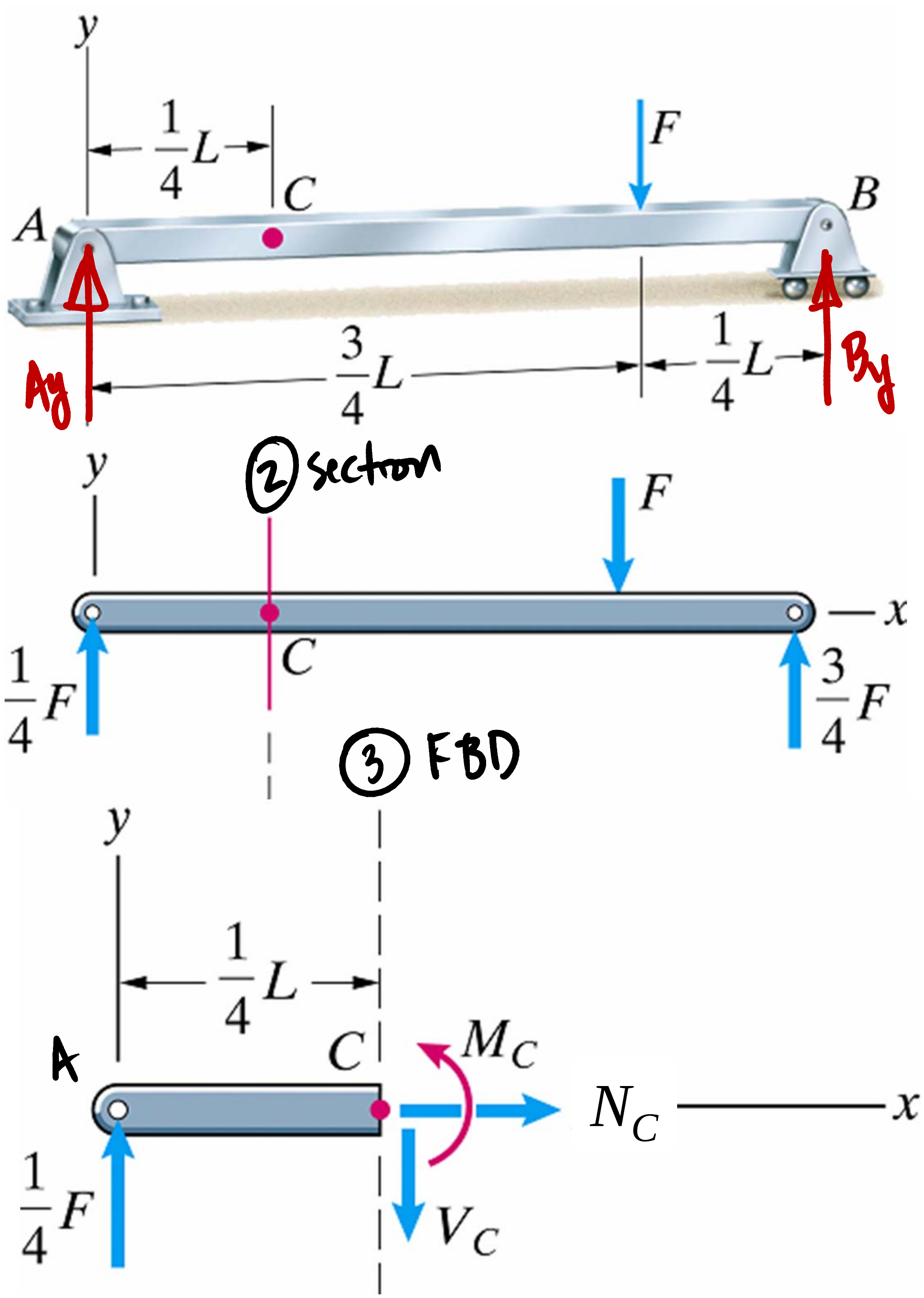


Bending Moment, M:

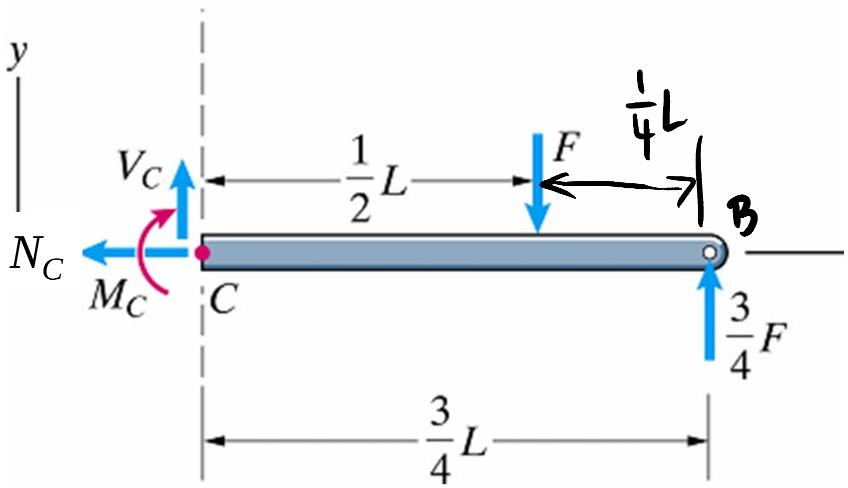
Positive – bends beam concave up ("smile")

Negative – bends beam concave down ("frown")

Example1. Find the internal forces and moment at C.



Take the right part.



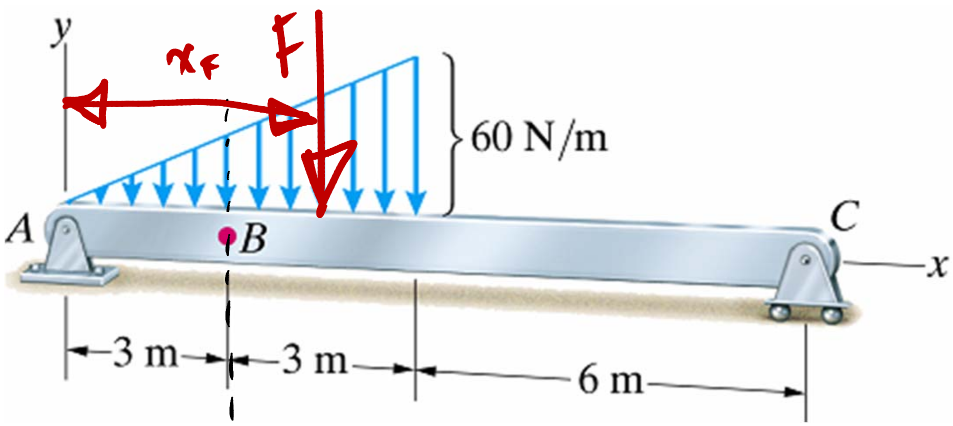
$$\sum F_x = 0 \Rightarrow N_c = 0 \checkmark$$

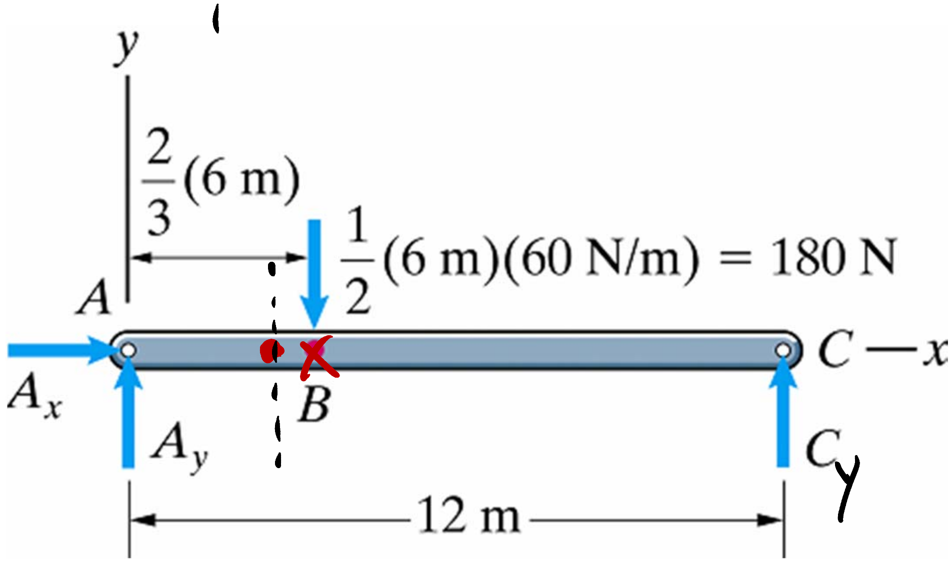
$$\sum F_y = 0 \Rightarrow V_c - F + \frac{3}{4}F$$

$$\Rightarrow V_c = \frac{1}{4}F \checkmark$$

$$\sum M_B = 0 = F\left(\frac{1}{4}L\right) - V_c\left(\frac{3}{4}L\right) - M_c \Rightarrow M_c = \frac{1}{4}FL - \left(\frac{1}{4}F\right)\left(\frac{3}{4}L\right) = \frac{1}{16}FL \checkmark$$

Different FBD’s give same magnitude and sign of internal forces and bending moment.





Example2. Find the internal forces and moment at B.

① find equiv F of distr. load

$$F = \text{area} = \frac{1}{2} bh = \frac{1}{2}(6\text{m})(60\tfrac{\text{N}}{\text{m}})$$

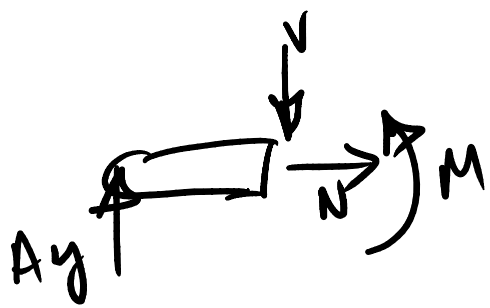
$$x_F = \text{centroid} = \frac{2}{3}(6\text{m}) = 4\text{m}$$

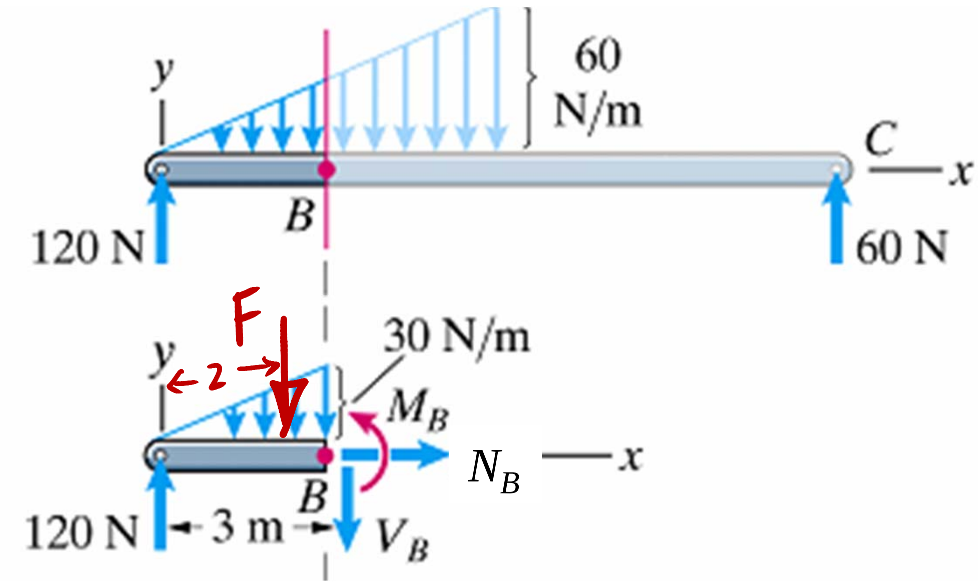
② Find Rxns

$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

$$\Sigma M_A = 0 = -4\text{m}(180\text{N}) + 12\text{m}\, C_y \Rightarrow C_y = 60\text{ N}$$

$$\Sigma F_y = 0 = A_y + F_c - 180\text{N} \Rightarrow A_y = 120\text{ N}$$



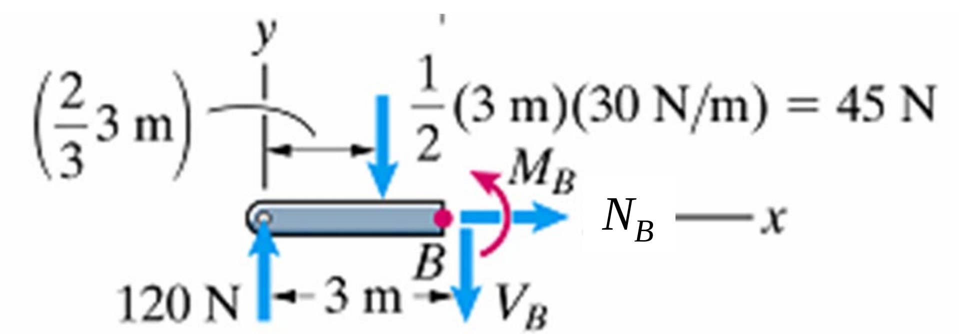


$$\Sigma F_x = 0 \Rightarrow N_B = 0$$

$$F = \text{area} = \frac{1}{2} b h = \frac{1}{2}(3\text{m})(30\,\tfrac{\text{N}}{\text{m}})$$

$$F = 45\,\text{N}$$

$$x_F = \text{centroid} = \frac{2}{3}(3\text{m}) = 2\text{m}$$



$$\Sigma F_y = 0 = 120\text{N} - 45\text{N} - V_B$$

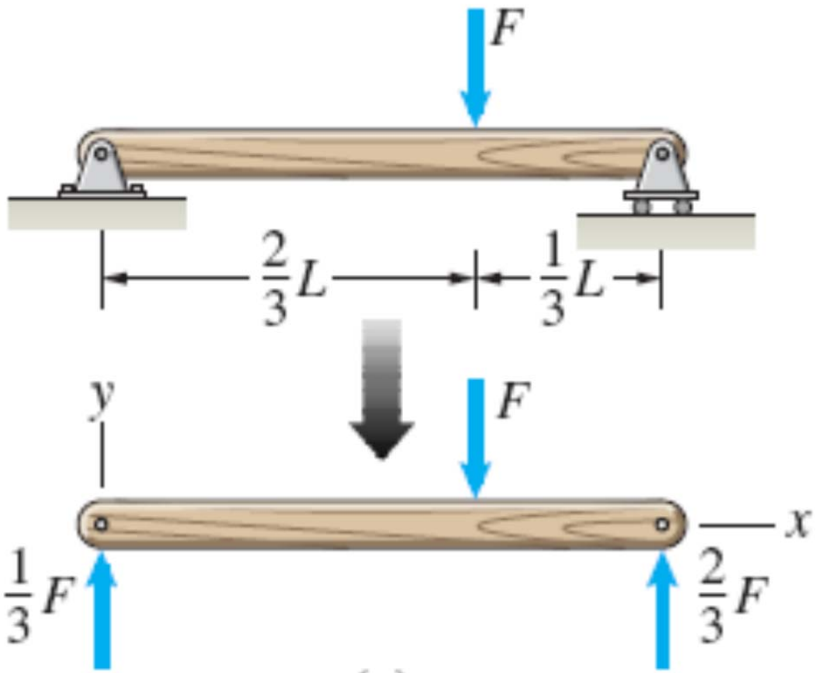
$$\Rightarrow V_B = 75\,\text{N}$$

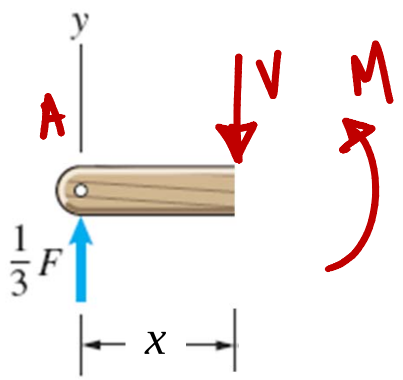
$$\Sigma M_A = 0 = -45\text{N}(2\text{m}) - V_B(3\text{m}) + M_B$$

$$\Rightarrow M_B = 315\,\text{Nm}$$

Shear and Bending Moment Diagrams

Shear Diagram – graph of V vs. x (position along the beam).
Bending Moment Diagram – graph of M vs. x.
- Can be obtained by cutting beam at variable x-values.





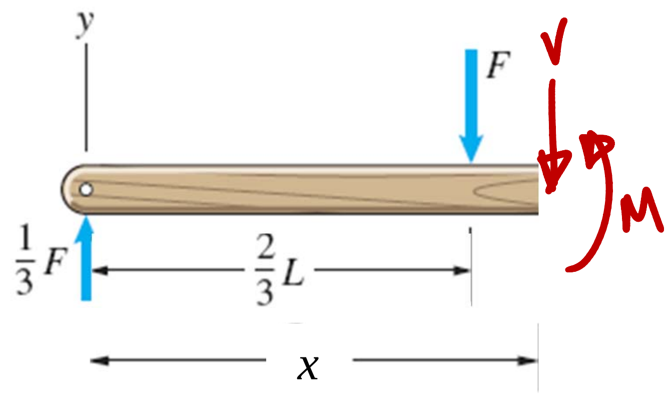
$0<x<\frac{2}{3}L$

$\sum F_y = 0 = \frac{1}{3}F - V \Rightarrow V = \frac{1}{3}F$

$\sum M_A = 0 = M - Vx \Rightarrow M = \frac{1}{3}Fx$

$\frac{2}{3}L<x<L$

@ $x=\frac{2}{3}L$, $M=\frac{2}{9}FL$

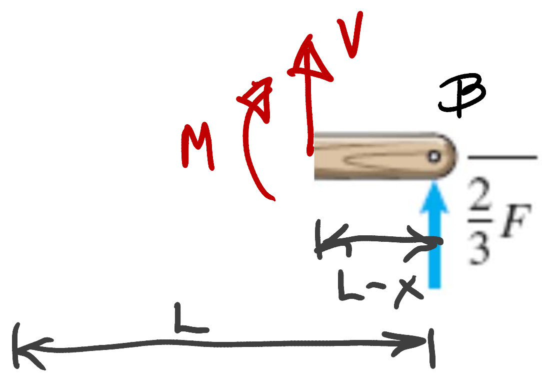


$\sum F_y = 0 = \frac{1}{3}F - F - V \Rightarrow V = -\frac{2}{3}F$

$\sum M_A = 0 = -F\left(\frac{2}{3}L\right) - Vx + M$

$\Rightarrow M = \frac{2}{3}FL - \frac{2}{3}Fx \Rightarrow M = \frac{2}{3}F(L-x)$

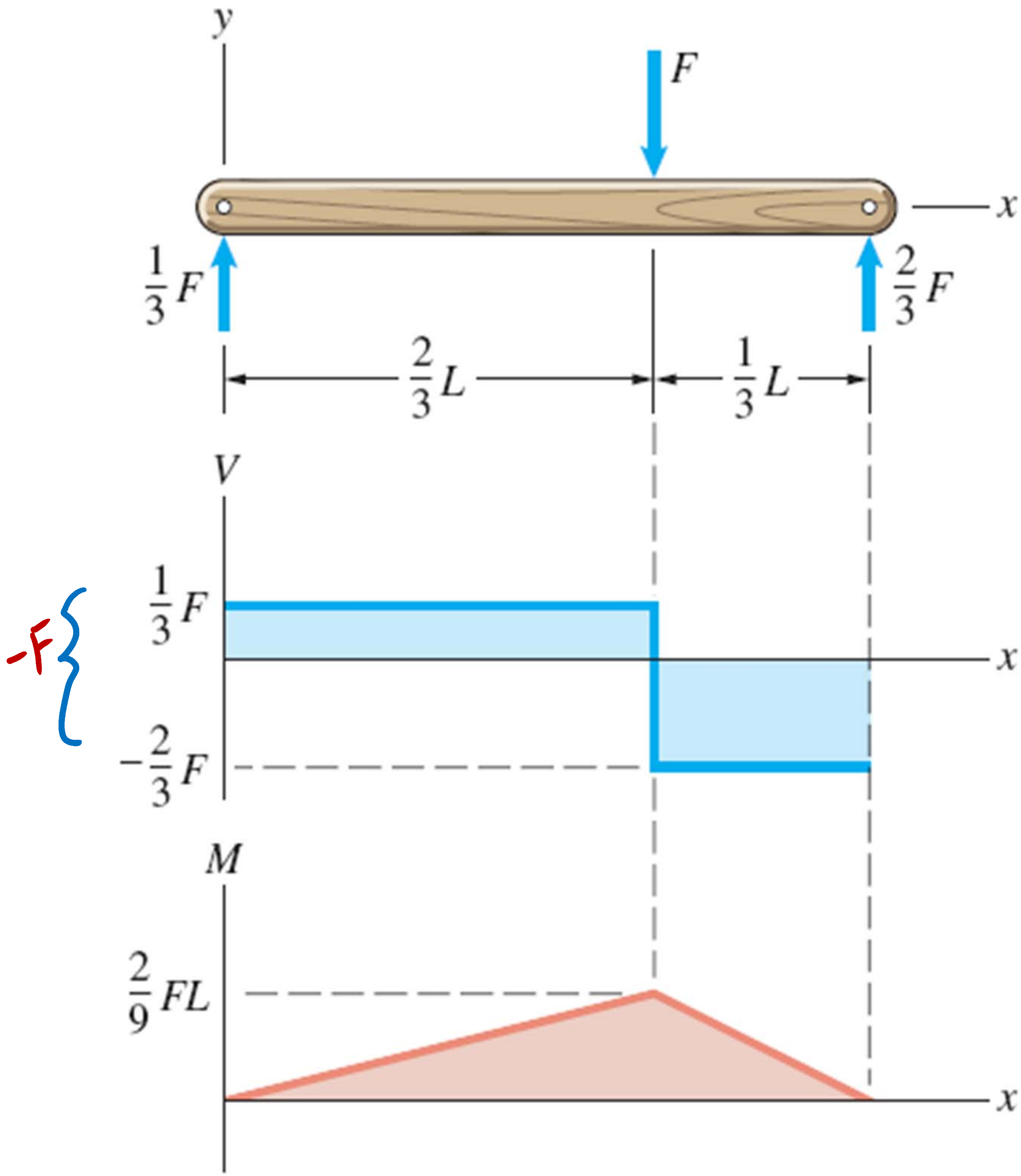
@ $x=\frac{2}{3}L \Rightarrow M = \frac{2}{9}FL$ ✓



$\sum F_y = 0 \Rightarrow V = -\frac{2}{3}F$ ✓

$\sum M_B = 0 = -M - V(L-x) \Rightarrow M = \frac{2}{3}F(L-x)$ ✓

For Concentrated (Point) Load

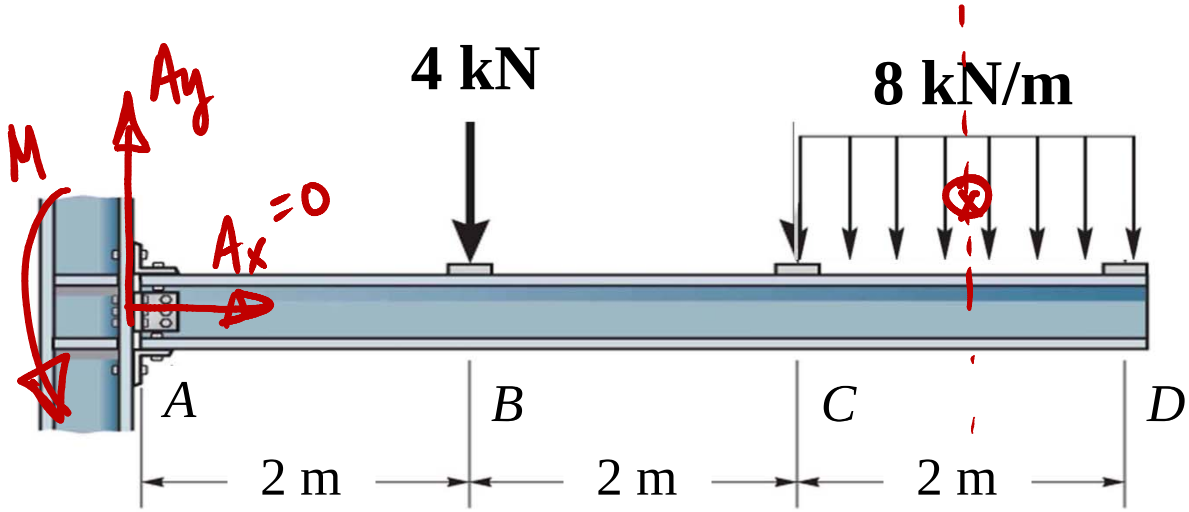


V discontinuous

jumps F

M continous

but changes slope

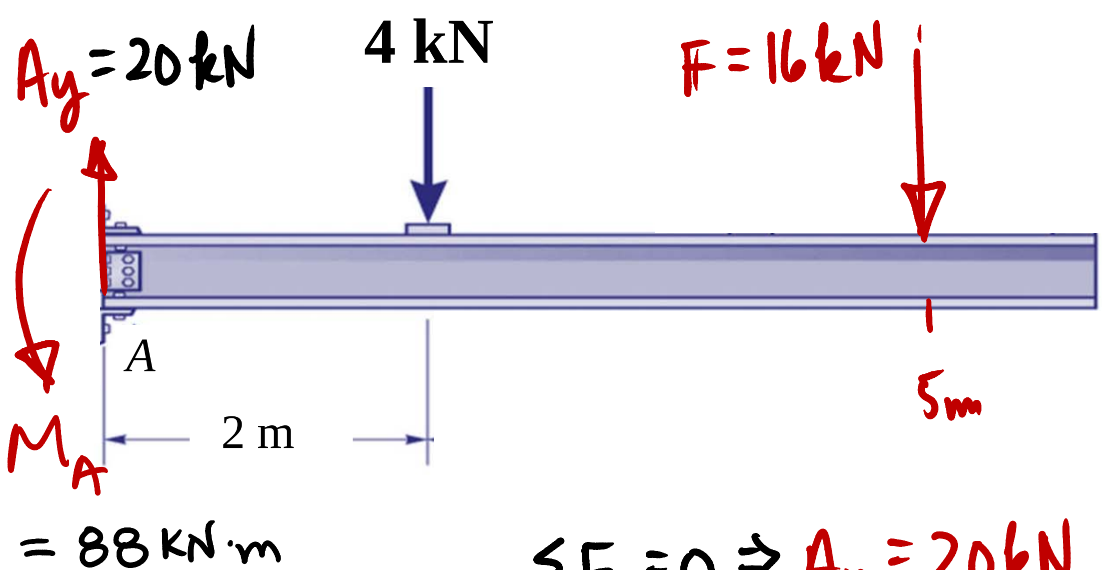


Ex. Draw the shear and bending moment diagrams.

Dist. Load

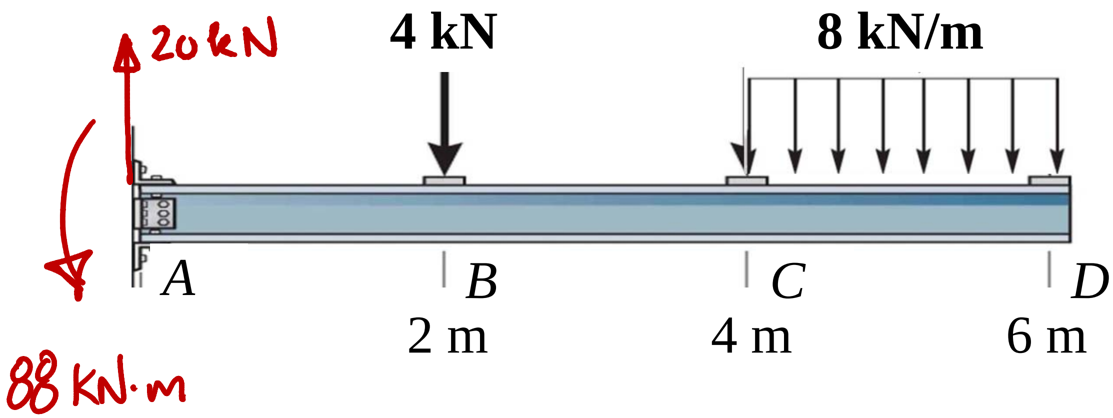
$F = bh = 8(2) = 16\ \text{kN}$

$x_F = \text{midpoint}$

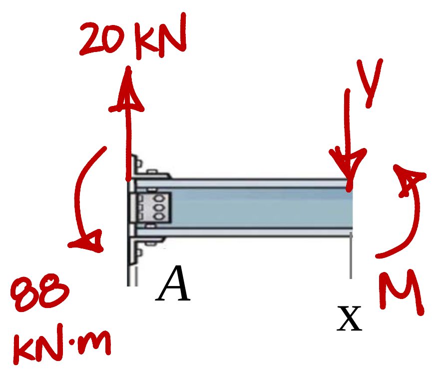


$\Sigma F_y = 0 \Rightarrow A_y = 20\ \text{kN}$

$\Sigma M_A = 0 = -4(2) - 16(5) + M_A \Rightarrow M_A = 88\ \text{kNm}$



$0 < x < 2m$

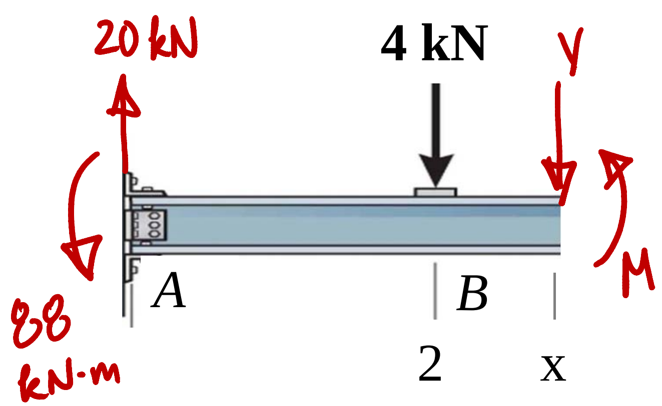


$\Sigma F_y = 0 = 20kN - V \Rightarrow V = 20kN$

$\Sigma M_A = 0 = 88 - Vx + M \Rightarrow M = -88 + 20x$

$M_{x=2} = -48 \text{ kN·m}$

$2 < x < 4m$

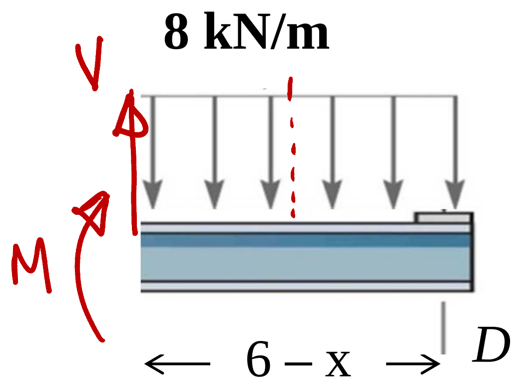


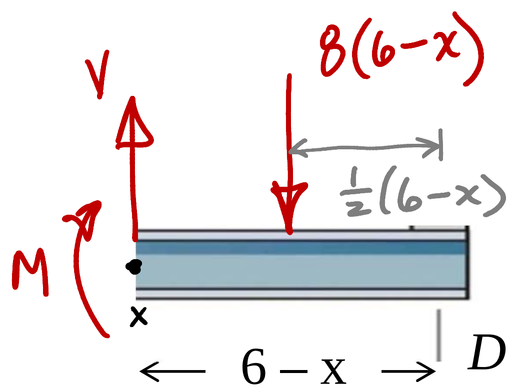
$\Sigma F_y = 0 = 20 - 4 - V \Rightarrow V = 16 \text{ kN}$

$\Sigma M_A = 0 = 88 - 4(2) - Vx + M$

$\Rightarrow M = -80 + 16x$

$M_{x=2} = -48 \text{ kN·m}$ $\quad M_{x=4} = -16 \text{ kN·m}$





$4 < x < 6\,\text{m}$

$$\Sigma F_y = 0 = V - 8(6-x)$$

$$\Rightarrow V = 48 - 8x$$

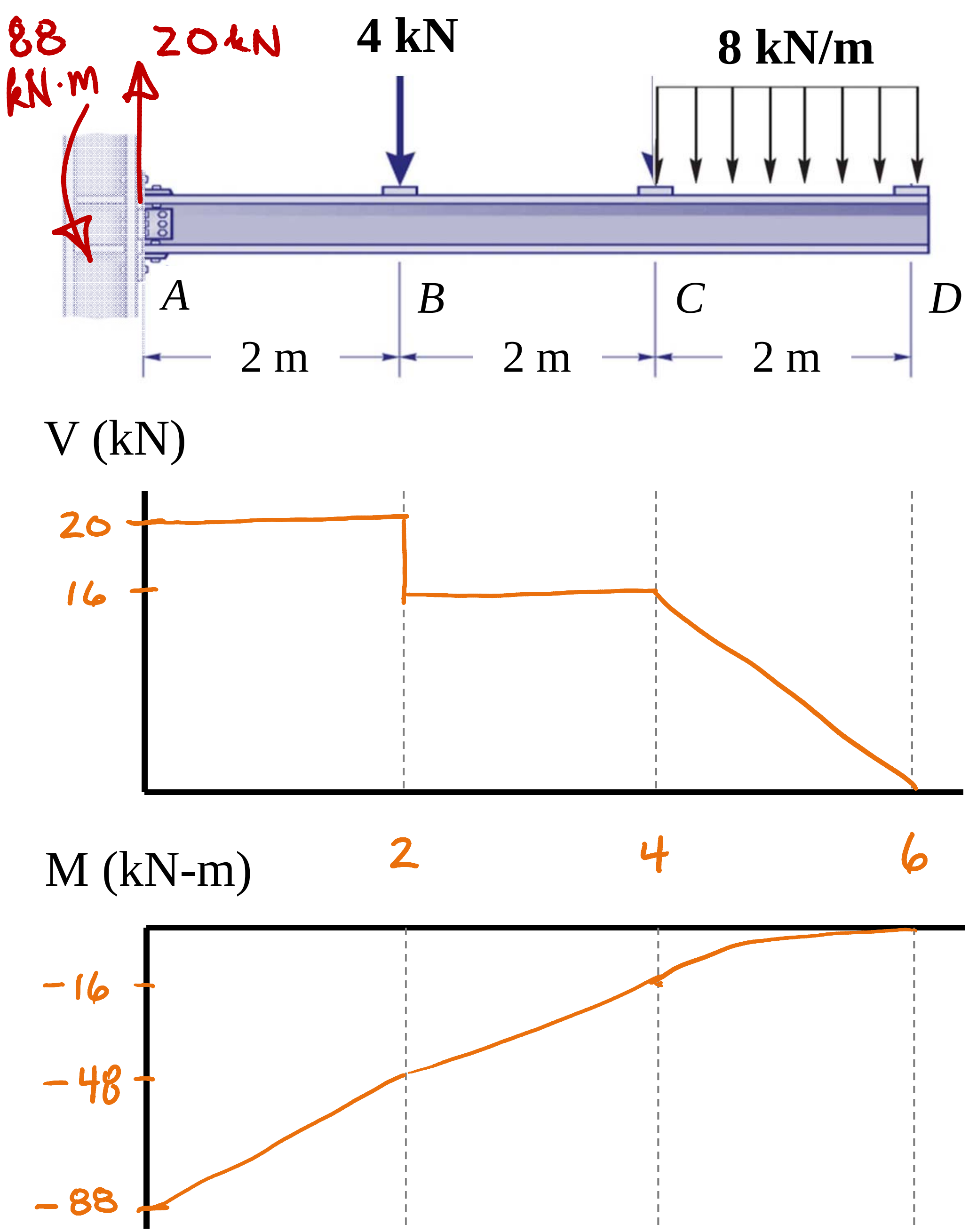
$$\Sigma M_x = 0 = -M - 8(6-x)\left(\tfrac{1}{2}\right)(6-x)$$

$$\Rightarrow M = -4(6-x)^2$$

$$M_{x=4} = -16\ \text{kN}\cdot\text{m}$$

$$M_{x=6} = 0$$

Shear and Moment Diagrams



$0 < x < 2\text{m}$ $V = 20\text{kN}$

$2 < x < 4\text{m}$ $V = 16\text{ kN}$

$4 < x < 6\text{m}$ $V = 48 - 8x$

$0 < x < 2\text{m}$ $M = -88 + 20x$

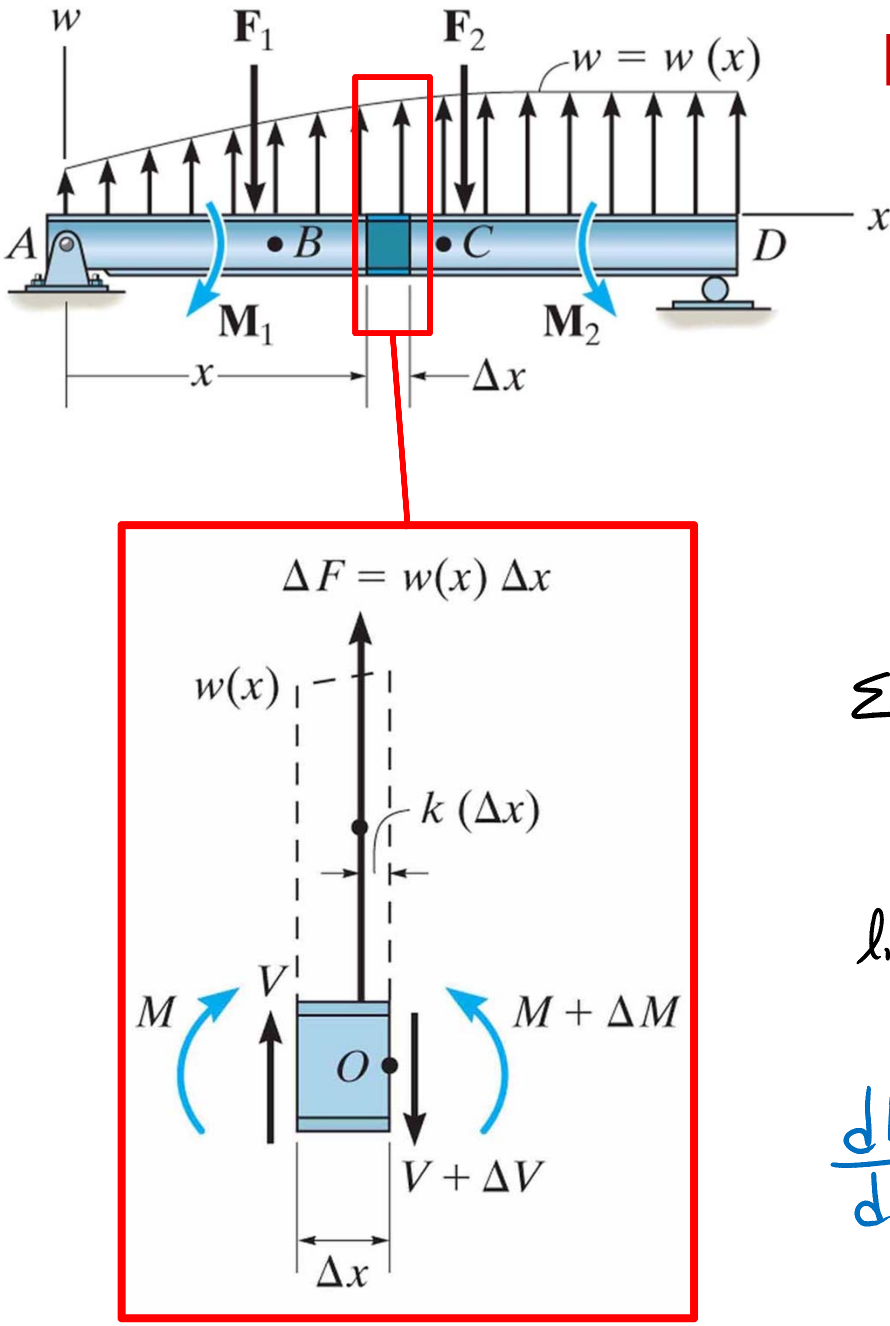
$M_{x=2} = -48\text{ kN·m}$

$2 < x < 4\text{m}$ $M = -80 + 16x$

$M_{x=4} = -16\text{ kN·m}$

$4 < x < 6\text{m}$ $M = -4(6-x)^2$

$M_{x=6} = 0$



Relations between w, V, M

$$\sum F_y = 0 = V - (V + \Delta V) + w(x)\Delta x$$

$$\Rightarrow \Delta V = w(x)\Delta x \Rightarrow \frac{\Delta V}{\Delta x} = w(x)$$

$$\frac{dV}{dx} = w \qquad V_{x_2} = V_{x_1} + \int_{x_1}^{x_2} w(x)\,dx$$

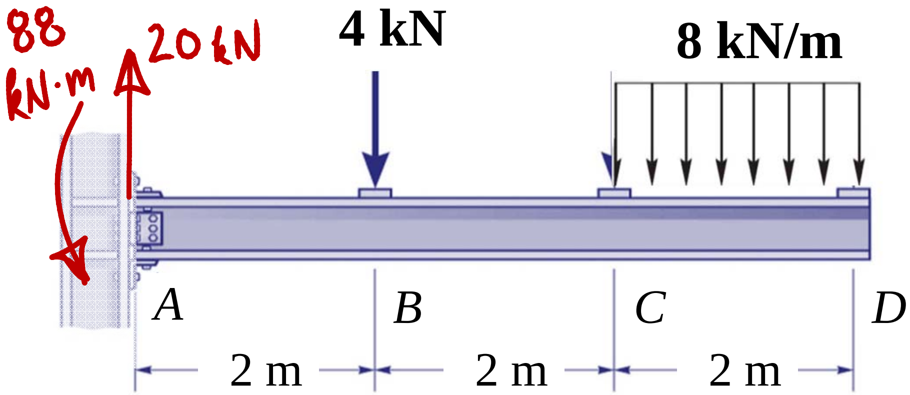
$$\sum M_o = 0 = -\cancel{M} + \cancel{M} + \Delta M - V\Delta x - \Delta F\, k\Delta x$$

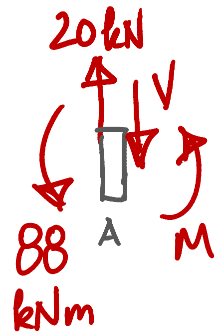
$$\Rightarrow \Delta M = V\Delta x + (w(x)\Delta x)\, k\Delta x$$

$$\lim_{\Delta x \to 0} \frac{\Delta M}{\Delta x} = V + k\,w(x)\,\cancelto{0}{\Delta x}$$

$$\frac{dM}{dx} = V \qquad M_{x_2} = M_{x_1} + \int_{x_1}^{x_2} V\,dx$$

Example 1: Relations Between w, V, M





@ $x=0$, $V=20$, $M=-88$

$0<x<2$ $\quad w=0=\frac{dV}{dx} \Rightarrow V=20=\frac{dM}{dx}$

$$M_2 = M_0 + \int_0^2 20\,dx = -48$$

@ $x=2$, V jumps $-4 \Rightarrow V=16$

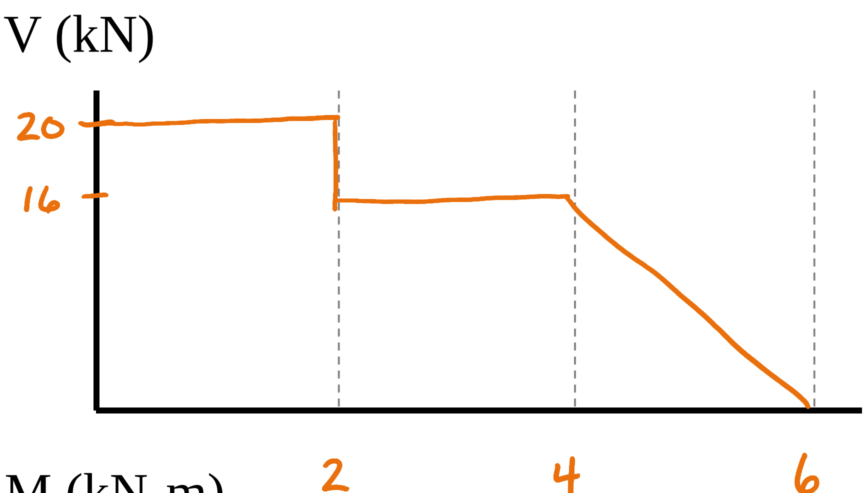
$2<x<4$ $\quad w=0=\frac{dV}{dx} \Rightarrow V=16=\frac{dM}{dx}$

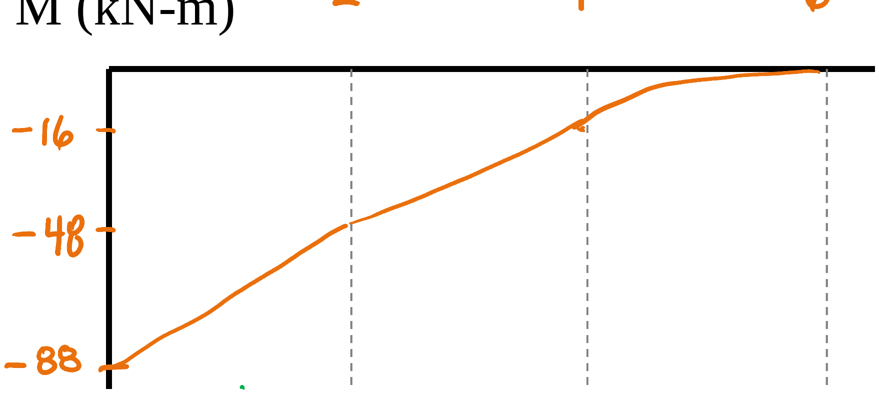
$$M_4 = M_2 + \int_2^4 16\,dx = -16$$

$4<x<6$ $\quad w=-8=dV/dx$ (decr. lin. to zero)

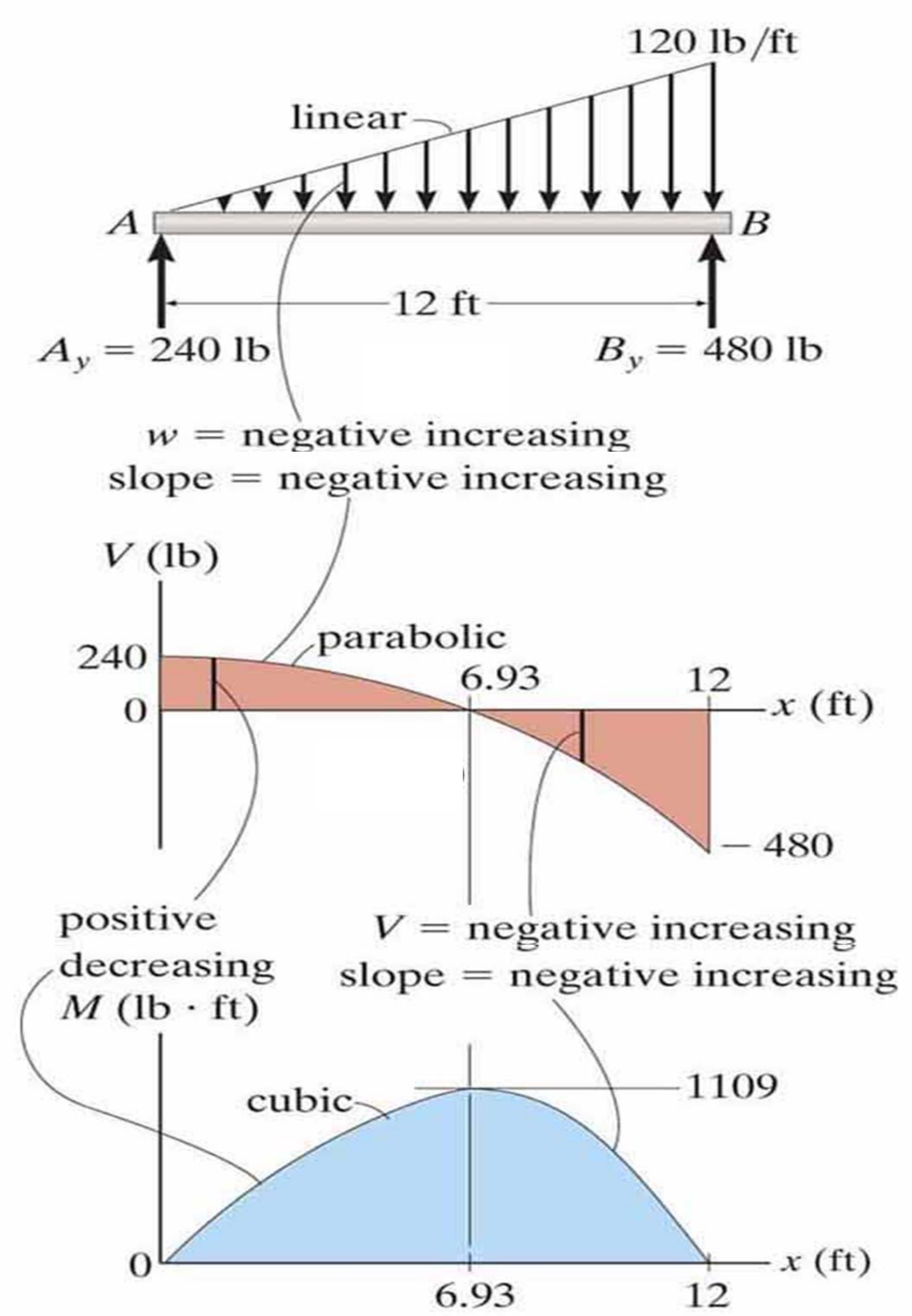
$$V(x) = V_4 + \int_4^x -8\,dx = dM/dx$$

$$M(x) = M_4 + \int_4^x V(x)\,dx$$ (incr. parab. to zero)





Example 2: Relations between w, V, M



$$w(x) = -\frac{120}{12}x + 0$$

$$w = -10x = \frac{dV}{dx}$$

$$\Rightarrow V = V_0 + \int_0^x -10x\,dx$$

$$V = 240 - 5x^2$$

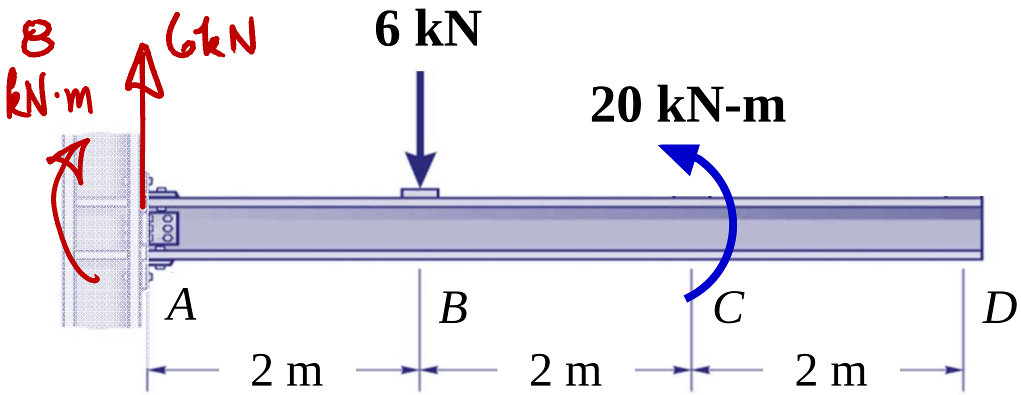
crosses axis @ $240 - 5x^2 = 0$

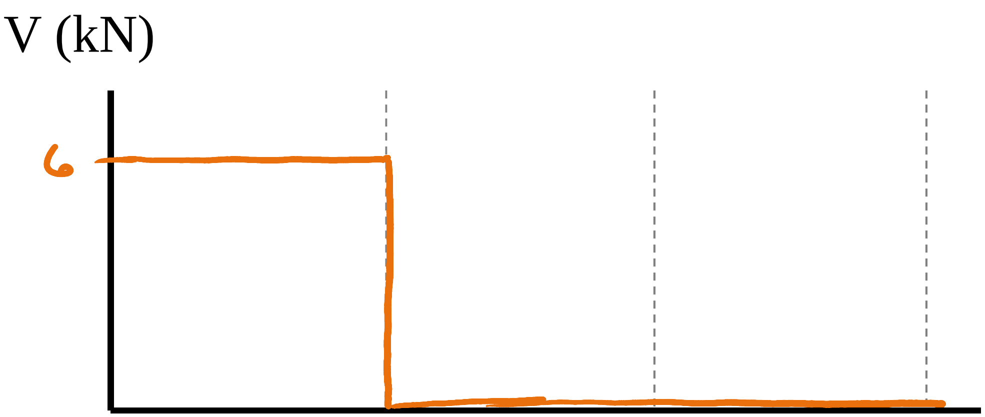
$$\Rightarrow x = \sqrt{240/5} = 6.93$$

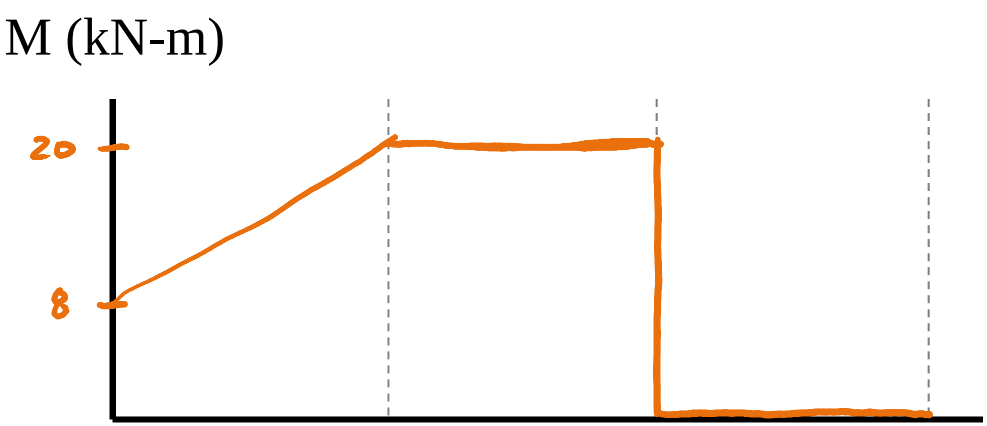
$$\frac{dM}{dx} = V = 240 - 5x^2$$

$$\Rightarrow M = \cancel{M_0}^{\,0} + 240x - \frac{5}{3}x^3$$

@ $x = 6.93 \Rightarrow M_{max} = 1109$







Effect of a Couple

$V_0 = +6 \quad M_0 = +8$

$0<x<2 \quad w=0 \Rightarrow V = \text{const} = 6$

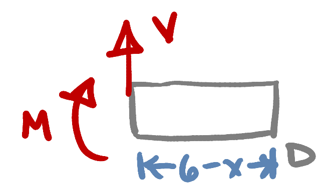
$\frac{dM}{dx} = 6 \Rightarrow M = 8 + 6x$

@ $x=2 \quad V$ jumps $-6 \Rightarrow V \to 0$

$2<x<4 \quad w=0 \Rightarrow V = \text{const} = 0$

$\frac{dM}{dx} = V = 0 \Rightarrow M = \text{const} = 20$

@ $x=4$?

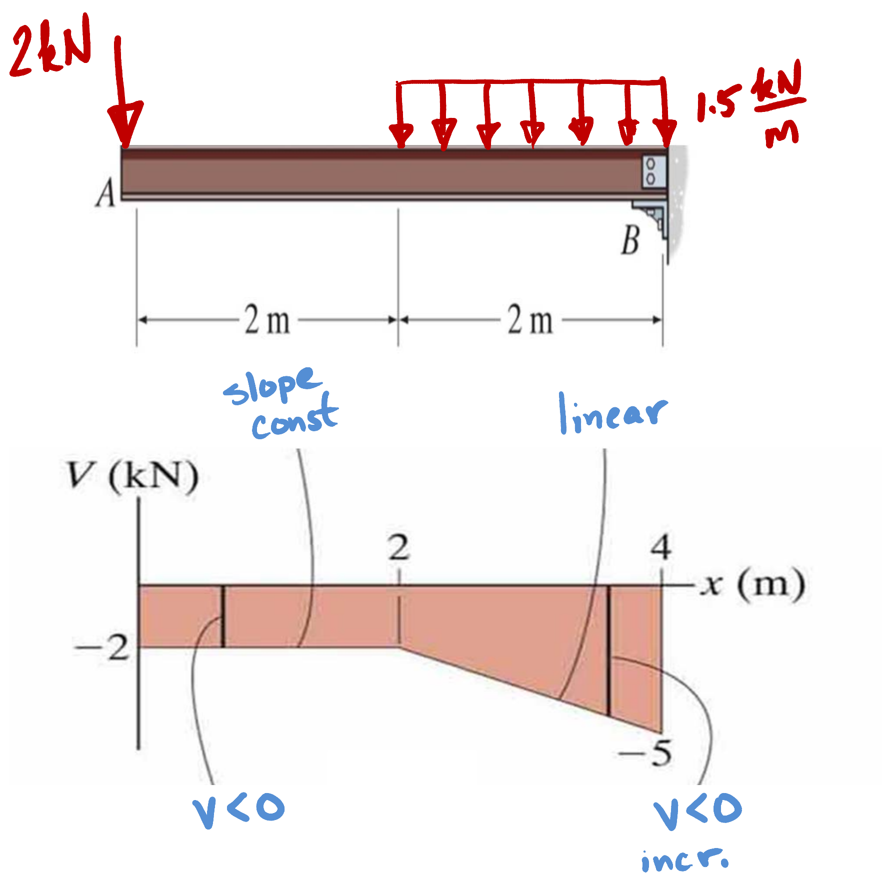


$\Sigma F_y = 0 \Rightarrow V = 0$

$\Sigma M_D = 0 \Rightarrow M = 0$

$\Rightarrow$ M jumps -20 @ $x=4$

$\Rightarrow$ M jumps $-C$



Example 3: Determine the Loading

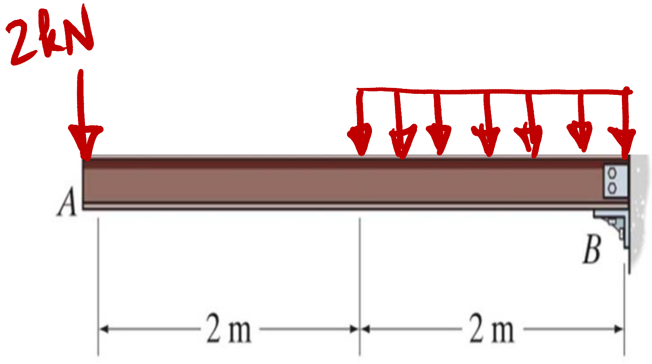
$0<x<2$ V const $\Rightarrow$ $w=0$

$2<x<4$ V linear $\Rightarrow$ w const.

slope neg $\Rightarrow$ $w<0$

$$\frac{dV}{dx} = -\frac{3}{2} = w$$

Couple? can't say since no effect on V



Example 4: Determine the Loading

$0 < x < 2$

M linear $\Rightarrow$ V const $\Rightarrow$ $w = 0$

$M = -2x \Rightarrow V = -2$

$2 < x < 4$

M parabolic $\Rightarrow$ V linear $\Rightarrow$ w const

$$\frac{dV}{dx} = w \quad \Rightarrow \quad V = V_2 + \int_2^x w \, dx$$

$$V = -2 + w(x-2) = \frac{dM}{dx}$$

$$M = M_2 + \int_2^x -2 + w(x-2) \, dx$$

integrate & use $M = -11$ @ $x = 4$ to solve for w