# FREE-BODY DIAGRAMS, EQUATIONS OF EQUILIBRIUM & CONSTRAINTS FOR A RIGID BODY

**Today's Objective:**

Students will be able to:

a) Identify support reactions in 3-D and draw a free body diagram, and,

b) apply the equations of equilibrium.

**In-Class Activities:**

- Check Homework, if any
- Reading Quiz
- Applications
- Support Reactions in 3-D
- Equations of Equilibrium
- Concept Quiz
- Group Problem Solving
- Attention quiz

# READING QUIZ

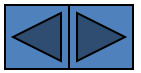
1. If a support prevents rotation of a body about an axis, then the support exerts a _____ on the body about that axis.

   A) couple moment  B) force

   C) Both A and B.  D) None of the above.

2. When doing a 3-D problem analysis, you have _____ scalar equations of equilibrium.

Enter a number corresponding to your answer.



# 3D Rigid Body Equilibrium (Ch 5)

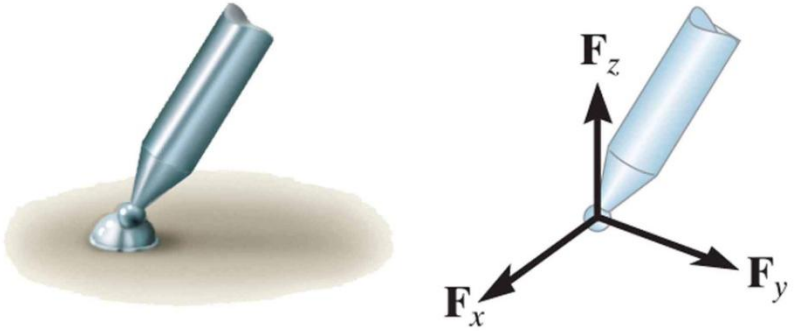
$$\sum \vec{F} = 0 \Rightarrow \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0$$

$$\sum \vec{M}_P = 0 \Rightarrow \sum M_{Px} = 0 \quad \sum M_{Py} = 0 \quad \sum M_{Pz} = 0$$

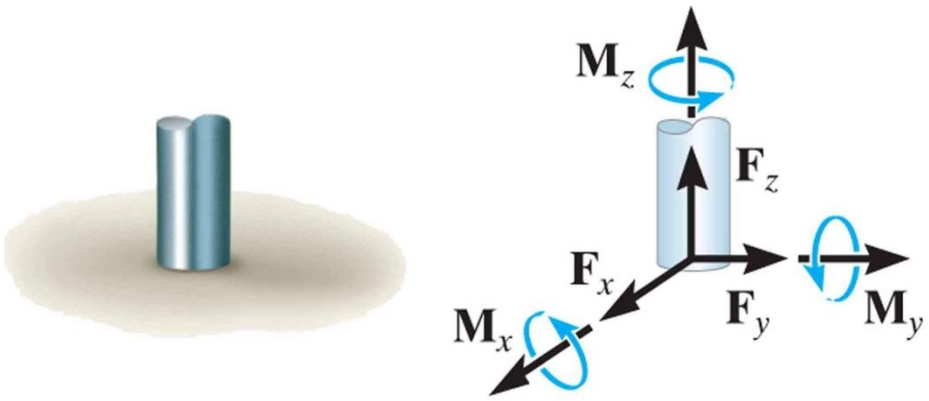
6 equations → **6 unknowns** that can be solved for a single rigid body in 3D

# Support Reactions in 3D


**roller or smooth surface**
single F normal to surface



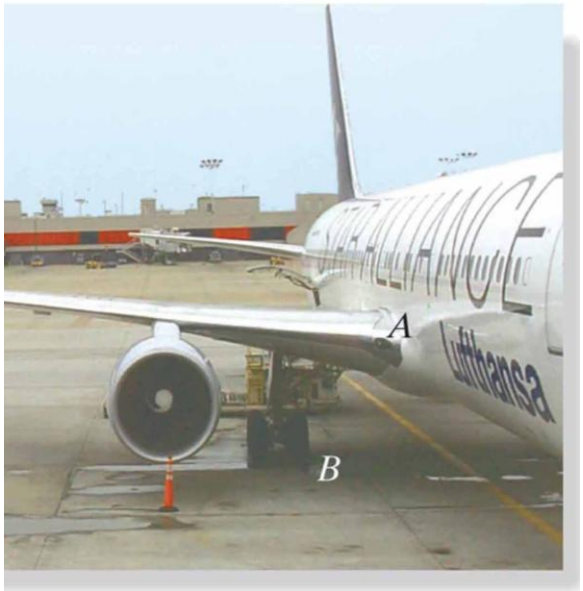

**Ball-and-socket joint**
$F_x$, $F_y$ and $F_z$



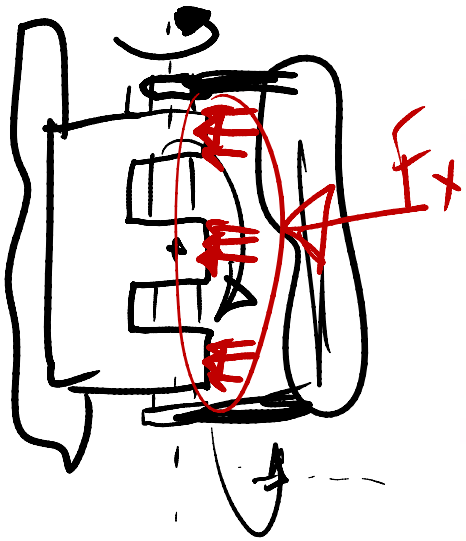

**fixed support**
$F_x$, $F_y$, $F_z$ and $M_x$, $M_y$, $M_z$

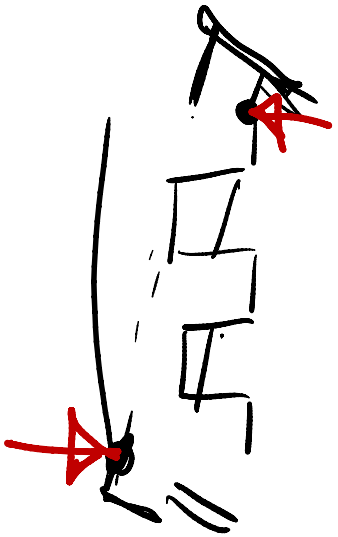
*more complete table in text

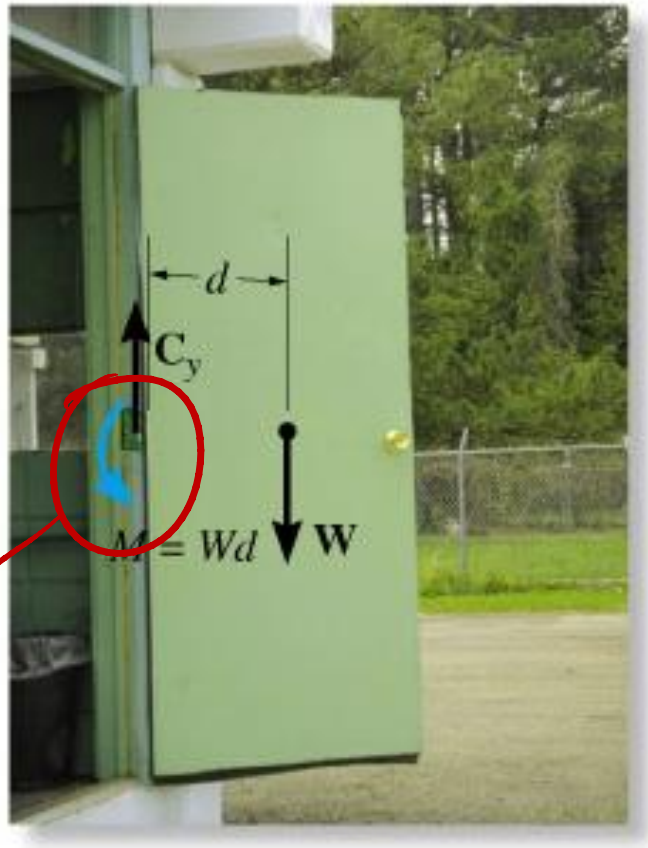
# Real Supports



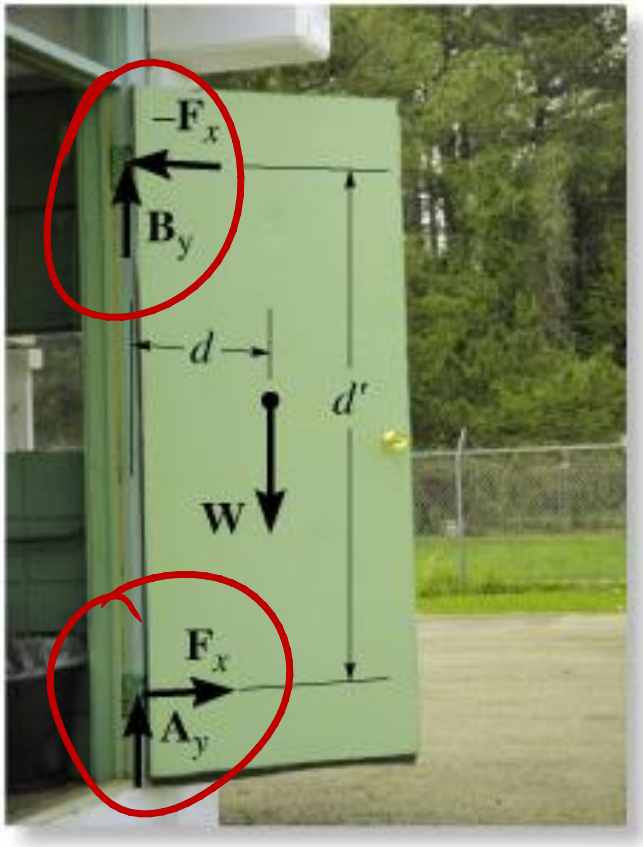
# The Door Hinge & "Proper Alignment"





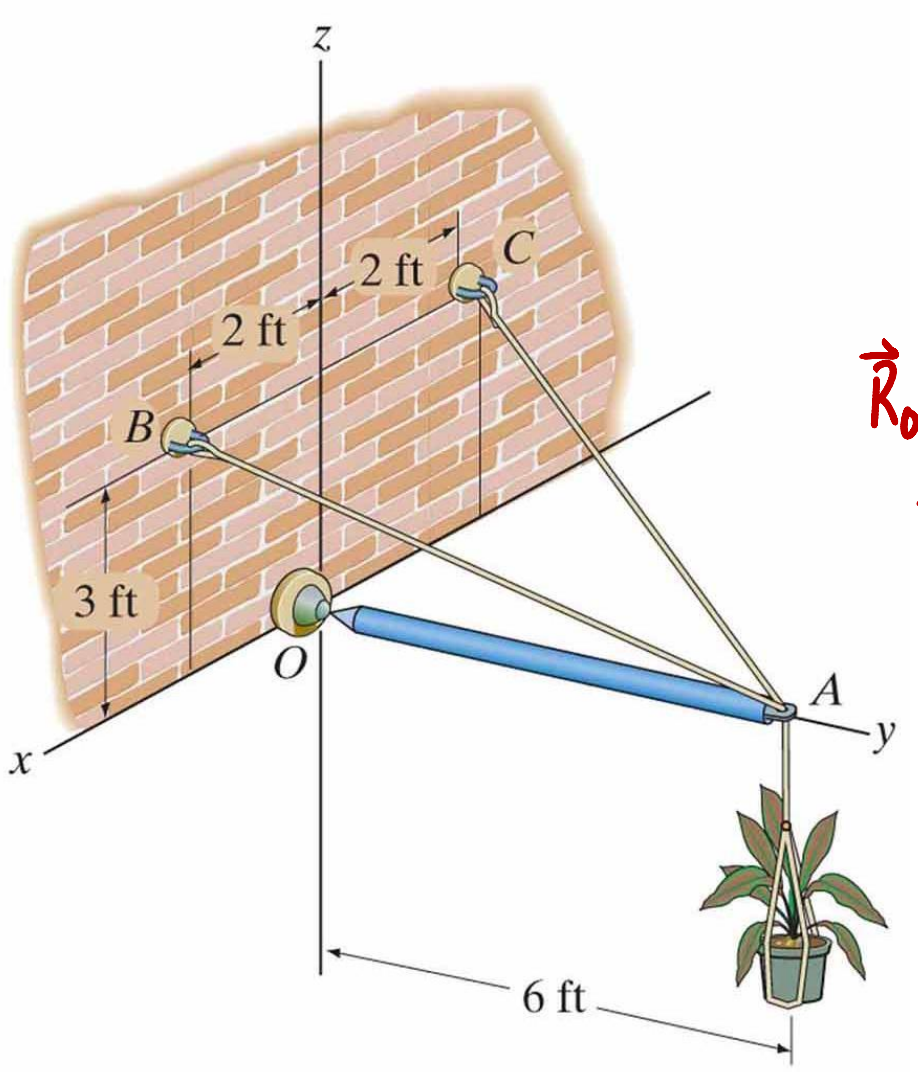


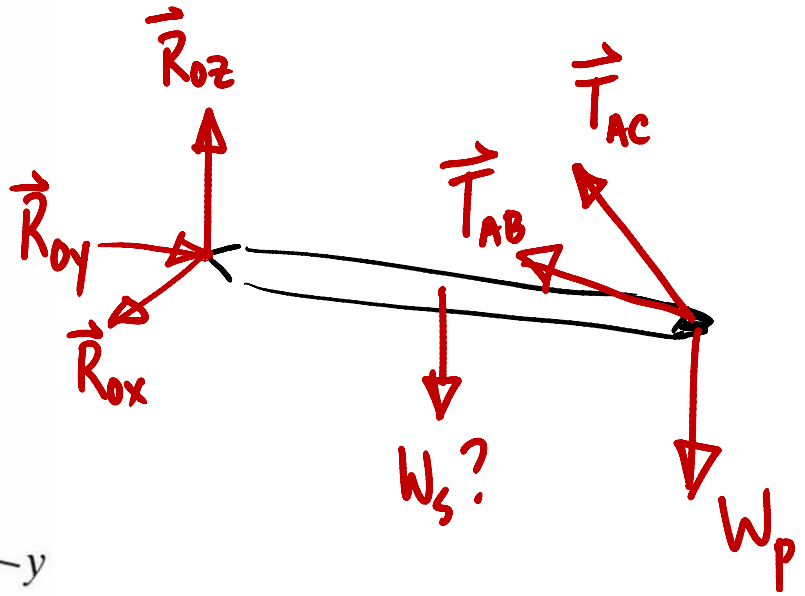

(a)



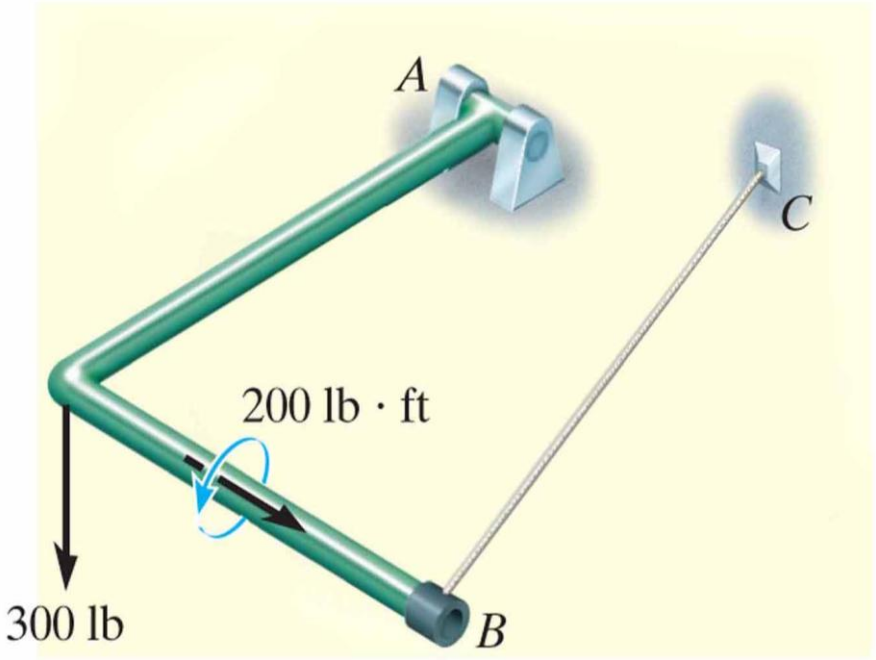

(b)

# FBD Example 1 in 3D

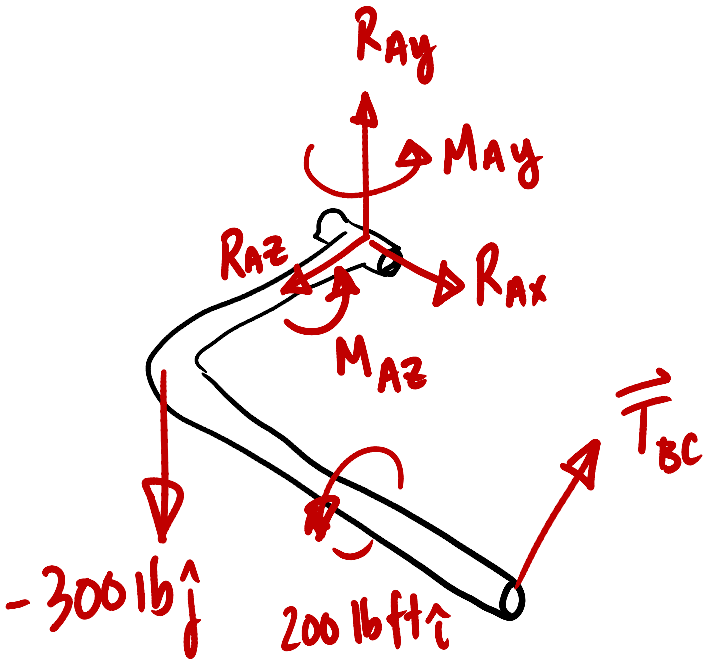


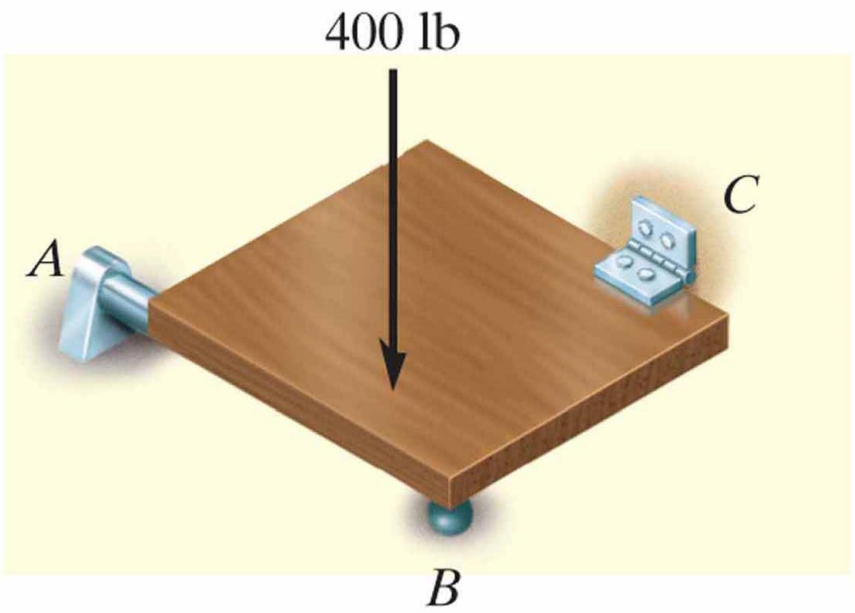
# FBD Example 2 in 3D



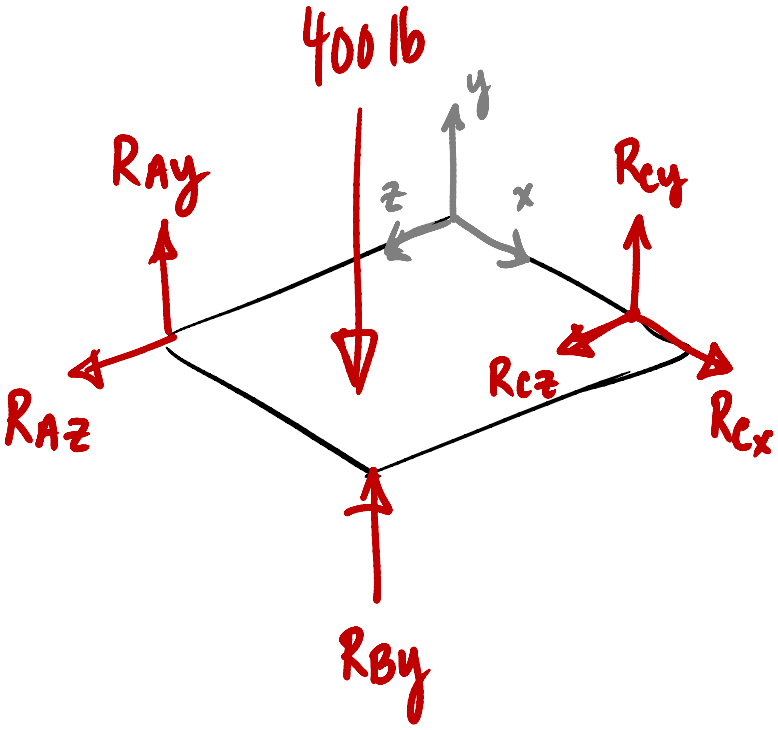

Pin at $A$ and cable $BC$.



# FBD Example 3 in 3D

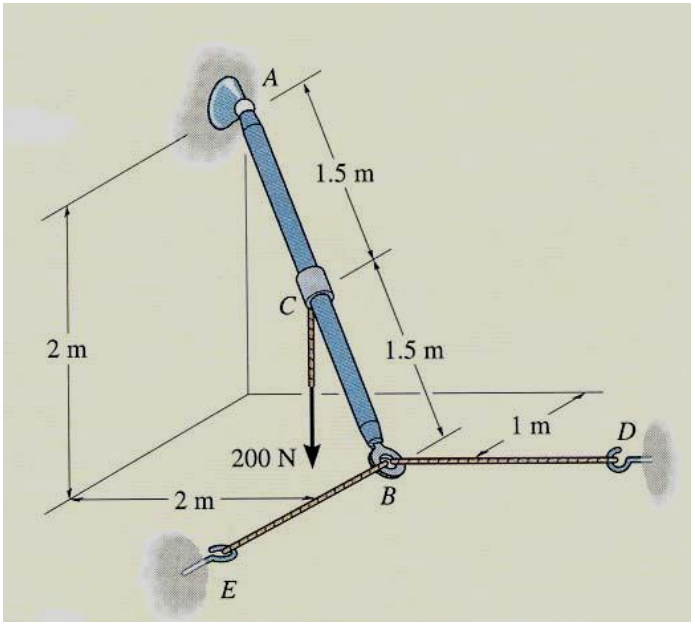



Properly aligned journal bearing at $A$ and hinge at $C$. Roller at $B$.



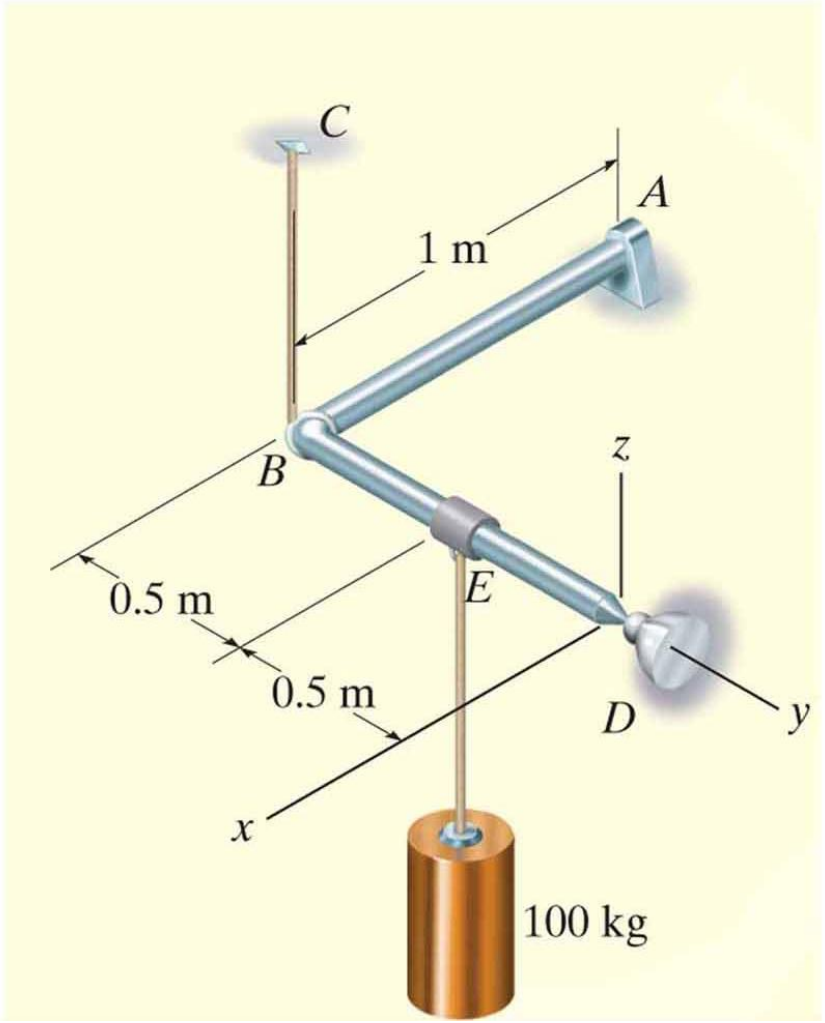
# CONCEPT QUIZ

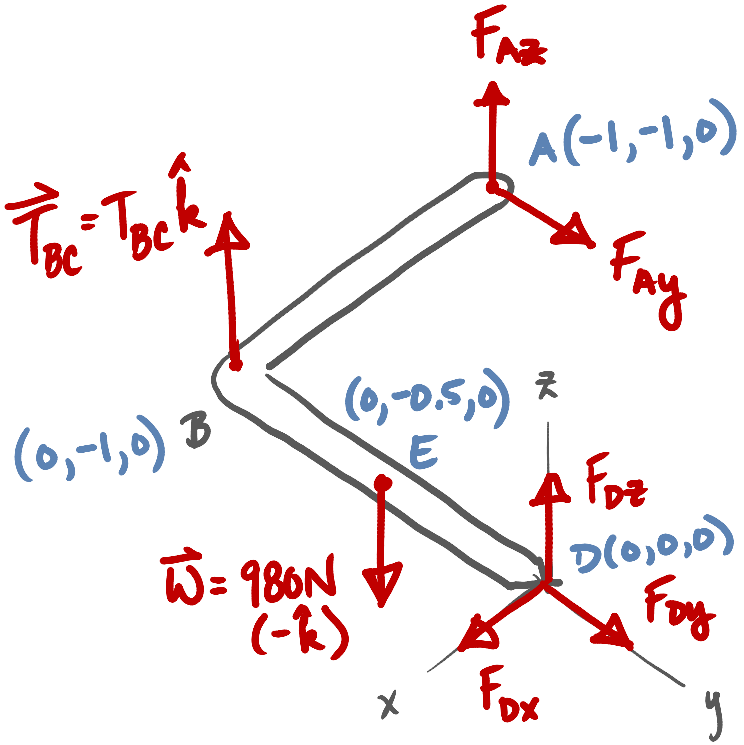
1. The rod AB is supported using two cables at B and a ball-and-socket joint at A. How many unknown support reactions exist in this problem?




   1) 5 force and 1 moment reaction
   2) 5 force reactions
   3) 3 force and 3 moment reactions
   4) 4 force and 2 moment reactions

# FBD Example 4 in 3D

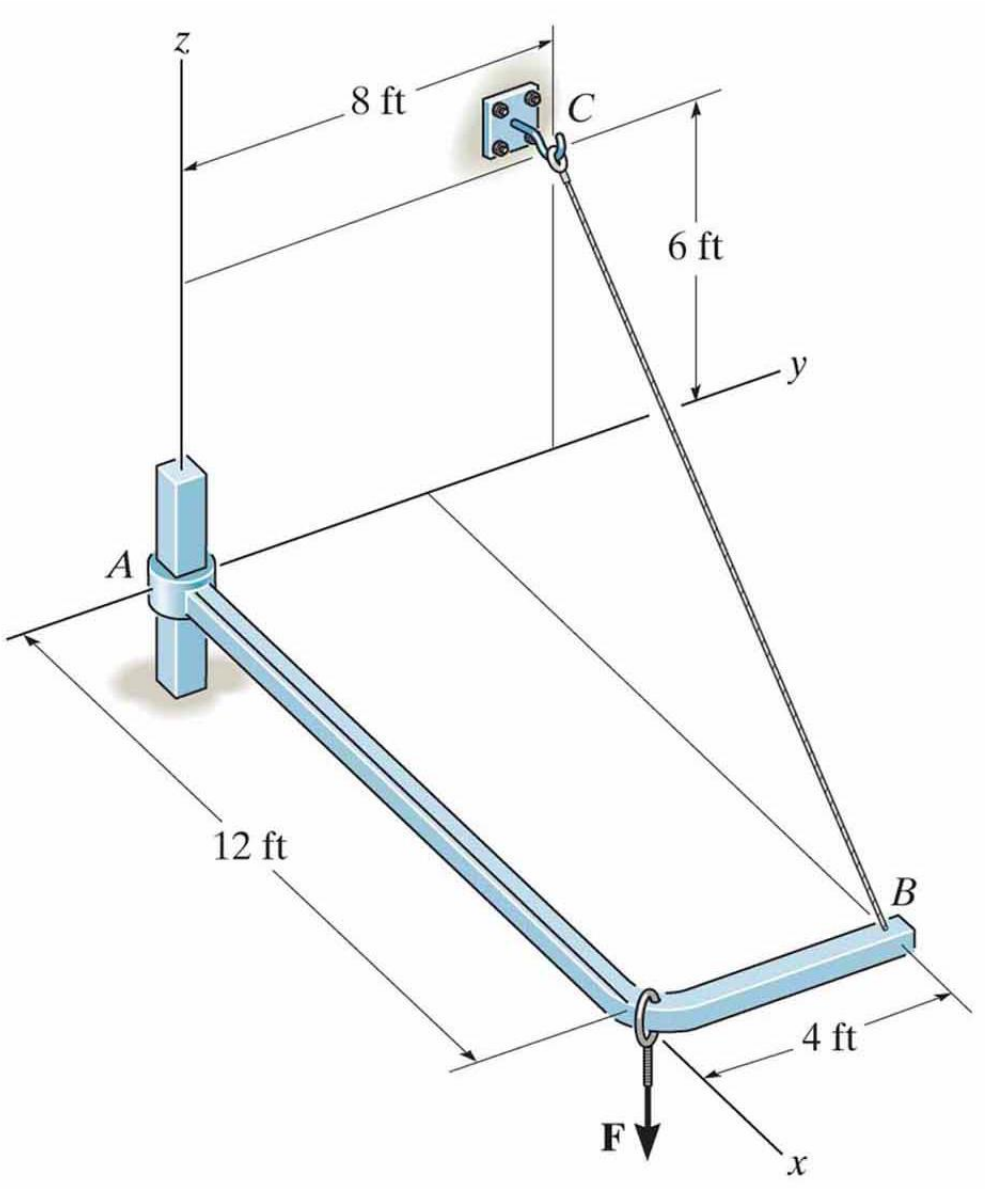


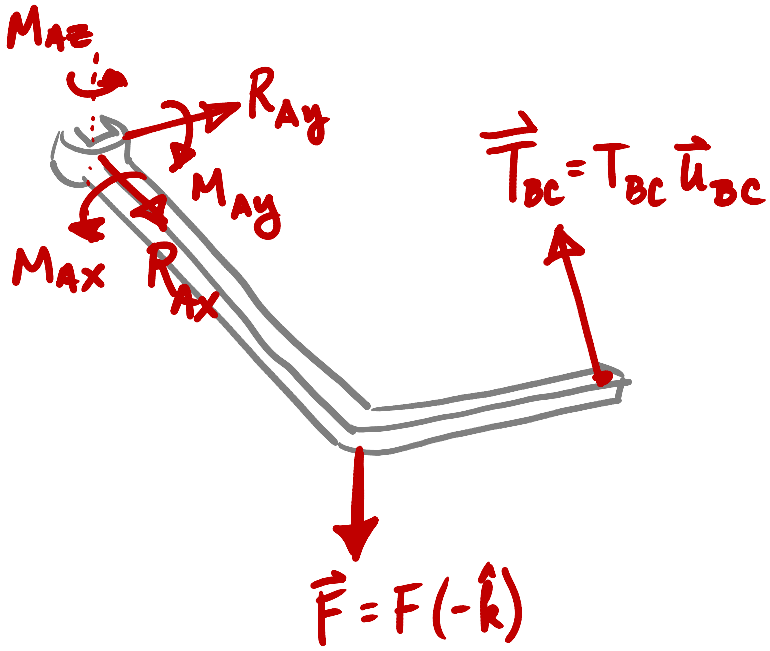




Assume: properly aligned journal bearing at A

$\Rightarrow$ 6 rxns

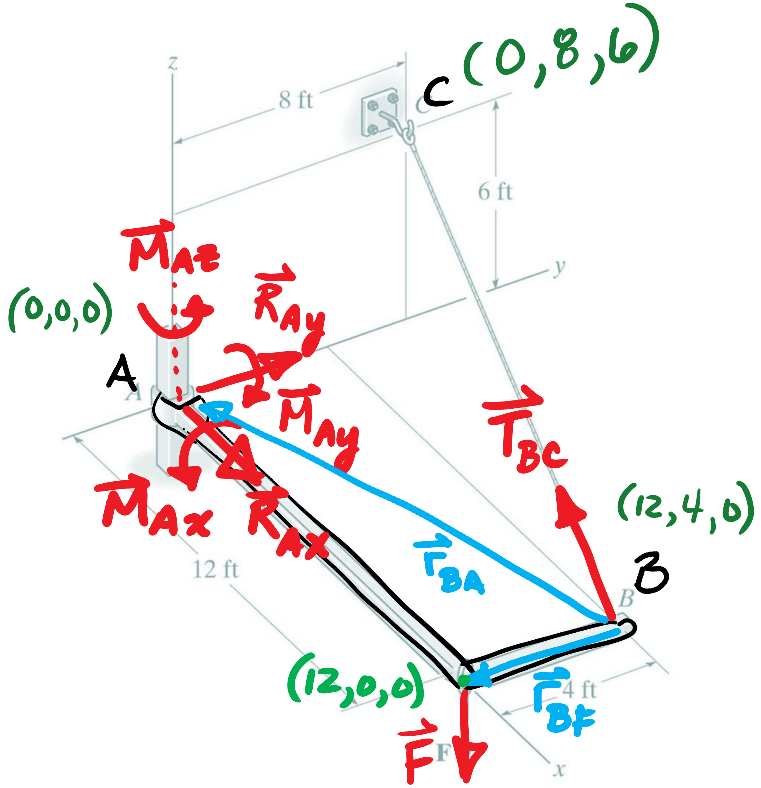
# FBD Example 5 in 3D



# Solution to Example 5 in 3D (cont)

Determine the reaction forces at A and the tension in the cable, if F = 120N.




Find: $R_{Ax}, R_{Ay}, M_{Ax}, M_{Ay}, M_{Az}, T_{BC}$

$$\vec{F} = -120\text{N}\,\hat{k}$$

$$\vec{T}_{BC} = T_{BC}\vec{u}_{BC} = T_{BC}\left(\frac{-12\hat{i}+4\hat{j}+6\hat{k}}{\sqrt{12^2+4^2+6^2}}\right)$$

$$= T_{BC}\left(-\frac{6}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{3}{7}\hat{k}\right)$$

$$\Sigma\vec{F} = 0 = \vec{R}_A + \vec{F} + \vec{T}_{BC}$$

$$0 = R_{Ax}\hat{i} + R_{Ay}\hat{j} - 120\text{N}\,\hat{k} - \frac{6}{7}T_{BC}\hat{i} + \frac{2}{7}T_{BC}\hat{j} + \frac{3}{7}T_{BC}\hat{k}$$

$\Rightarrow \hat{i}: \quad R_{Ax} = \frac{6}{7}T_{BC}$ $\qquad R_{Ax} = 420\text{N}$

$\hat{j}: \quad R_{Ay} = -\frac{2}{7}T_{BC}$ $\qquad R_{Ay} = -80\text{N}$

$\hat{k}: \quad 120\text{N} = \frac{3}{7}T_{BC} \Rightarrow T_{BC} = 280\text{N}$

# Solution to Example 5 in 3D (cont)

$$\sum \vec{M}_B = 0 = \vec{r}_{BF} \times \vec{F} + \vec{r}_{BA} \times \vec{R}_A + \vec{M}_A$$

$$0 = -4\hat{j} \times -120\hat{k}$$

$$+(-12\hat{i} - 4\hat{j}) \times (420\hat{i} - 80\hat{j})$$

$$+ M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k}$$

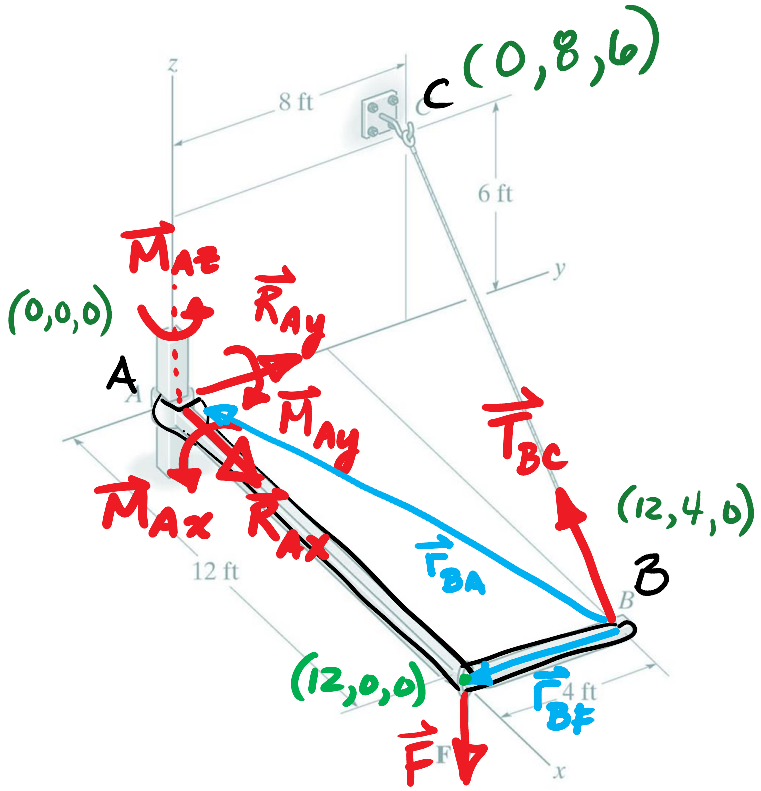
$$= 480\hat{i} + 960\hat{k} + 1680\hat{k}$$

$$+ M_{Ax}\hat{i} + M_{Ay}\hat{j} + M_{Az}\hat{k}$$

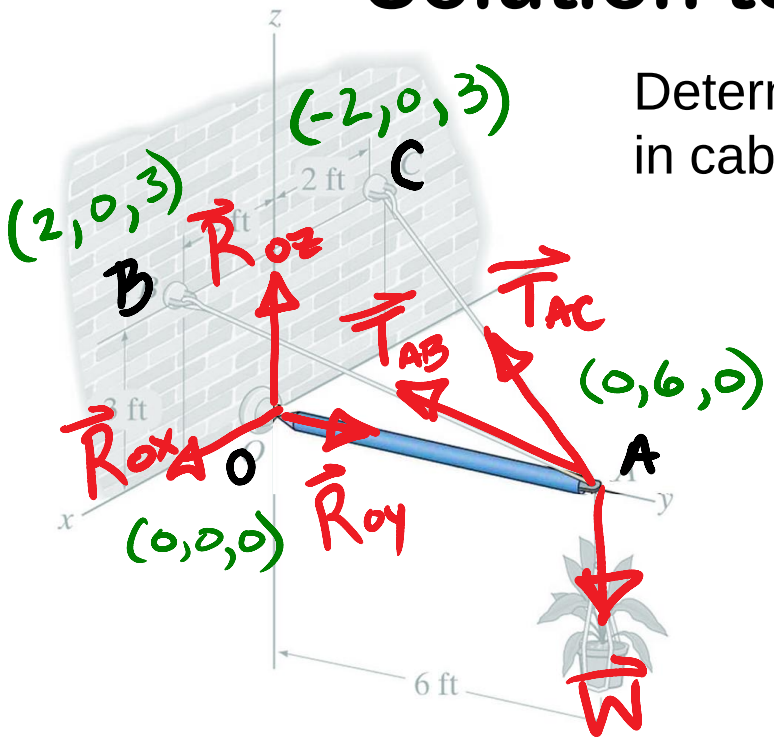
$$\Rightarrow \quad M_{Ax} = -480 \text{ lb ft}$$

$$M_{Ay} = 0$$

$$M_{Az} = -2640 \text{ lb ft}$$



# Solution to Example 1 in 3D




Determine the reaction forces at O and tension in cables AB and AC if the plant weighs 30 lb.

Find: $R_{ox}, R_{oy}, R_{oz}, T_{AB}, T_{AC}$

Given: $\vec{W} = -30\text{ lb}\,\hat{k}$

$$\vec{T}_{AB} = T_{AB}\vec{u}_{AB} = T_{AB}\left(\frac{2\hat{i}-6\hat{j}+3\hat{k}}{\sqrt{2^2+6^2+3^2}}\right)$$

$$\vec{T}_{AC} = T_{AC}\vec{u}_{AC} = T_{AC}\left(\frac{-2\hat{i}-6\hat{j}+3\hat{k}}{7}\right)$$

$$\vec{R}_o = R_{ox}\hat{i} + R_{oy}\hat{j} + R_{oz}\hat{k}$$

$$\Sigma \vec{M}_A = 0 = \vec{r}_{AO} \times \vec{R}_o = -6\hat{j} \times (R_{ox}\hat{i} + R_{oy}\hat{j} + R_{oz}\hat{k}) = 6R_{ox}\hat{k} - 6R_{oz}\hat{i}$$

$$\Rightarrow R_{ox} = 0,\ R_{oz} = 0$$

$$\Sigma F_x = 0 = \frac{2}{7}T_{AB} - \frac{2}{7}T_{AC} + \cancel{R_{ox}}^{\,0} \Rightarrow T_{AB} = T_{AC}$$

$$\Sigma F_y = 0 = -\frac{6}{7}T_{AB} - \frac{6}{7}T_{AC} + R_{oy} \Rightarrow R_{oy} = 60\text{ lb}$$

$$\Sigma F_z = 0 = \frac{3}{7}T_{AB} + \frac{3}{7}T_{AC} + \cancel{R_{oz}}^{\,0} - 30\text{ lb} \Rightarrow T_{AB} = T_{AC} = 35\text{ lb}$$

# ATTENTION QUIZ

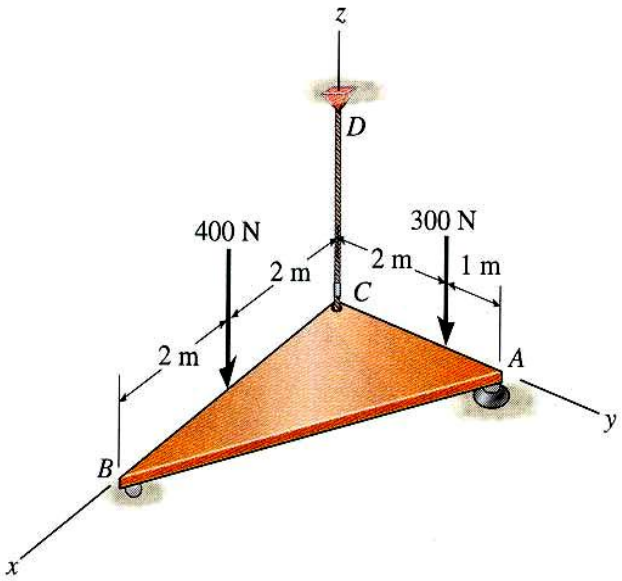
1. A plate is supported by a ball-and-socket joint at A, a roller joint at B, and a cable at C. How many unknown support reactions are there in this problem?

   A) 4 forces and 2 moments

   B) 6 forces

   C) 5 forces

   D) 4 forces and 1 moment



# ATTENTION QUIZ

2. What will be the easiest way to determine the force reaction $B_Z$ ?

A) Scalar equation $\sum F_Z = 0$

B) Vector equation $\sum \boldsymbol{M_A} = 0$

C) Scalar equation $\sum M_Z = 0$

D) Scalar equation $\sum M_Y = 0$