EQUATIONS OF EQUILIBRIUM & TWO- AND THREE-FORCE MEMEBERS

Today's Objectives:

Students will be able to:

a) Apply equations of equilibrium to solve for unknowns, and,

b) Recognize two-force members.

In-Class Activities:

- Check Homework, if any
- Reading Quiz
- Applications
- Equations of Equilibrium
- Two-Force Members
- Concept Quiz
- Group Problem Solving
- Attention Quiz

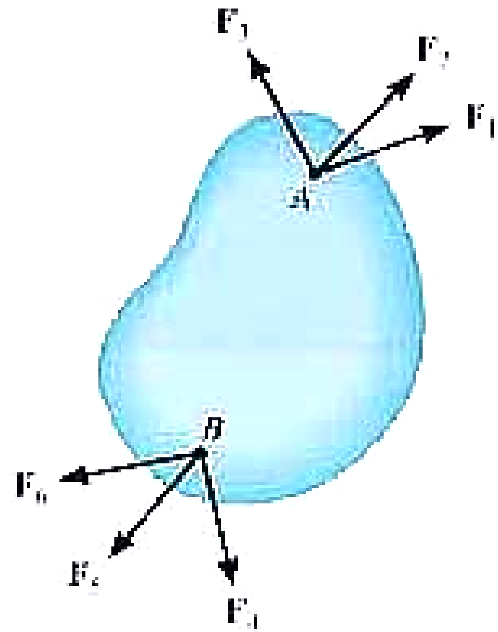
READING QUIZ

1. The three scalar equations $\sum F_X = \sum F_Y = \sum M_O = 0$, are ____ equations of equilibrium in two dimensions.

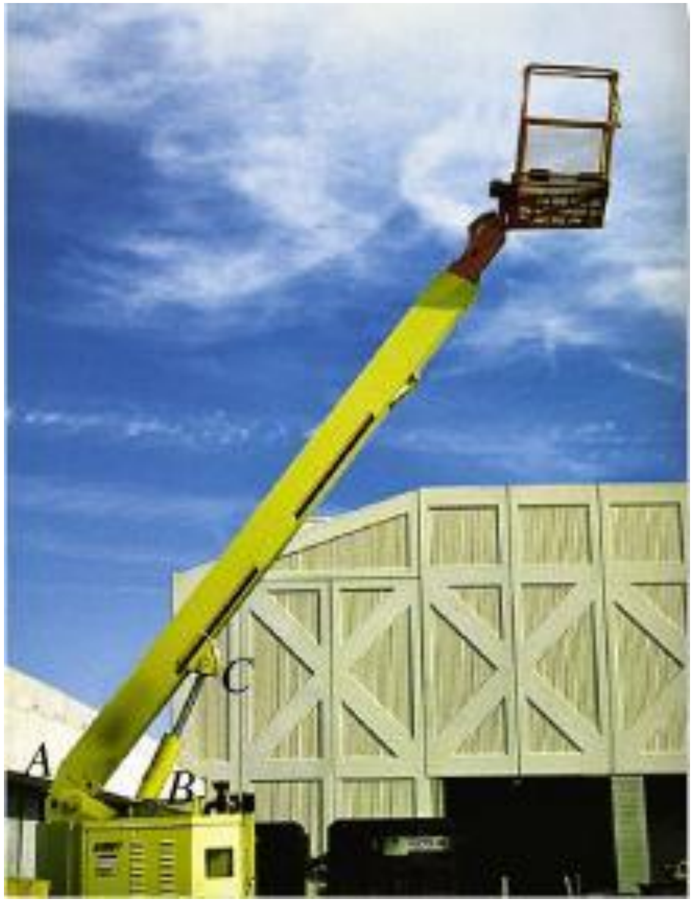
   1) incorrect
   2) the only correct
   3) the most commonly used
   4) not sufficient

2. A rigid body is subjected to forces as shown. This body can be considered as a _______ member.

   A) single-force
   B) two-force
   C) three-force
   D) six-force

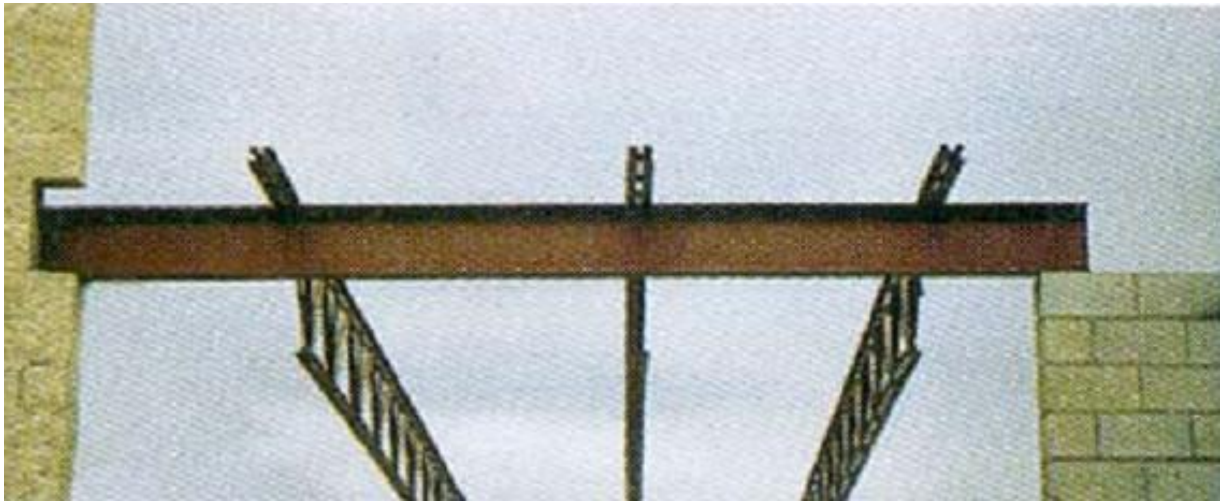


APPLICATIONS



For a given load on the platform, how can we determine the forces at the joint A and the force in the link (cylinder) BC?

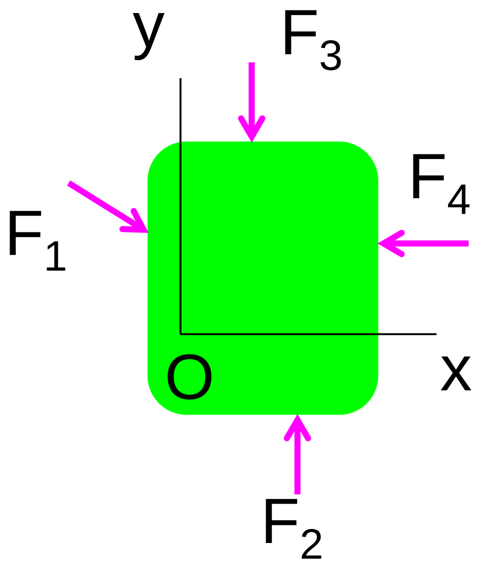
APPLICATIONS
(continued)



A steel beam is used to support roof joists. How can we determine the support reactions at each end of the beam?

EQUATIONS OF EQUILIBRIUM
(Section 5.3)

A body is subjected to a system of forces that lie in the x-y plane. When in equilibrium, the net force and net moment acting on the body are zero:



$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad \sum M_O = 0$$

Where point O is any arbitrary point.

Please note that these equations are the ones most commonly used for solving 2-D equilibrium problems. Another possibility:

$$\sum F_x = 0 \qquad \sum M_A = 0 \qquad \sum M_B = 0$$

Where points A, B are two arbitrary points.

STEPS FOR SOLVING 2-D EQUILIBRIUM PROBLEMS

1. If not given, establish a suitable x - y coordinate system.

2. Draw a free body diagram (FBD) of the object under analysis.

3. Apply the three equations of equilibrium (EofE) to solve for the unknowns.

IMPORTANT NOTES

1. If we have more unknowns than the number of independent equations, then we have a statically indeterminate situation. We cannot solve these problems using just statics.

2. The order in which we apply equations may affect the simplicity of the solution. For example, if we have two unknown vertical forces and one unknown horizontal force, then solving $\sum F_X = O$ first allows us to find the horizontal unknown quickly.

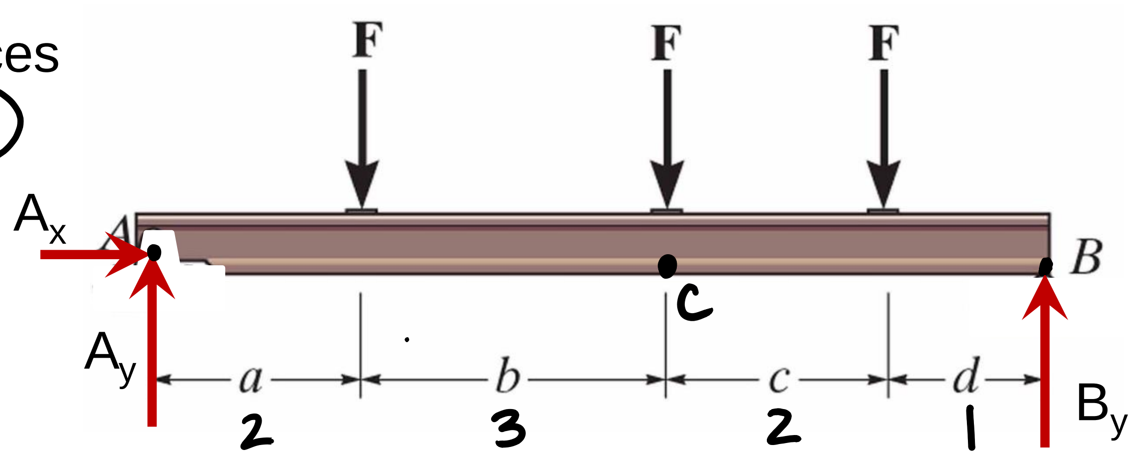
3. If the answer for an unknown comes out as negative number, then the sense (direction) of the unknown force is opposite to that assumed when starting the problem.

find: A_x, A_y, B_y

FBD Example 1 in 2D

Determine the reaction forces at A and B if each F is 5kN.

a = 2m
b = 3m
c = 2m
d = 1m



$\Sigma F_x, \Sigma F_y, \Sigma M_P = 0$

or

$$\Sigma M_A = 0 = -5(2) - 5(5) - 5(7) + B_y(8) \Rightarrow B_y = \frac{70}{8} \text{ kN}$$

$$\Sigma M_B = 0 = 5\text{kN}(1+3+6)\text{m} - A_y(8\text{m}) \Rightarrow A_y = \frac{50}{8} \text{ kN}$$

~~$\Sigma M_C = 0$~~ no new info

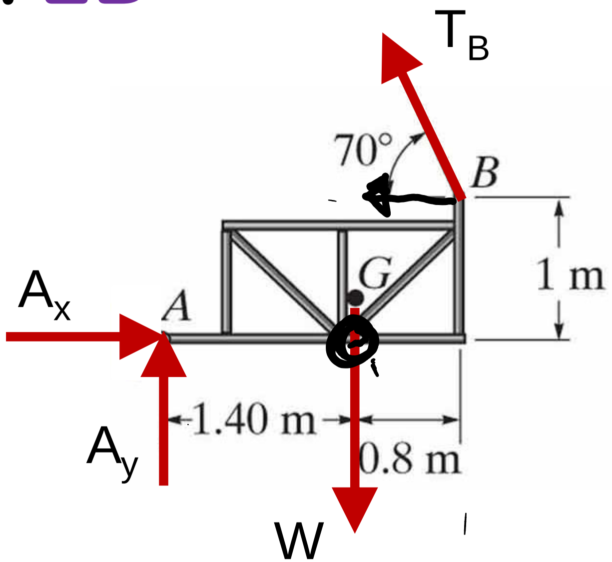
~~$\Sigma F_y = 0$~~ no new info

$$\Sigma F_x = 0 = A_x$$

FBD Example 2 in 2D

Determine the reaction forces at A and the tension in the cable, if the weight of the basket is 3kN.

Find: A_x, A_y, T_B



$$\sum F_x = 0 = A_x - T_B \cos 70°$$

$$\sum F_y = 0 = -W + A_y + T_B \sin 70°$$

choose one

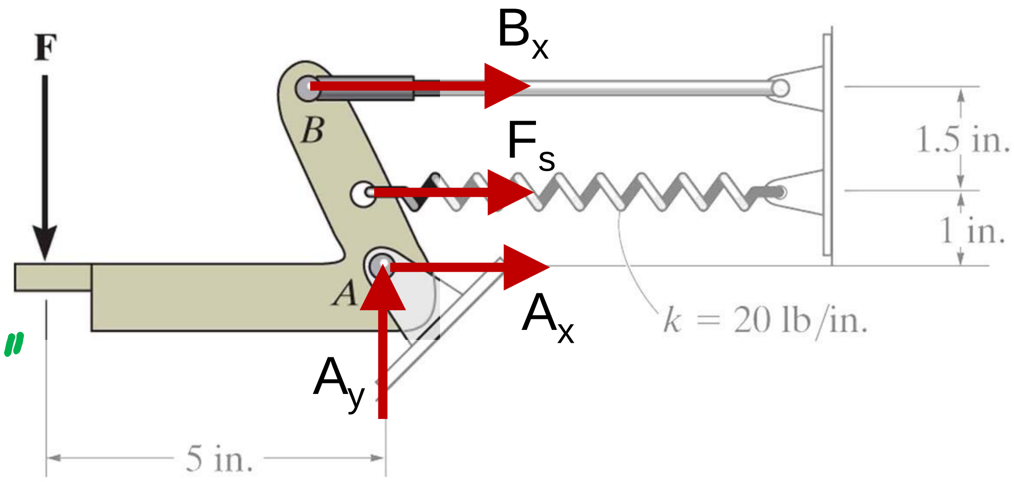
$$\sum M_G = 0$$ .... don't know vert. location of G .... problem

$$\sum M_A = 0 = -W(1.4m) + T_B \sin 70°(2.2m) + T_B \cos 70°(1m)$$

$$\sum M_B = 0 = -A_y(2.2m) + A_x(1m) + W(0.8m)$$

FBD Example 3 in 2D

What is the applied force F on the pedal that causes the spring to stretch by 1.5 in, and creates a force of 20 lb in link B?



Find: F   Given: $B_x = 20\,lb$, $s = 1.5''$

$$F_s = ks = 20\frac{lb}{in}(1.5\,in) = 30\,lb$$

$$\Sigma M_A = 0 = F(5in) - F_s(1\,in) - B_x(2.5\,in) \Rightarrow F = \frac{(30\,lb)(1\,in) + 20\,lb(2.5\,in)}{5\,in}$$

$$\Rightarrow \boxed{F = 16\,lb}$$

Note: if we were asked to find A_x & A_y,

$$\Sigma F_x = 0 = A_x + F_s + B_x \Rightarrow A_x = -50\,lb$$

$$\Sigma F_y = 0 = A_y - F \Rightarrow A_y = 16\,lb$$

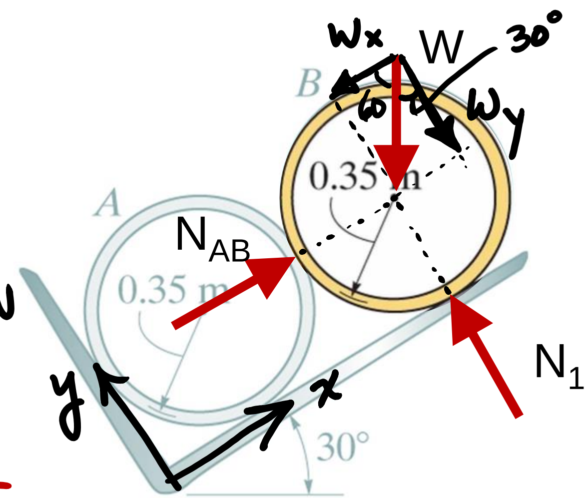
FBD Example 5 in 2D

Each pipe weighs 3kN. Determine the reactions between the pipes and the forklift blades.

Find: N_1, N_2, N_3 Given: $W_1 = W_2 = 3\,kN$

$$\Sigma F_x = 0 = N_{AB} - W_x \Rightarrow N_{AB} = 3kN(\cos 60^\circ) = 1.5\,kN$$

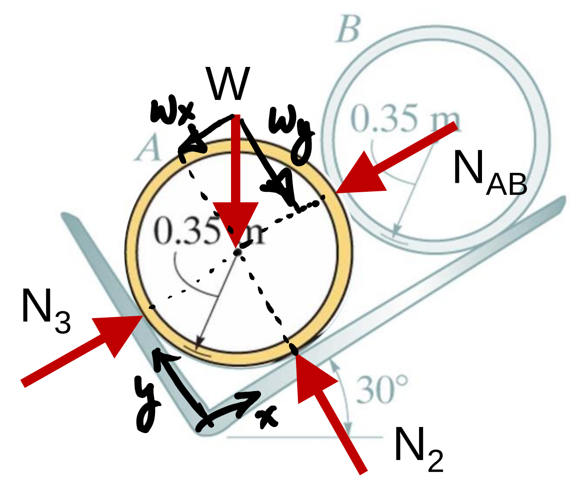
$$\Sigma F_y = 0 = N_1 - W_y \Rightarrow N_1 = 3kN(\cos 30^\circ) = 2.6\,kN$$



Pipe 2

$$\Sigma F_x = 0 = N_3 - W_x - N_{AB} \Rightarrow N_3 = 3\,kN$$

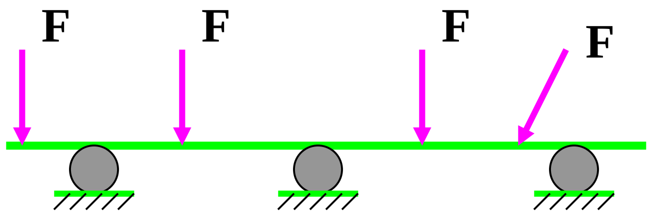
$$\Sigma F_y = 0 = N_2 - W_y \Rightarrow N_2 = 2.6\,kN$$



CONCEPT QUIZ

1. For this beam, how many support reactions are there and is the problem statically determinate?

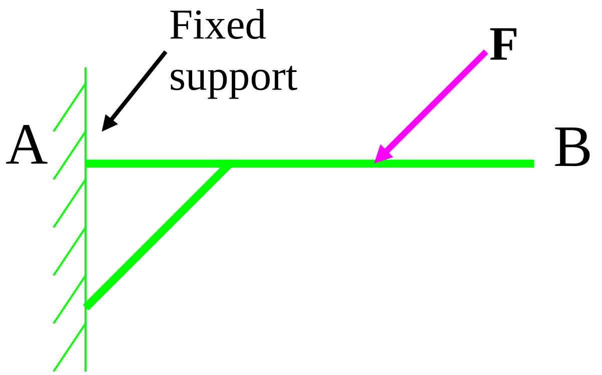
   1) (2, Yes) 2) (2, No)
   3) (3, Yes) 4) (3, No)



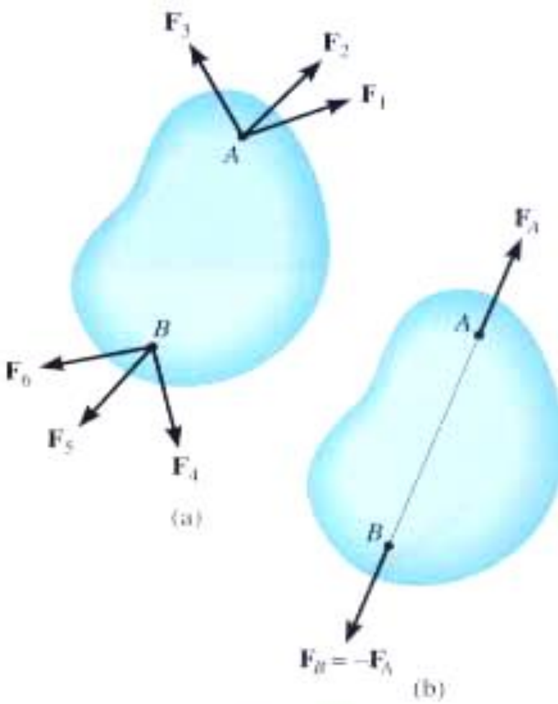
2. The beam AB is loaded and supported as shown: a) how many support reactions are there on the beam, b) is this problem statically determinate, and c) is the structure stable?

   A) (4, Yes, No) B) (4, No, Yes)

   C) (5, Yes, No) D) (5, No, Yes)



TWO-FORCE MEMBERS & THREE FORCE-MEMBERS (Section 5.4)

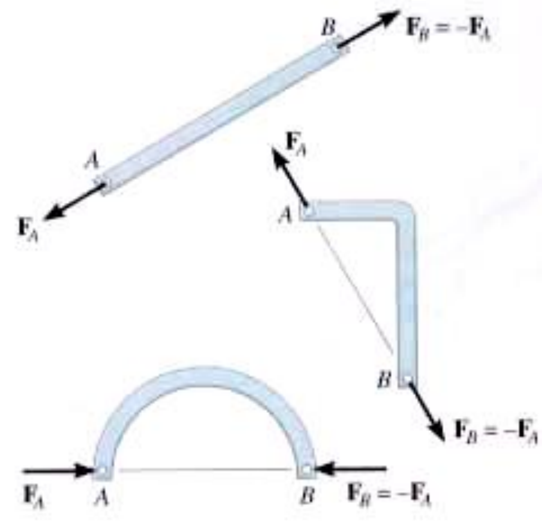


Two-force member

The solution to some equilibrium problems can be simplified if we recognize members that are subjected to forces at only two points (e.g., at points A and B).

If we apply the equations of equilibrium to such a member, we can quickly determine that the resultant forces at A and B must be equal in magnitude and act in the opposite directions along the line joining points A and B.

EXAMPLE OF TWO-FORCE MEMBERS

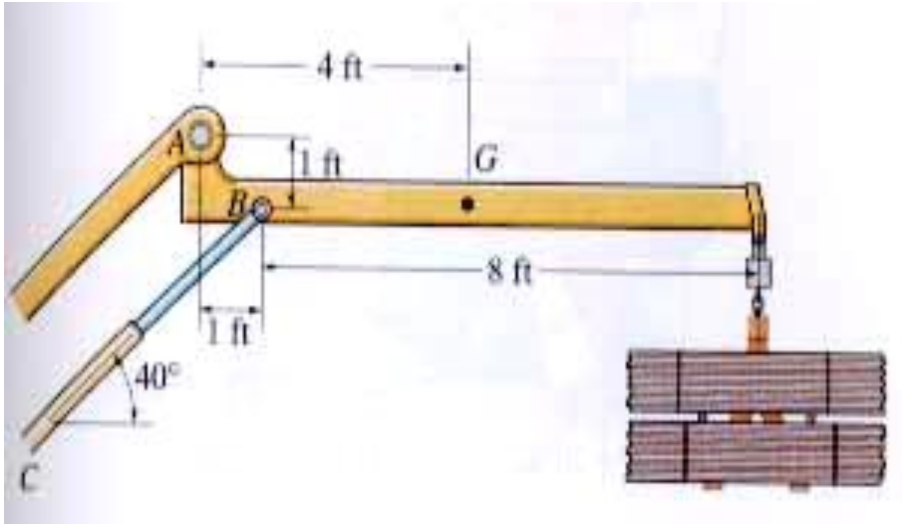


Two-force members

In the cases above, members AB can be considered as two-force members, provided that their weight is neglected.

This fact simplifies the equilibrium analysis of some rigid bodies since the directions of the resultant forces at A and B are thus known (along the line joining points A and B).

EXAMPLE



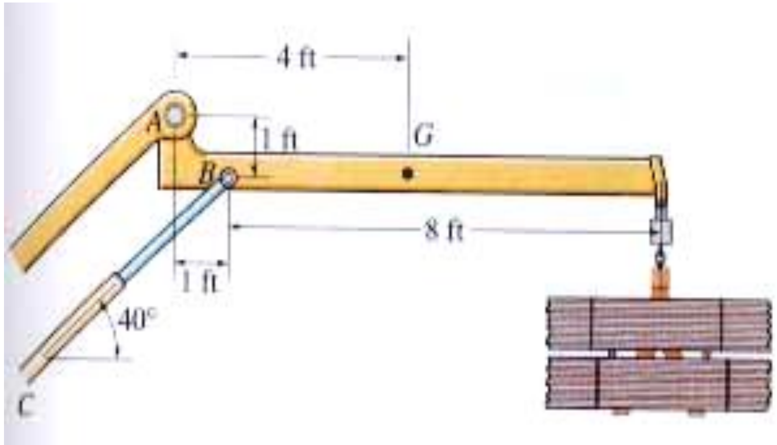
Given: Weight of the boom = 125 lb, the center of mass is at G, and the load = 600 lb.

Find: Support reactions at A and B.

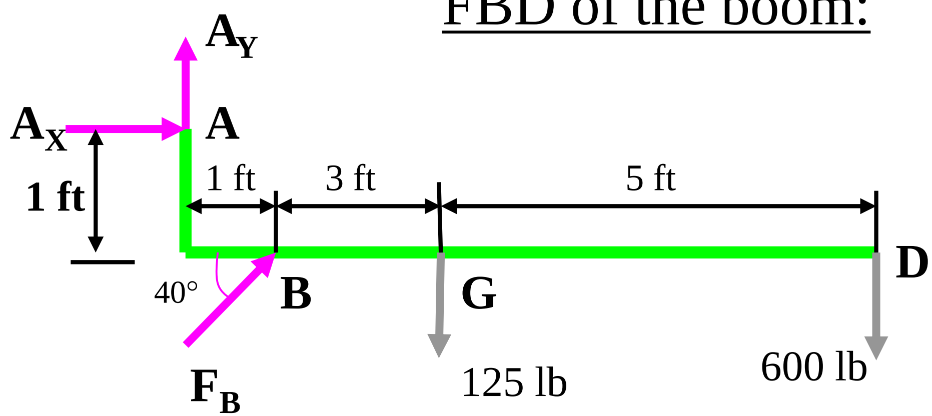
Plan:

1. Put the x and y axes in the horizontal and vertical directions, respectively.
2. Determine if there are any two-force members.
3. Draw a complete FBD of the boom.
4. Apply the E-of-E to solve for the unknowns.

EXAMPLE
(Continued)



FBD of the boom:



Note: Upon recognizing CB as a two-force member, the number of unknowns at B are reduced from two to one. Now, using Eof E, we get,

$$\circlearrowleft + \sum M_A = 125 * 4 + 600 * 9 - F_B \sin 40° * 1 - F_B \cos 40° * 1 = 0$$

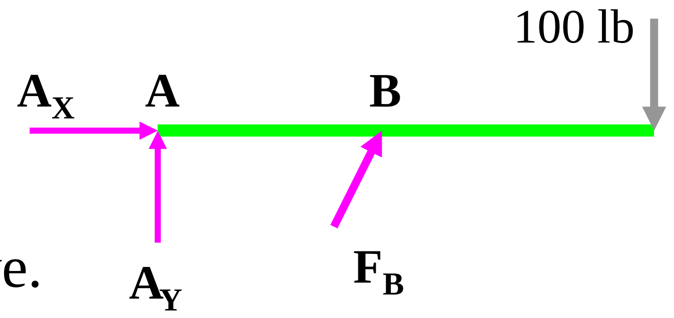
$F_B = 4188$ lb or 4190 lb

$\rightarrow + \sum F_X = A_X + 4188 \cos 40° = 0;$ $A_X = -3210$ lb

$\uparrow + \sum F_Y = A_Y + 4188 \sin 40° - 125 - 600 = 0;$ $A_Y = -1970$ lb

ATTENTION QUIZ

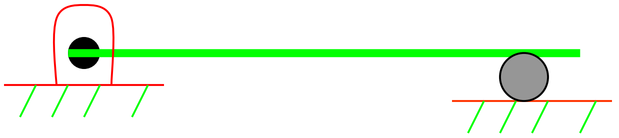
1. Which equation of equilibrium allows you to determine F_B right away?



A) $\sum F_X = 0$ B) $\sum F_Y = 0$

C) $\sum M_A = 0$ D) Any one of the above.

2. A beam is supported by a pin joint and a roller. How many support reactions are there and is the structure stable for all types of loadings?



A) (3, Yes) B) (3, No)

C) (4, Yes) D) (4, No)