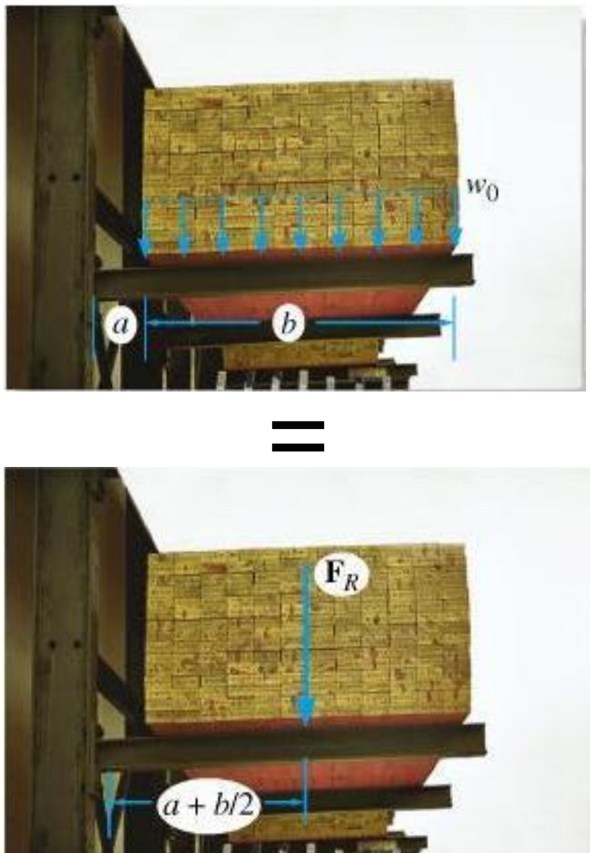
REDUCTION OF A SIMPLE DISTRIBUTED LOADING

Today's Objectives:

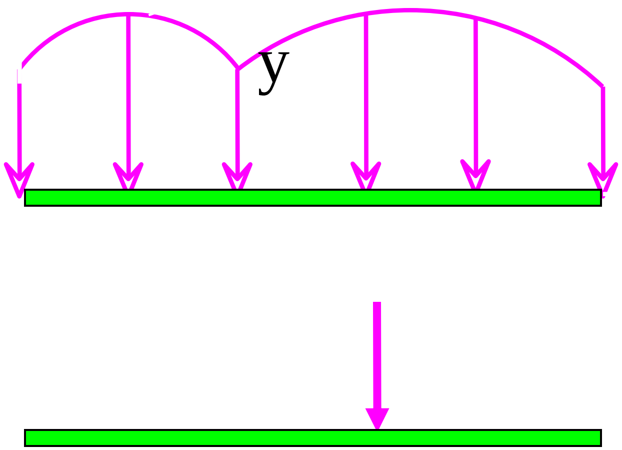
Students will be able to determine an equivalent force for a distributed load.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equivalent Force
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ



1. The resultant force (F_R) due to a distributed load is equivalent to the ______ under the distributed loading curve, w = w(x).

A) centroid B) arc length

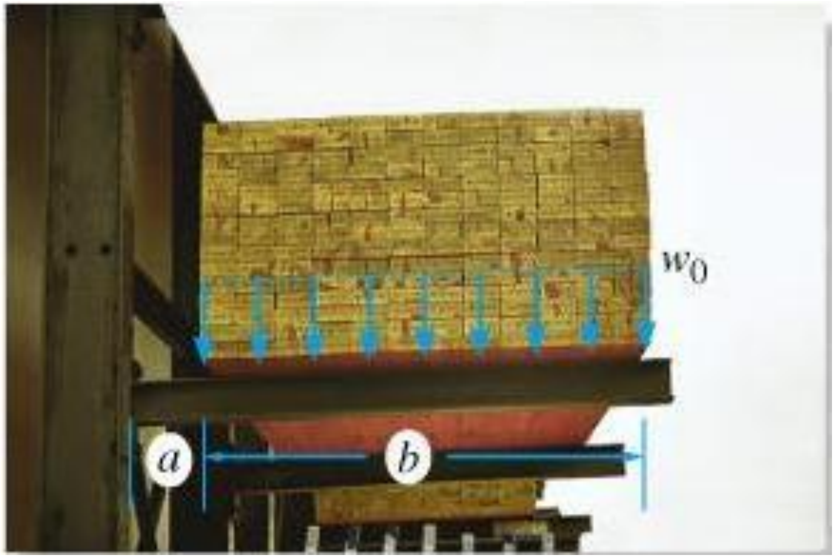
C) area D) volume

2. The line of action of the distributed load's equivalent force passes through the _______ of the distributed load.

A) centroid B) mid-point

C) left edge D) right edge

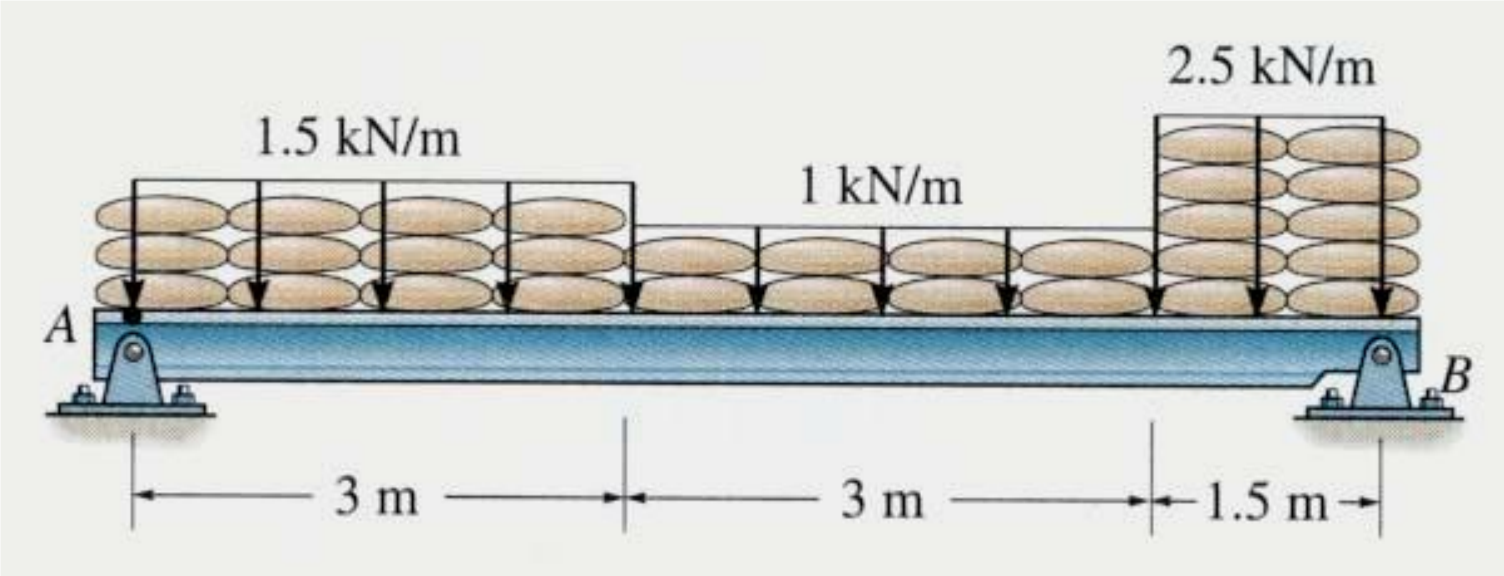
APPLICATIONS



A distributed load on the beam exists due to the weight of the lumber.

Is it possible to reduce this force system to a single force that will have the same external effect?
If yes, how?

APPLICATIONS
(continued)

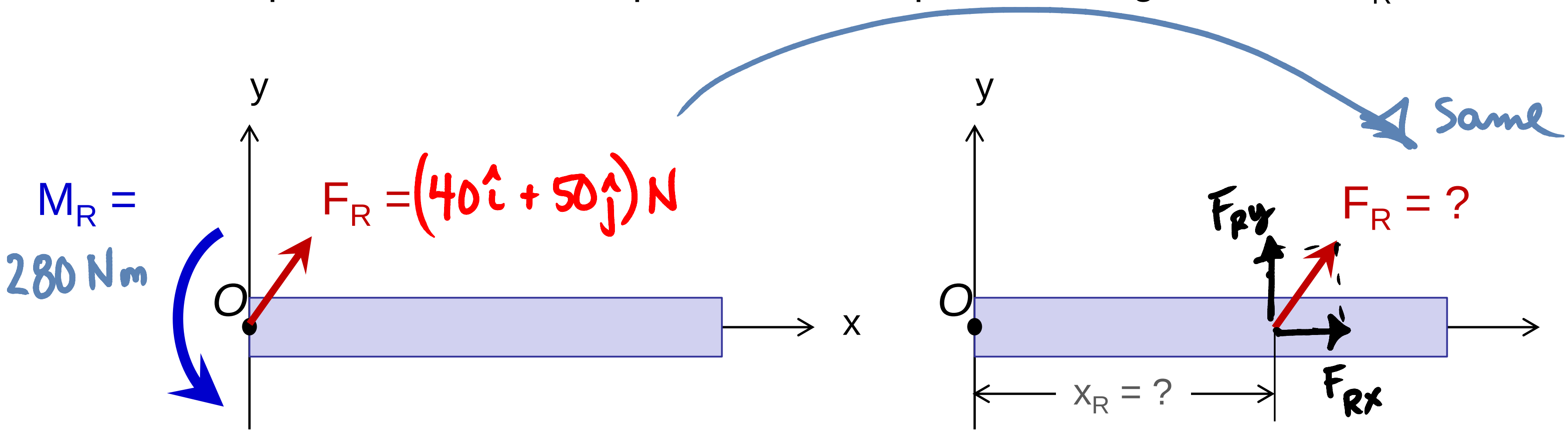


The sandbags on the beam create a distributed load.

How can we determine a single equivalent resultant force and its location?

Review -- Single Equivalent Force

Replace force and couple at O with equivalent single force at x_R

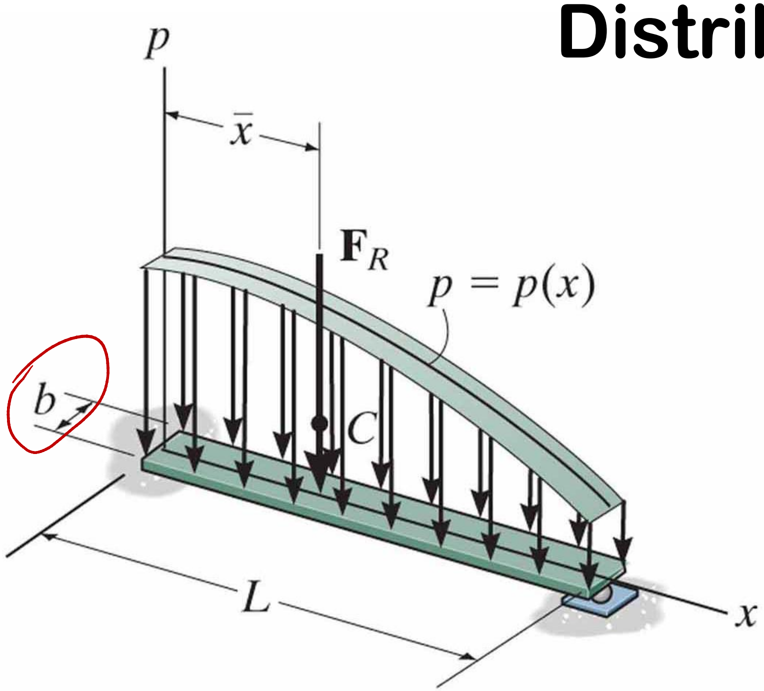


$$\sum_1 \vec{F} = \sum_2 \vec{F} \Rightarrow$$

$$\sum_1 M_O = \sum_2 M_O \Rightarrow +280\ \text{Nm} = +F_{Ry}\, x_R \Rightarrow x_R = \frac{280\ \text{Nm}}{50\ \text{N}}$$

$$\boxed{x_R = 5{,}6\ \text{m}}$$

Distributed Loading



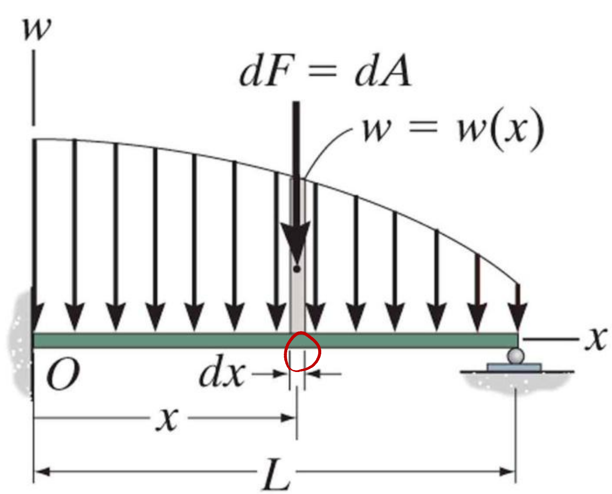
- Represented by pressure function *p*.
- If load varies only in *x* direction, use *intensity* function $w(x) = b\,p(x)$.

$$dF = w(x)\,dx$$

- Replace distributed load by F_R at x_R.

$$\sum_1 \vec{F} = \sum_2 \vec{F}\ ;\quad \sum_1 \vec{M}_O = \sum_2 \vec{M}_O$$

$$F_R = \int dF = \int_{x=0}^{x=L} w(x)\,dx \quad \text{= Area of diagram}$$

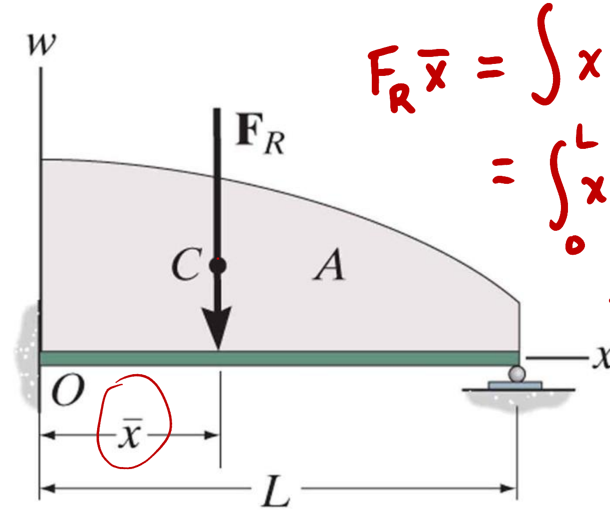


$$F_R \bar{x} = \int x\,dF$$

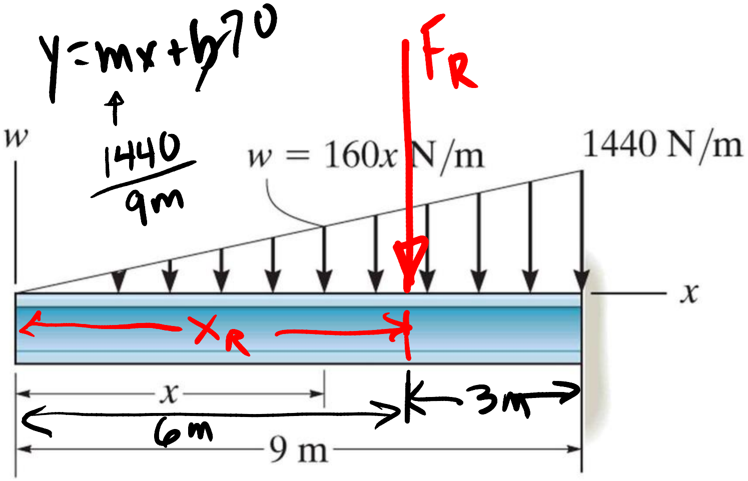
$$= \int_0^L x\,w(x)\,dx$$

$$\Rightarrow \bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)\,dx}$$

centroid



Example 1. Determine the single force F_R and its x location equivalent to the distributed loading shown below.



Find: F_R, x_R

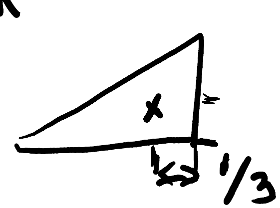
$$F_R = \int_{x=0}^{x=L} w(x)\,dx = \int_0^9 160x\,dx = 80x^2\Big|_0^9$$

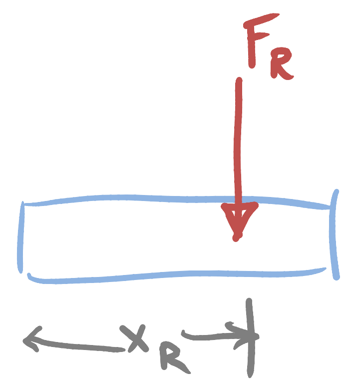
$$F_R = 6480\text{ N}$$

$$x_R = \frac{\int_0^L x\,w(x)\,dx}{F_R} = \int_0^9 160x^2\,dx \Big/ 6480 = \frac{160}{6480}\left(\frac{x^3}{3}\right)\Big|_0^9 = \underline{6\text{ m}}$$

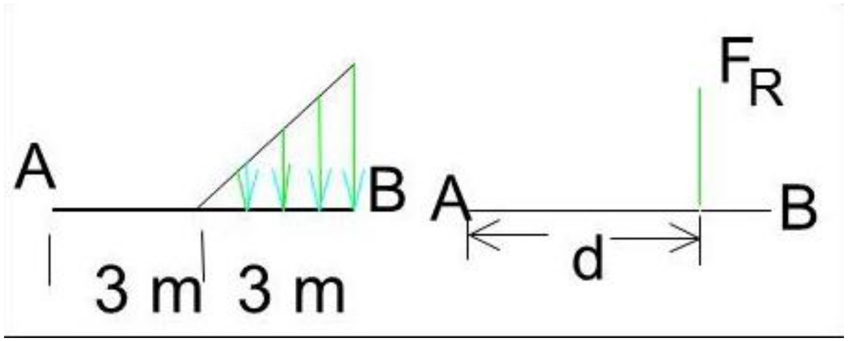
$$F_R = \text{Area} = \tfrac{1}{2}bh = \tfrac{1}{2}(9\text{m})(1440\text{ N/m}) = \underline{6480\text{ N}}$$

$$x_R = \text{Centroid} = 9\text{m} - \tfrac{1}{3}(9\text{m}) = \underline{6\text{m}}$$





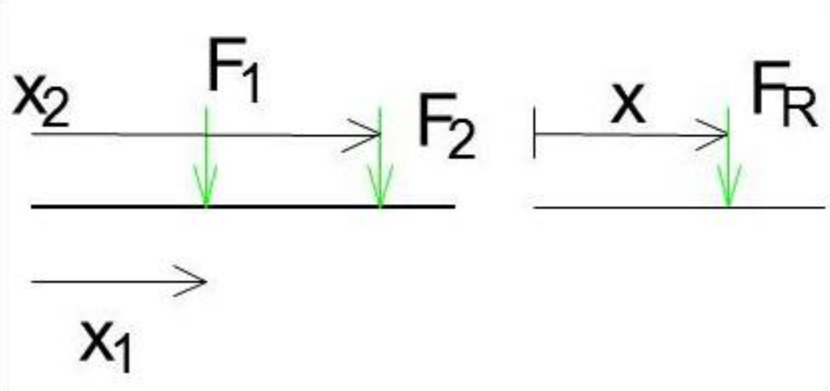
CONCEPT QUIZ



1. What is the location of F_R, i.e., the distance d?

A) 2 m B) 3 m C) 4 m

D) 5 m E) 6 m

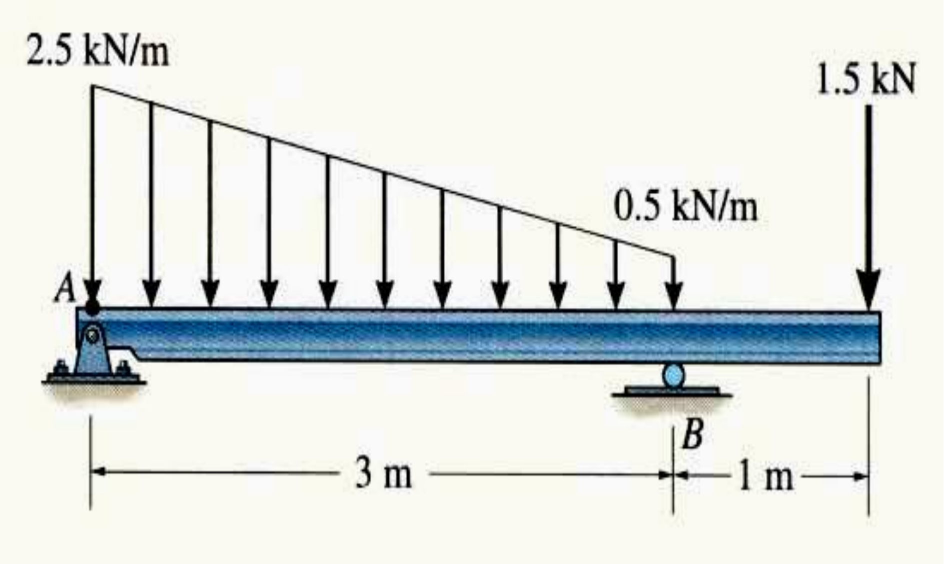


2. If $F_1 = 1$ N, $x_1 = 1$ m, $F_2 = 2$ N and $x_2 = 2$ m, what is the location of F_R, i.e., the distance x.

A) 1 m B) 1.33 m C) 1.5 m

D) 1.67 m E) 2 m

Example 2



Given: The loading on the beam as shown.

Find: The equivalent force and its location from point A.

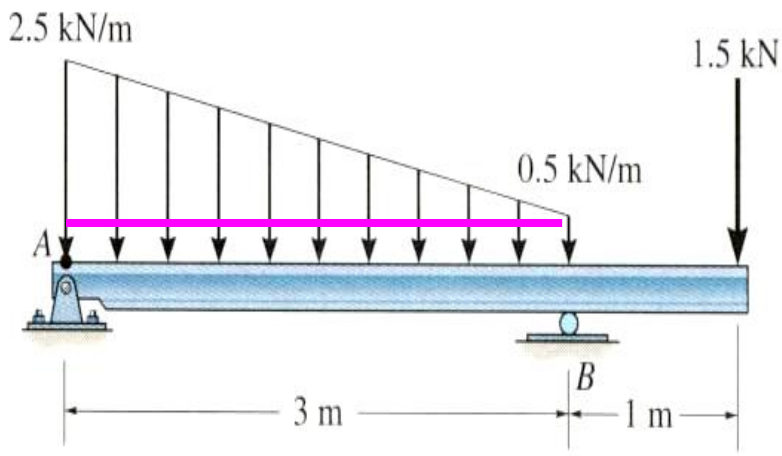
Plan:

1) Consider the trapezoidal loading as two separate loads (one rectangular and one triangular).

2) Find F_R and $\bar{x}$ for each of these two distributed loads.

3) Determine the overall F_R and $\bar{x}$ for the three point loadings.

Example 2 (continued)



For the rectangular loading of height 0.5 kN/m and width 3 m,

F_{R1} = 0.5 kN/m × 3 m = 1.5 kN

= 1.5 m from A

For the triangular loading of height 2 kN/m and width 3 m,

F_{R2} = (0.5) (2 kN/m) (3 m) = 3 kN

and its line of action is at = 1 m from A

For the combined loading of the three forces,

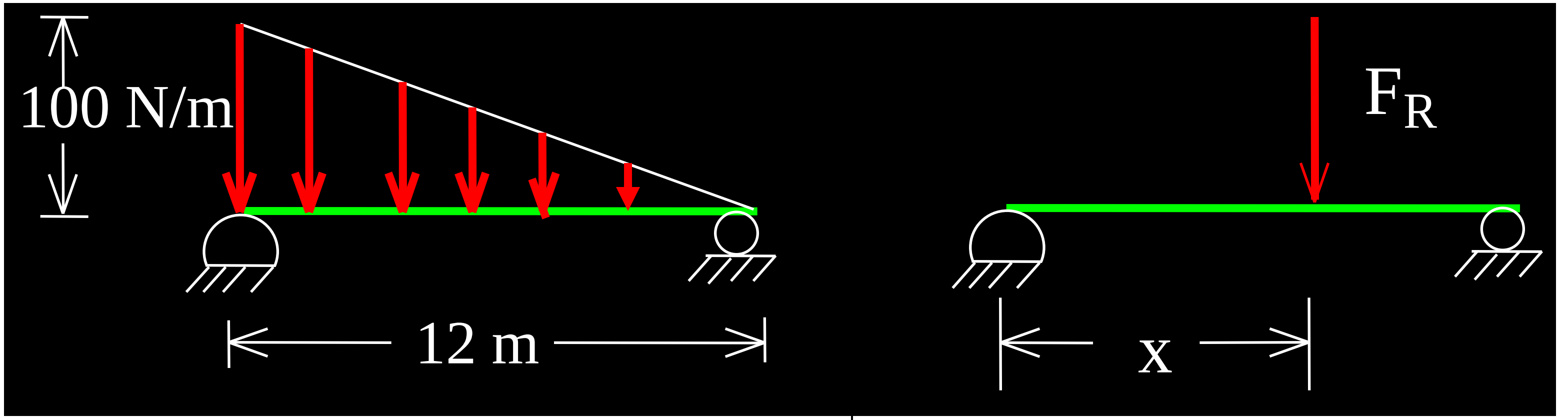
F_R = 1.5 kN + 3 kN + 1.5 kN = 6 kN

↺+ M_{RA} = (1.5) (1.5) + 3 (1) + (1.5) 4 = 11.25 kN • m

Now, F_R = 11.25 kN • m

Hence, $\bar{x}$ = (11.25) / (6) = 1.88 m from A.

ATTENTION QUIZ



1. F_R = ________________

A) 12 N B) 100 N

C) 600 N D) 1200 N

2. x = ______________.

A) 3 m B) 4 m

C) 6 m D) 8 m