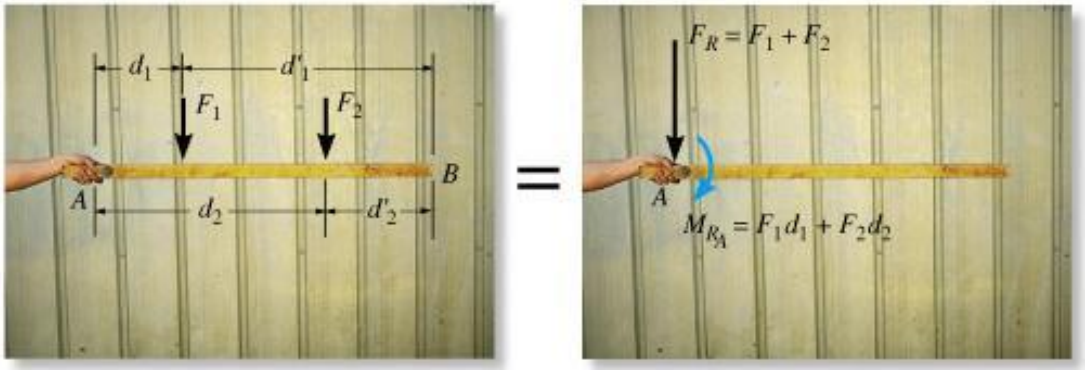
EQUIVALENT SYSTEMS, RESULTANTS OF FORCE AND COUPLE SYSTEM, & FURTHER REDUCTION OF A FORCE AND COUPLE SYSTEM

Today's Objectives:

Students will be able to:

a) Determine the effect of moving a force.

b) Find an equivalent force-couple system for a system of forces and couples.



In-Class Activities:

- Check Homework
- Reading Quiz
- Applications
- Equivalent Systems
- System Reduction
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

1. A general system of forces and couple moments acting on a rigid body can be reduced to a ___ .

   1) single force.

   2) single moment.

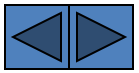
   3) single force and two moments.

   4) single force and a single moment.

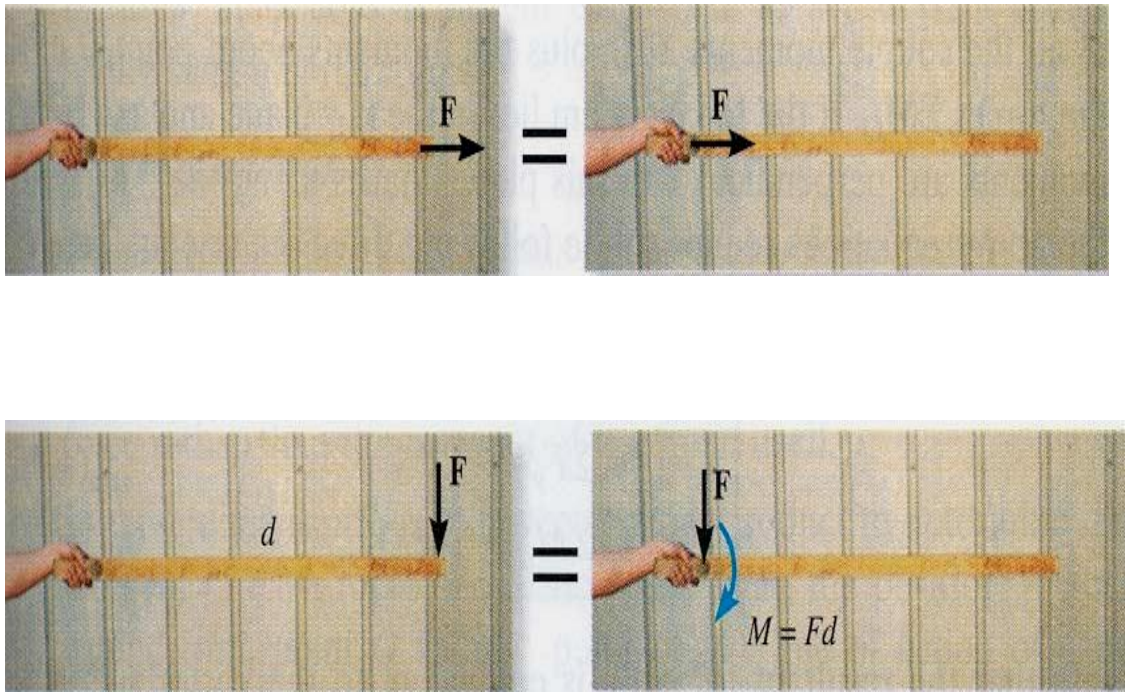
2. The original force and couple system and an equivalent force-couple system have the same _____ effect on a body.

   1) internal 2) external

   3) internal and external 4) microscopic

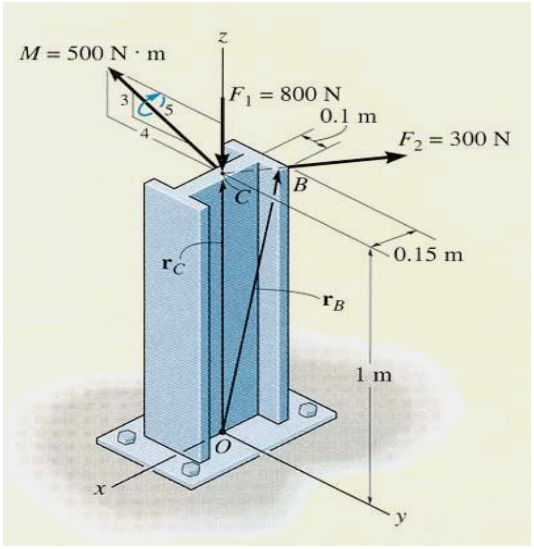


APPLICATIONS



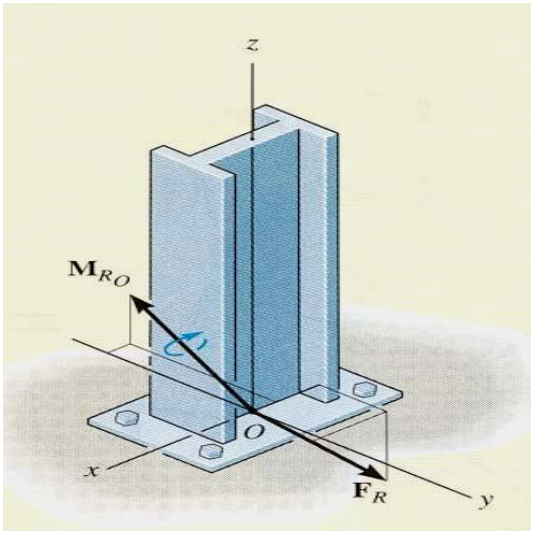
What is the resultant effect on the person's hand when the force is applied in four different ways ?

APPLICATIONS (continued)



Several forces and a couple moment are acting on this vertical section of an I-beam.

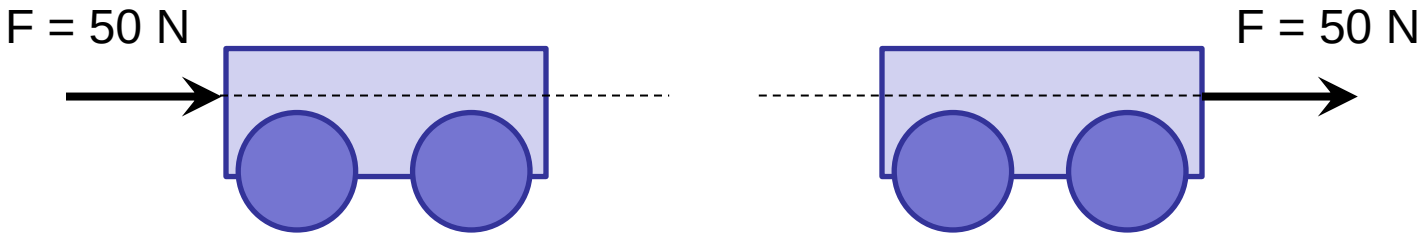
|| ??



Can you replace them with just one force and one couple moment at point O that will have the same external effect? If yes, how will you do that?

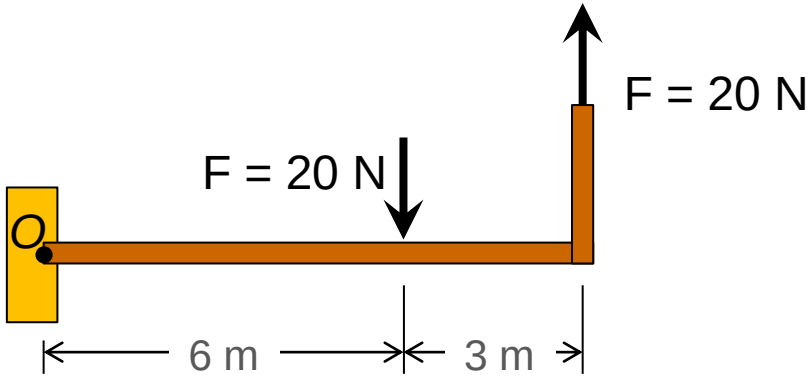
Equivalent Systems (4.7 - 4.8)

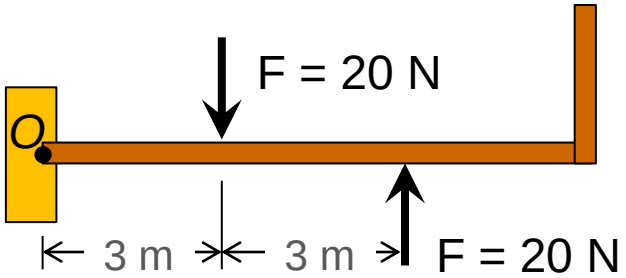
- Undergo same rigid body motion.
- Same resultant forces and moments.



principle of transmissibility

couple moments are free vectors

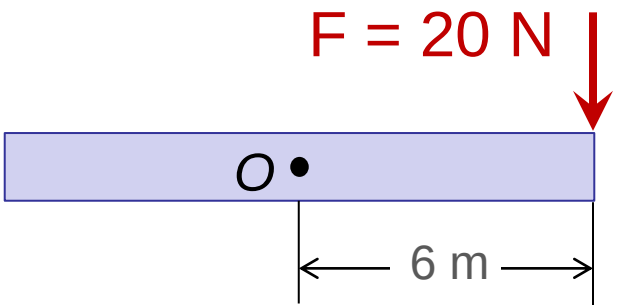




Establishing Equivalence

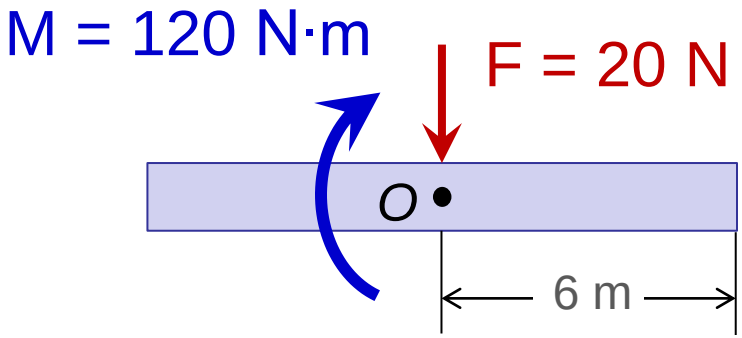
Equivalent.

translates & rotates about O



$F_R = 20$ N ↓ $M_{RO} = 120$ Nm ↻

translates & rotates about O

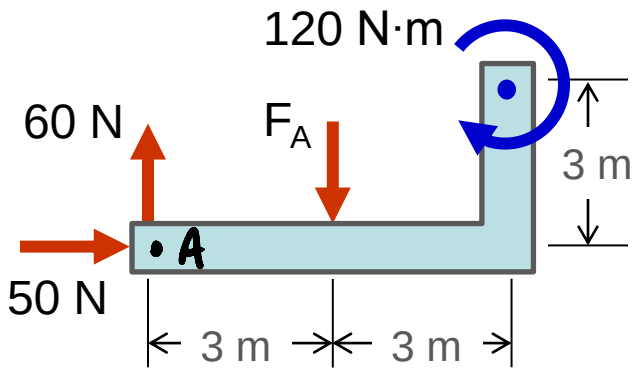


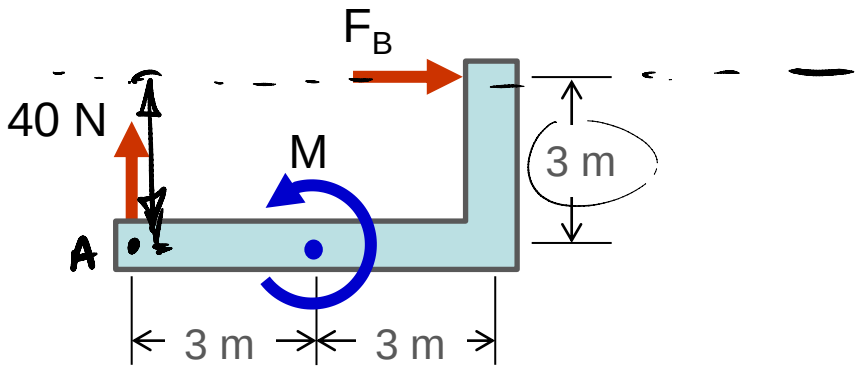
$F_R = 20$ N ↓ $M_{RO} = 120$ Nm ↻

$$\sum_1 \vec{F} = \sum_2 \vec{F}$$

$$\sum_1 \vec{M}_A = \sum_2 \vec{M}_A$$

Example 1. The two force systems are equivalent. Determine the forces F_A and F_B and the couple M. Answers: 20N, 50N, -30 N-m





$$\sum_1 F_x = \sum_2 F_x \Rightarrow 50N = F_B$$

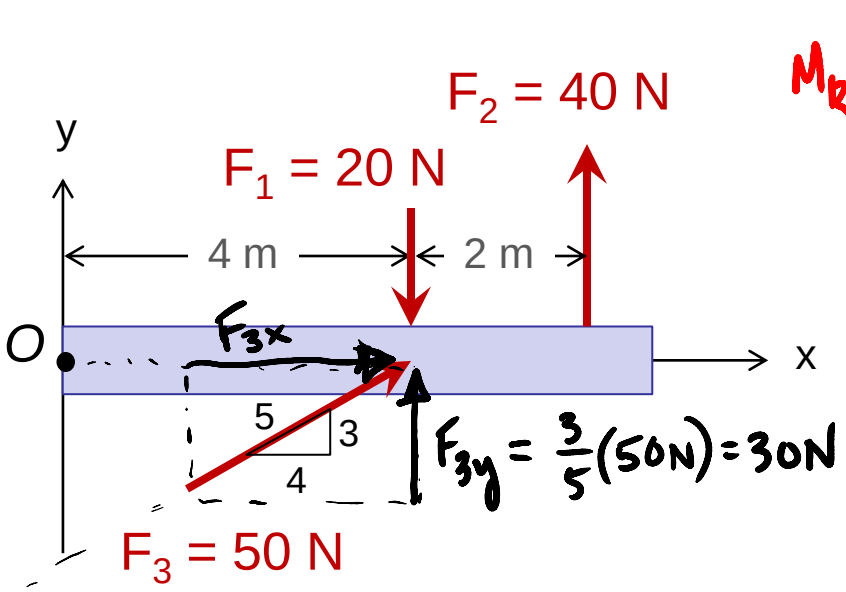
$$\sum_1 F_y = \sum_2 F_y \Rightarrow 60N - F_A = 40N \Rightarrow F_A = 20N$$

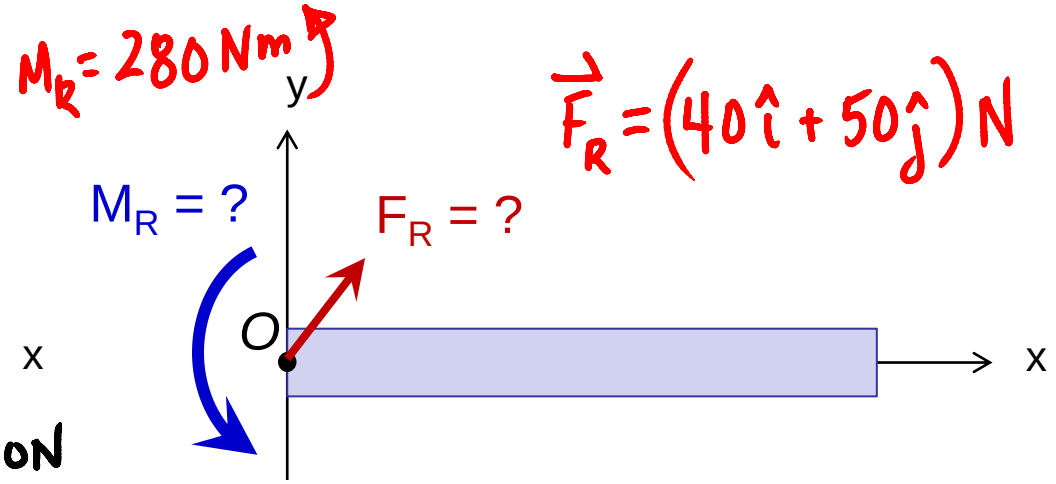
$$\sum_1 M_A = \sum_2 M_A \Rightarrow -F_A(3m) - 120\,Nm = -F_B(3m) + M$$

$$M = (50N)(3m) - (20N)(3m) - 120\,Nm = -30\,Nm$$

Example 2: Reduce to an Equivalent Single Force and Single Couple Moment

Replace multiple forces, couples with single force and couple at O





$\vec{F}_R = (40\hat{i} + 50\hat{j})\,N$

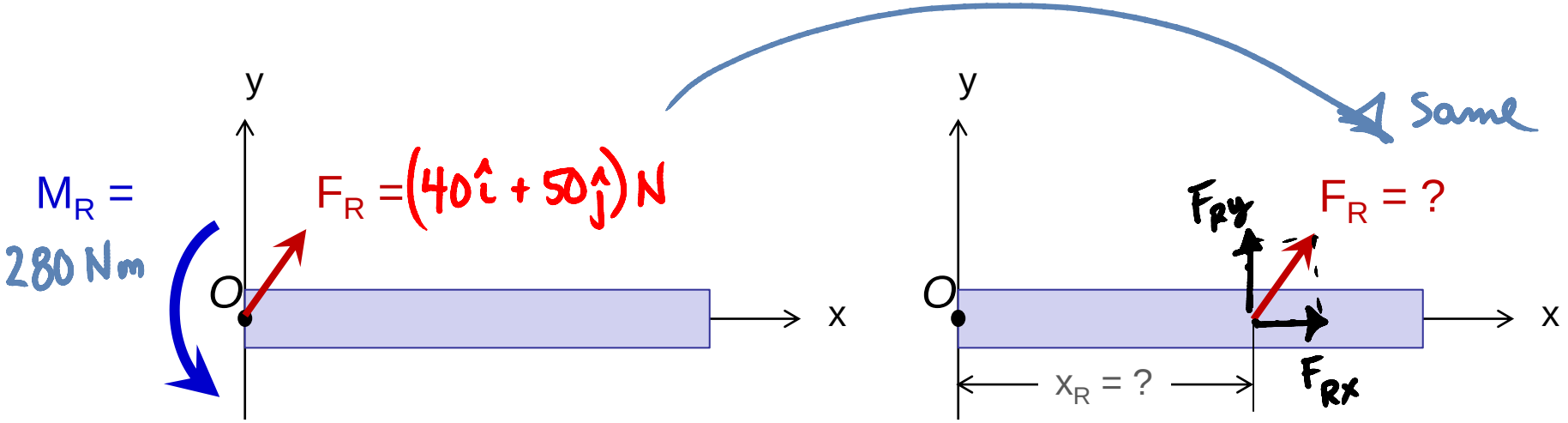
$$F_{Rx} = \Sigma F_{ix} = +\left(\frac{4}{5}\right)50N = 40N$$

$$F_{Ry} = \Sigma F_{iy} = -20N + 40N + \left(\frac{3}{5}\right)50N = 50N$$

$$M_{RO} = \Sigma M_{iO} = -20N(4m) + 40N(6m) + 30N(4m) = 280\ Nm$$

Example 3: Replace a force and couple with a single force

Step 2: Replace force and couple at O with equivalent single force at x_R

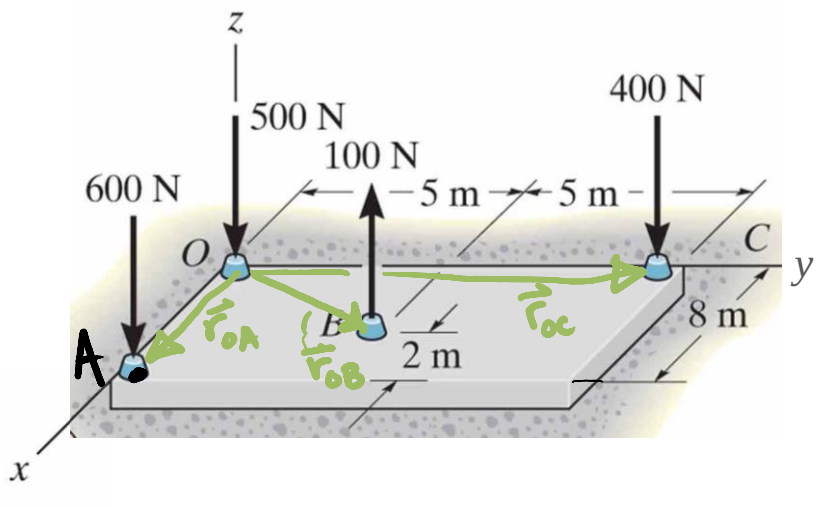


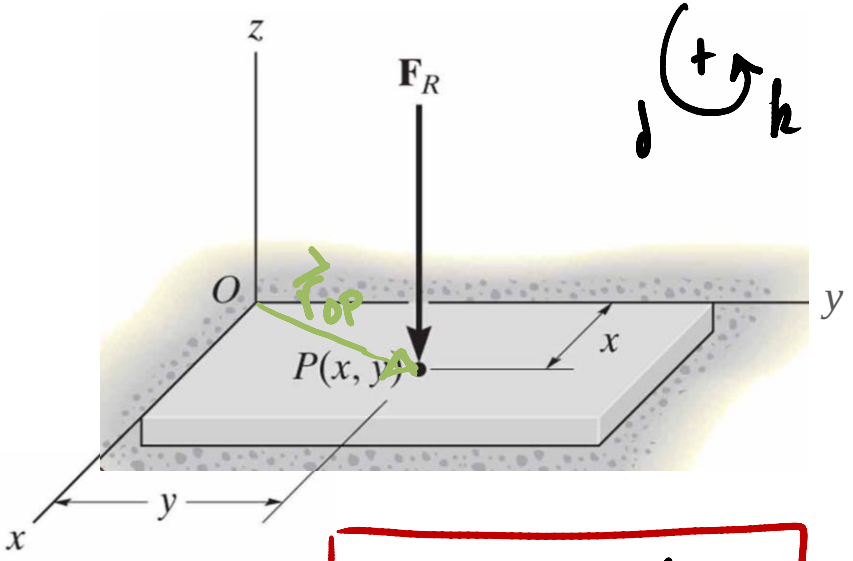
$$\sum_1 \vec{F} = \sum_2 \vec{F} \Rightarrow$$

$$\sum_1 M_O = \sum_2 M_O \Rightarrow +280\,\text{Nm} = +F_{Ry}\,x_R \Rightarrow x_R = \frac{280\,\text{Nm}}{50\,\text{N}}$$

$$\boxed{x_R = 5.6\,\text{m}}$$

Example 4. Determine the single force F_R and its location P that is equivalent to the force system on the left.





$$\sum_1 \vec{F} = \sum_2 \vec{F} \Rightarrow (-600 - 500 + 100 - 400)\hat{k}\ N = \vec{F}_R = -1400\,\hat{k}\ N$$

$$\sum_1 \vec{M}_O = \sum_2 \vec{M}_O \Rightarrow \vec{r}_{OA} \times (-600\hat{k}\ N) + \vec{r}_{OB} \times (100\hat{k}\ N) + \vec{r}_{OC} \times (-400\hat{k}\ N) = \sum_2 \vec{M}_O$$

$$= (8\hat{i}) \times (-600\hat{k}) + (6\hat{i} + 5\hat{j}) \times (100\hat{k}) + (10\hat{j}) \times (-400\hat{k}) = (x\hat{i} + y\hat{j}) \times (-1400\hat{k})$$

$$= 4800\hat{j} - 600\hat{j} + 500\hat{i} - 4000\hat{i} = 1400x\,\hat{j} - 1400y\,\hat{i}$$

$4800 - 600 = 1400x$ $\qquad$ $500 - 4000 = -1400y$

$x = 3\,m$

$y = 2.5\,m$

ATTENTION QUIZ

1. For this force system, the equivalent system at P is _____ .

A) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +60 ft ·lb

B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

C) F_{RP} = 30 lb (along +y-dir.) and M_{RP} = -30 ft ·lb

D) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +30 ft ·lb