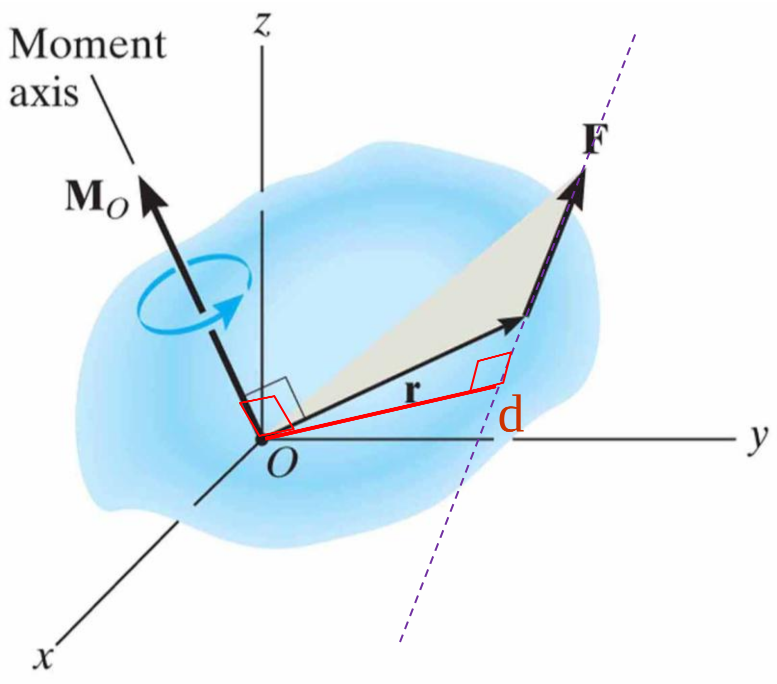
Vector Moment Analysis (4.2 - 4.4)



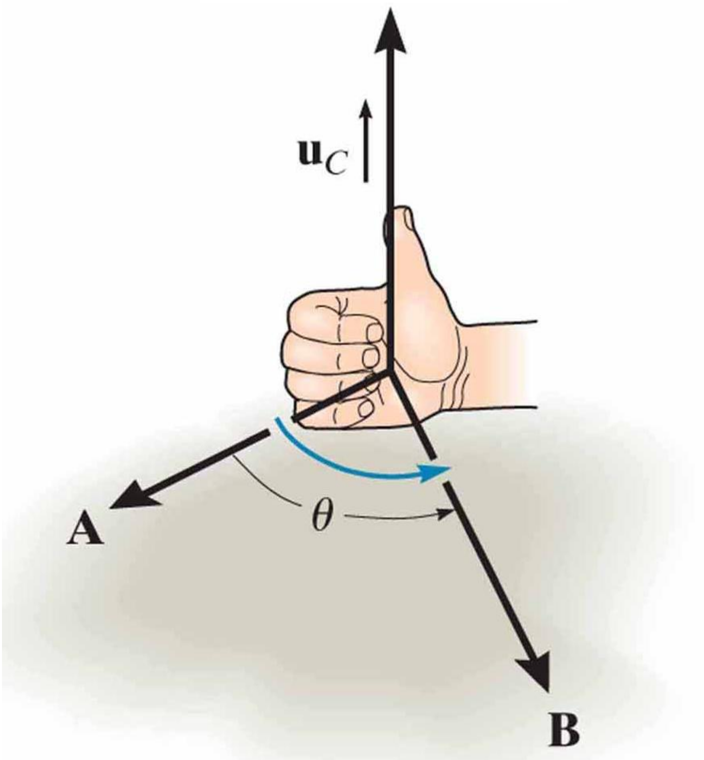
$$M_O = F\,d$$

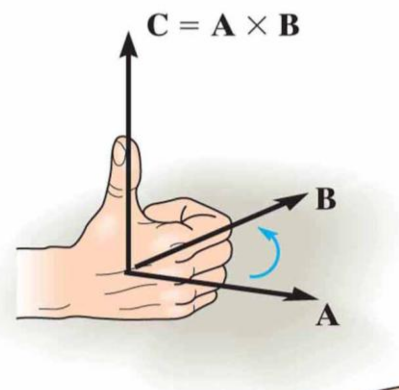
$$M_O \perp F \perp d$$

- Magnitude of moment is still given by product of force and perpendicular distance.
- Direction is perpendicular to line of F and d.
- Can be difficult to determine in 3D using scalars.

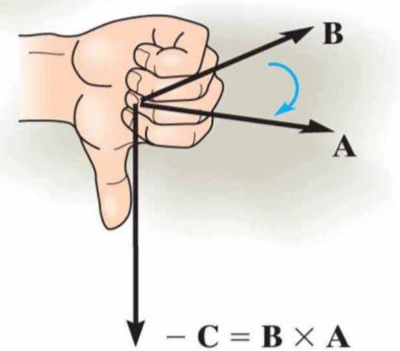
Vector (Cross) Product

$\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$





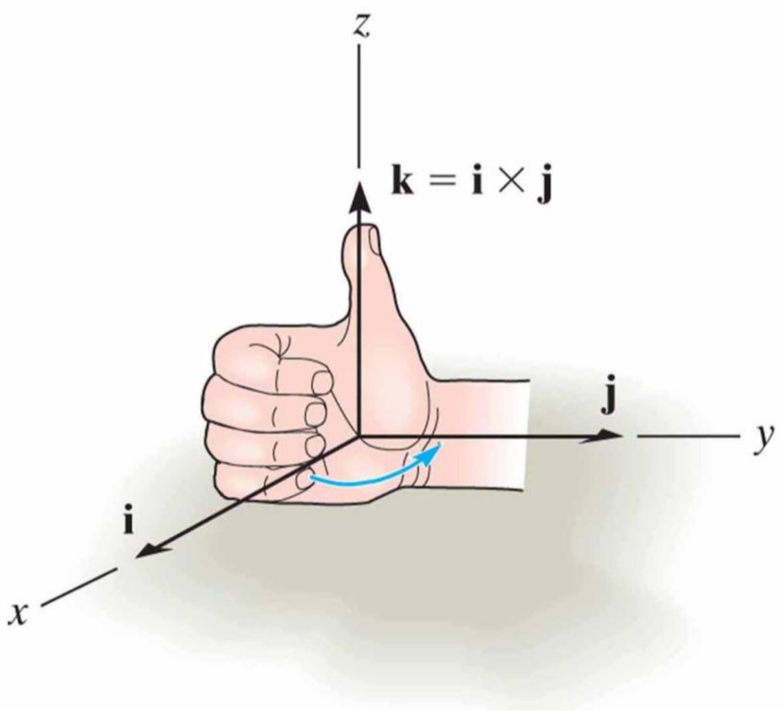
$$\boldsymbol{A} \times \boldsymbol{B} = -(\boldsymbol{B} \times \boldsymbol{A})$$

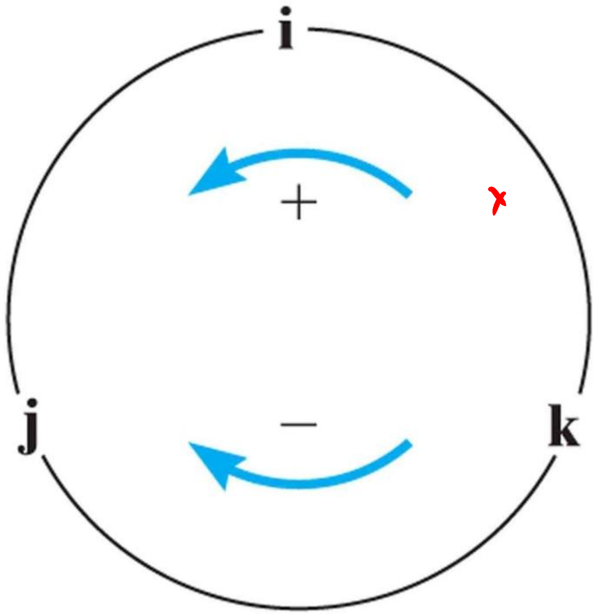


Vector product of two vectors ***A*** and ***B*** is vector ***C***:

1. Line of action of ***C*** is perpendicular to plane of ***A*** and ***B***.
2. Magnitude of ***C*** is $C = AB\sin\theta$
3. Direction of ***C*** is obtained from the right-hand rule.

Cartesian Unit Vectors





$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

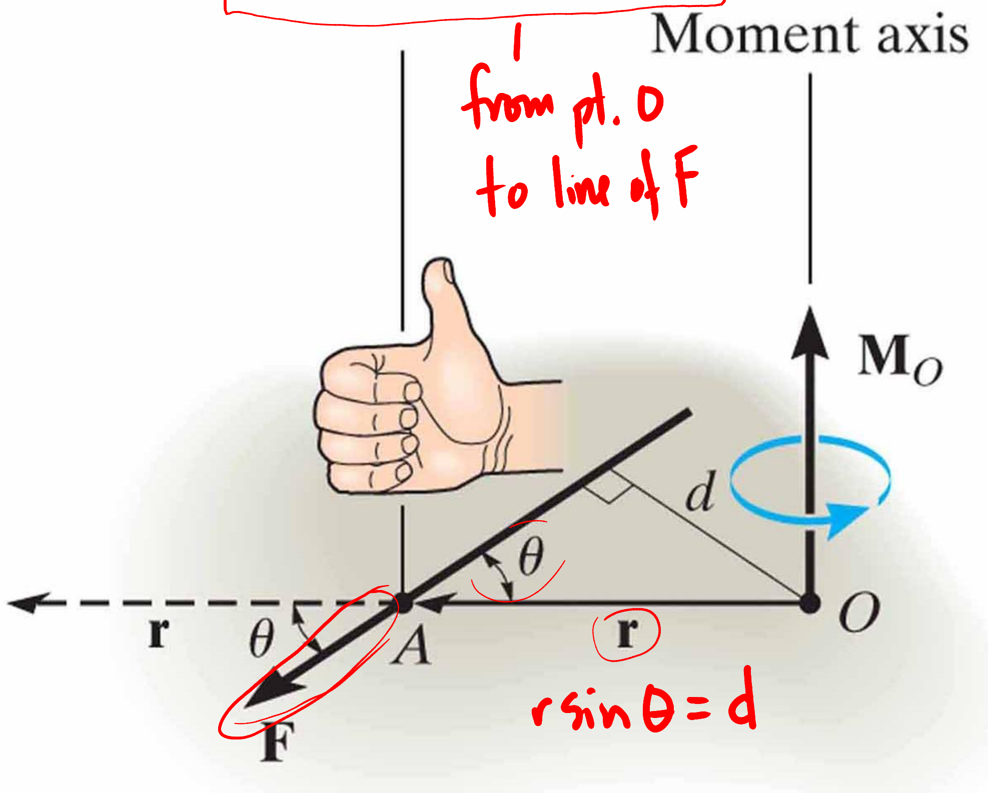
$$\boxed{\vec{M}_O = \vec{r} \times \vec{F}}$$

from pt. O to line of F

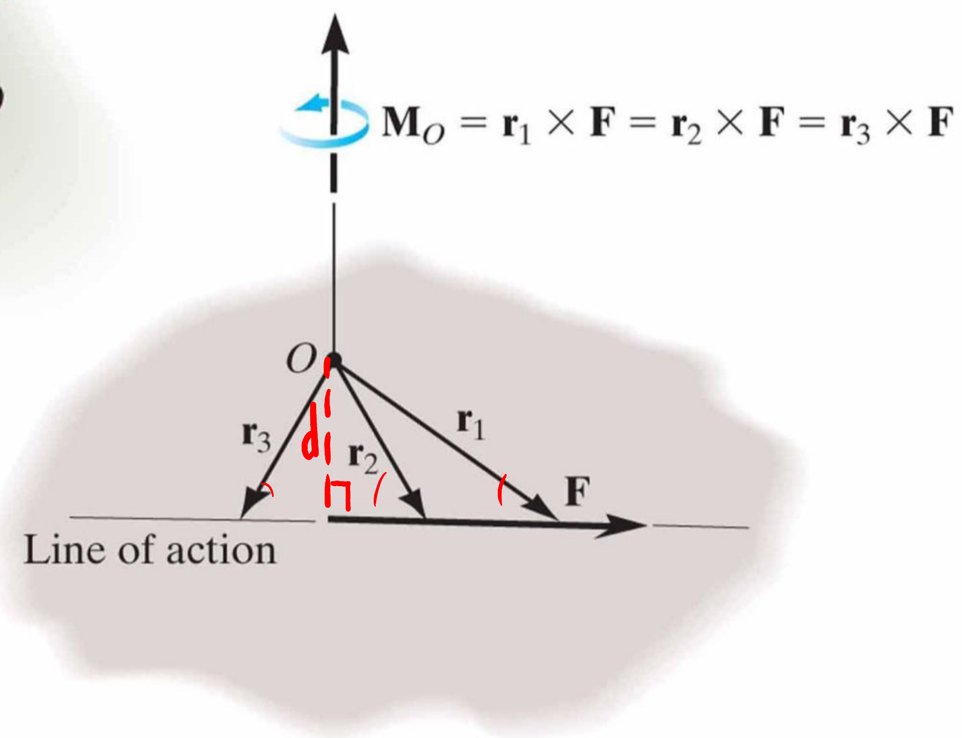
Magnitude of M_O

$$M_O = r\,F\,\sin\theta$$

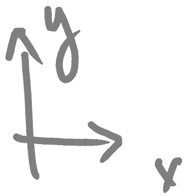
$$M_O = F\,d$$



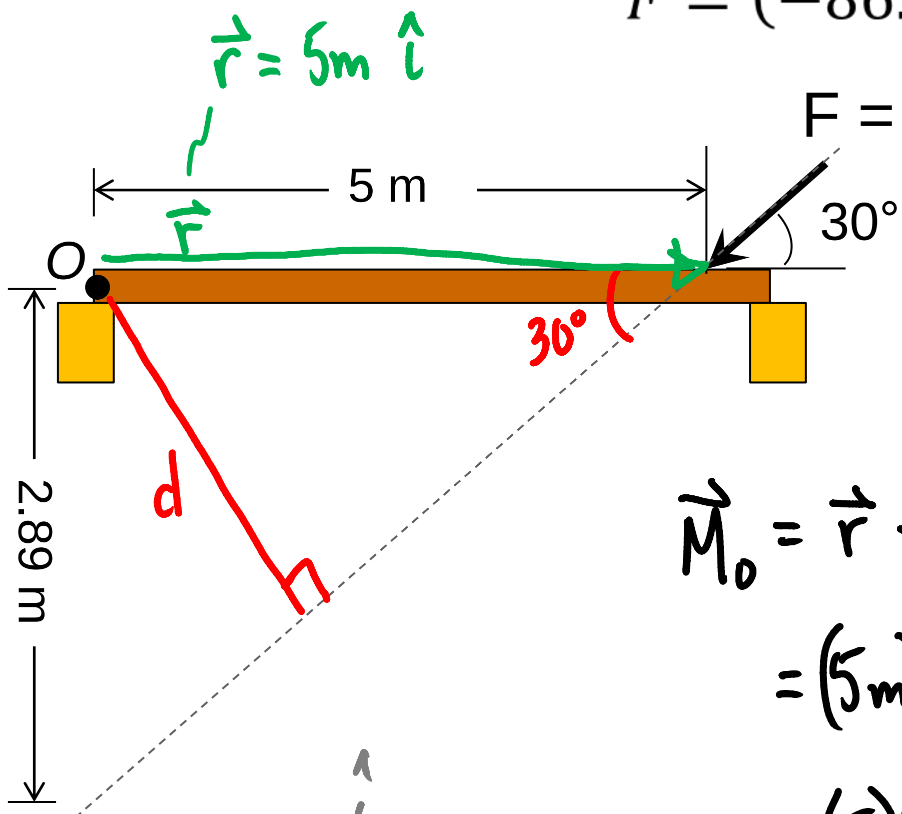
Any position vector **r** that connects O to line of **F** gives same result.



Example in 2D: Find M_O



$$\vec{F} = (-86.6\hat{\imath} - 50\hat{\jmath})\ \text{N}$$



$$d = 5\text{m} \sin 30^\circ = 2.5\text{m}$$

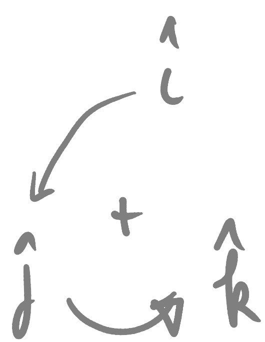
$$M_O = Fd = 100\text{N}(2.5\text{m}) \boxed{= 250\ \text{Nm} \circlearrowright \ (-)}$$

$$\vec{M}_O = \vec{r} \times \vec{F}$$

$$= (5\text{m})\hat{\imath} \times (-86.6\hat{\imath} - 50\hat{\jmath})\ \text{N}$$

$$= \cancel{(5)(-86.6)\hat{\imath}\times\hat{\imath}}^{\,0}$$

$$+ (5\text{m})(-50\text{N})\underbrace{\hat{\imath}\times\hat{\jmath}}_{+\hat{k}} = \boxed{-250\hat{k}\ \text{Nm}}$$



Evaluating Cross Products

$$\overrightarrow{M_O} = \vec{r} \times \vec{F}$$

$$\vec{r} = r_x\hat{\imath} + r_y\hat{\jmath} + r_z\hat{k}$$

$$\vec{F} = F_x\hat{\imath} + F_y\hat{\jmath} + F_z\hat{k}$$

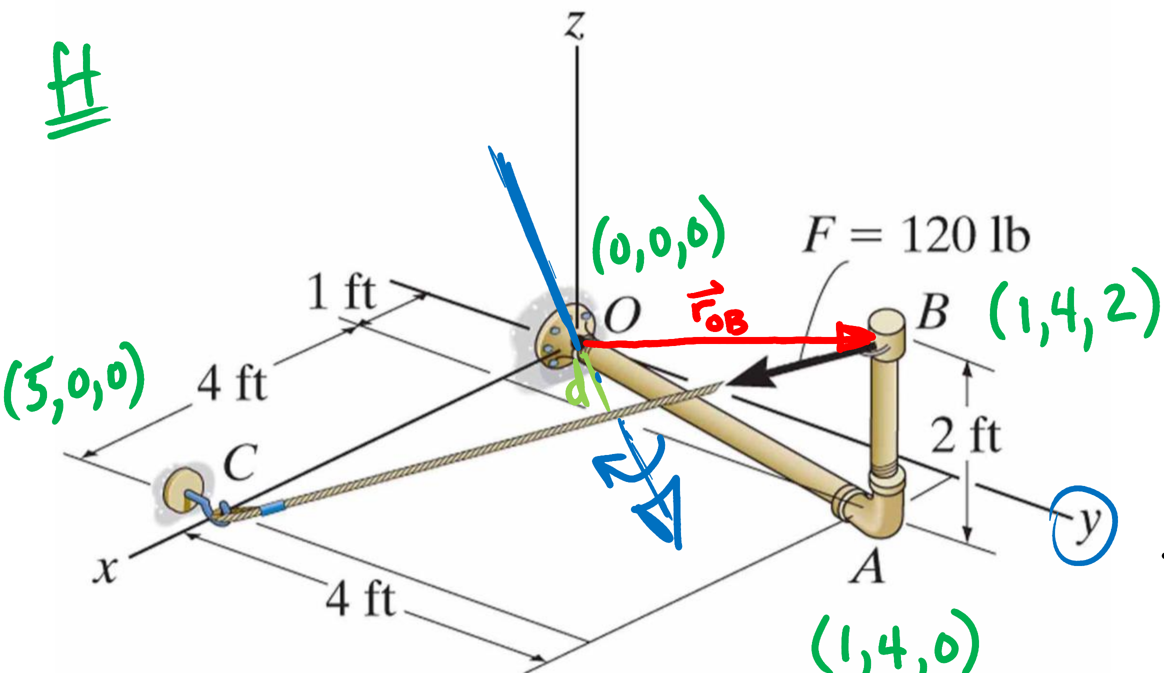
$$\begin{array}{cccccc} & (+) & & & (-) & \\ = \det & \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} & & & \begin{matrix} \hat{\imath} & \hat{\jmath} \\ r_x & r_y \\ F_x & F_y \end{matrix} & \end{array}$$

$$= (r_yF_z - r_zF_y)\hat{\imath} + (r_zF_x - r_xF_z)\hat{\jmath} + (r_xF_y - r_yF_x)\hat{k}$$

Example 2a: Find moment of F about point O

$\vec{M}_O$

ft



$$\vec{M}_O = \vec{r}_{OB} \times \vec{F}$$

$$\vec{r}_{OB} = (\hat{i} + 4\hat{j} + 2\hat{k})\ \text{ft}$$

$$\vec{F} = F\vec{u}_{BC}$$

$$\vec{u}_{BC} = \frac{\vec{r}_{BC}}{r_{BC}} = \frac{(C-B)}{r_{BC}}$$

$$= \frac{(5-1)\hat{i} + (0-4)\hat{j} + (0-2)\hat{k}}{\sqrt{4^2+4^2+2^2}}$$

$$\vec{u}_{BC} = \frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}$$

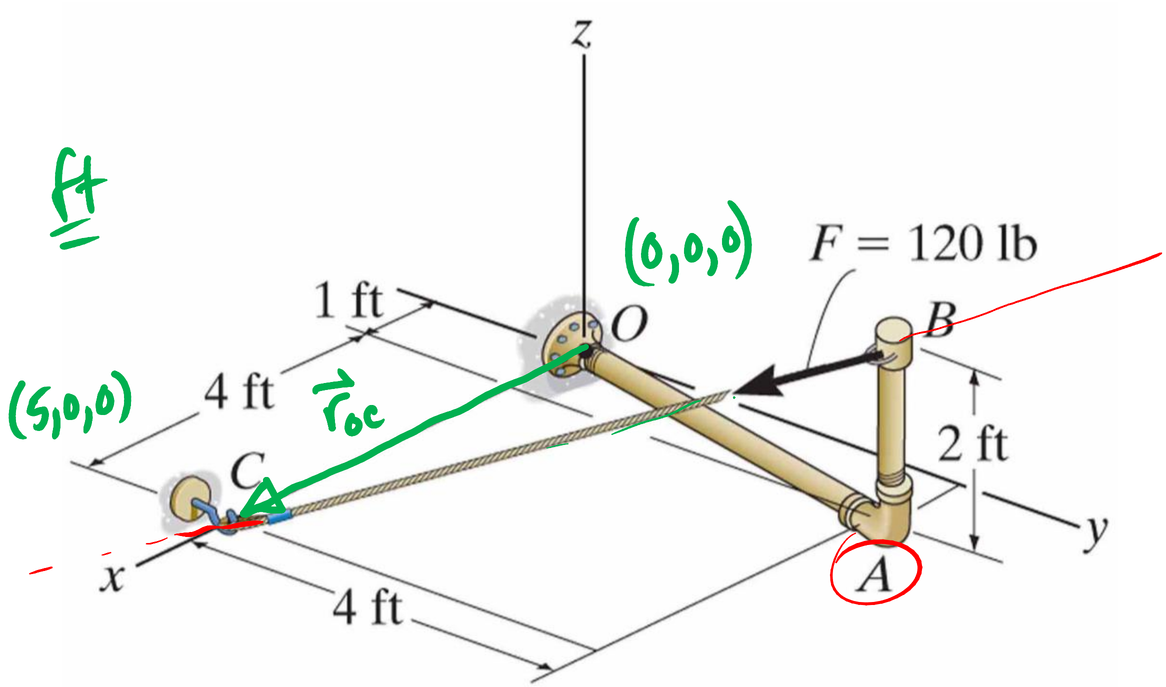
$$\vec{r}_{OB} \times \vec{F} = (\hat{i} + 4\hat{j} + 2\hat{k})^{\text{ft}} \times (120\ \text{lb})\left(\frac{2}{3}\hat{i} - \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k}\right)$$

$$= \begin{array}{c} \\ \text{ft} \\ \text{lb} \end{array} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 80 & -80 & -40 \end{vmatrix} = \begin{array}{l} (-160+160)\hat{i} \\ +(160+40)\hat{j} \\ +(-80-320)\hat{k} \end{array}$$

$$\boxed{\vec{M}_O = (200\hat{j} - 400\hat{k})\ \text{lb}\cdot\text{ft}}$$

$M_O = 447$ lb·ft

Example 2a': Find moment of F about point O



$$\vec{M}_o = \vec{r}_{oc} \times \vec{F}$$

$$\vec{F} = (80\hat{i} - 80\hat{j} - 40\hat{k}) \text{ lb}$$

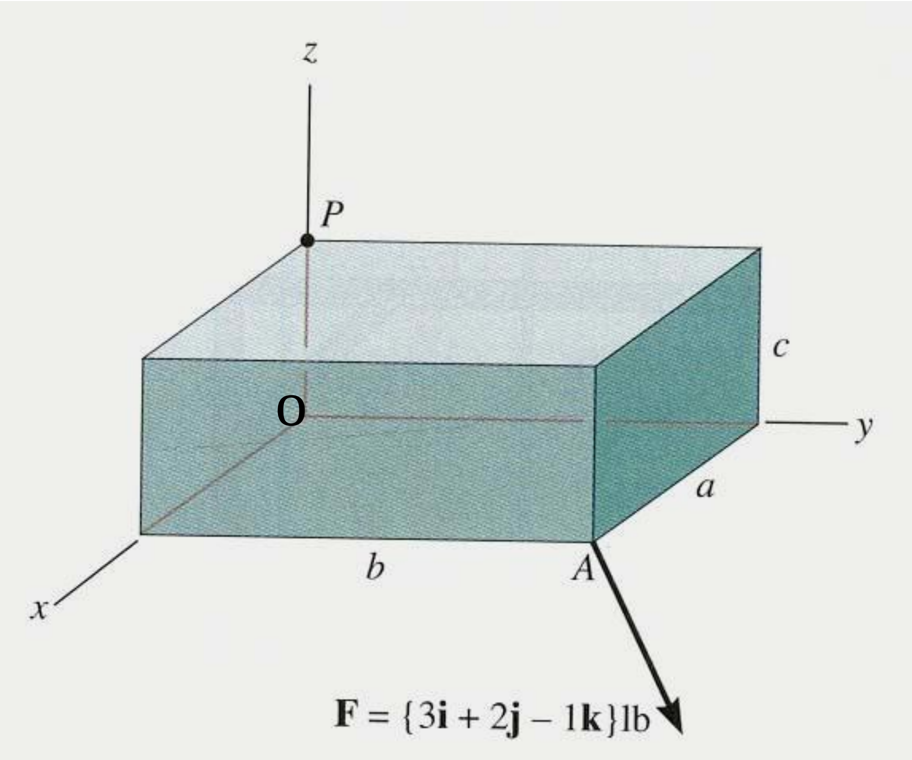
$$\vec{r}_{oc} = 5\hat{i} \text{ ft}$$

$$= 5\hat{i} \times (80\hat{i} - 80\hat{j} - 40\hat{k}) \text{ ft lb}$$

$$= (5)(-80)\underbrace{(\hat{i} \times \hat{j})}_{\hat{k}} + (5)(-40)\underbrace{(\hat{i} \times \hat{k})}_{-\hat{j}}$$

$$\boxed{\vec{M}_o = (200\hat{j} - 400\hat{k}) \text{ lb ft}}$$

EXAMPLE # 3



Given: a = 3 in, b = 6 in and c = 2 in.

Find: Moment of ***F*** about point O.

Plan:

1) Find $\boldsymbol{r}_{OA}$.

2) Determine $\boldsymbol{M}_O = \boldsymbol{r}_{OA} \times \boldsymbol{F}$.

Solution $\boldsymbol{r}_{OA} = \{3\,\boldsymbol{i} + 6\,\boldsymbol{j} - 0\,\boldsymbol{k}\}$ in

$$M_O = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 3 & 6 & 0 \\ 3 & 2 & -1 \end{vmatrix} = [\{6(-1) - 0(2)\}\,\boldsymbol{i} - \{3(-1) - 0(3)\}\,\boldsymbol{j} + \{3(2) - 6(3)\}\,\boldsymbol{k}]\text{ lb·in}$$

$$= \{-6\,\boldsymbol{i} + 3\,\boldsymbol{j} - 12\,\boldsymbol{k}\}\text{ lb·in}$$