(Scalar) Moments & Couples (4.1, 4.4, 4.6)

Moment of a Force (aka, Torque)**:**

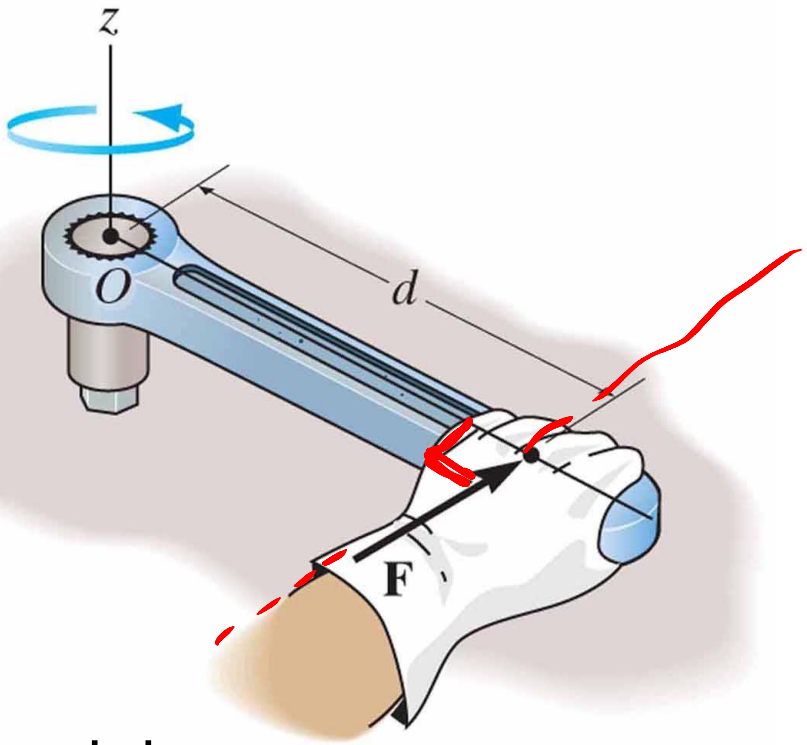
measure of the *tendency* of a force to cause rotation of a body about an axis.

$$M_O = F\,d$$

magnitude of moment about point O

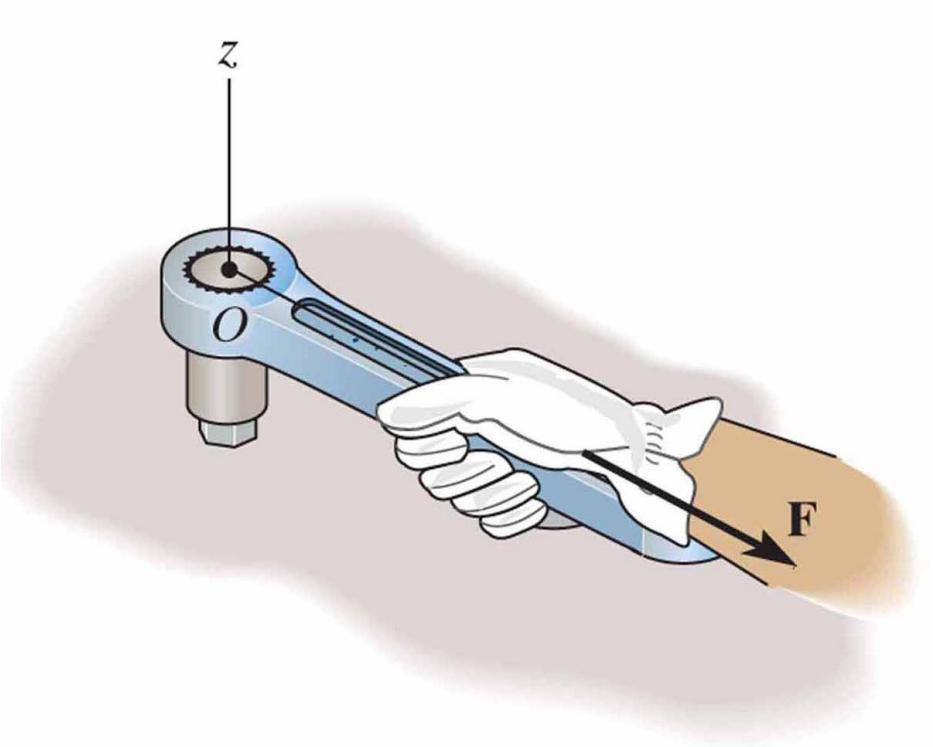
magnitude of F

perpendicular distance from F to O

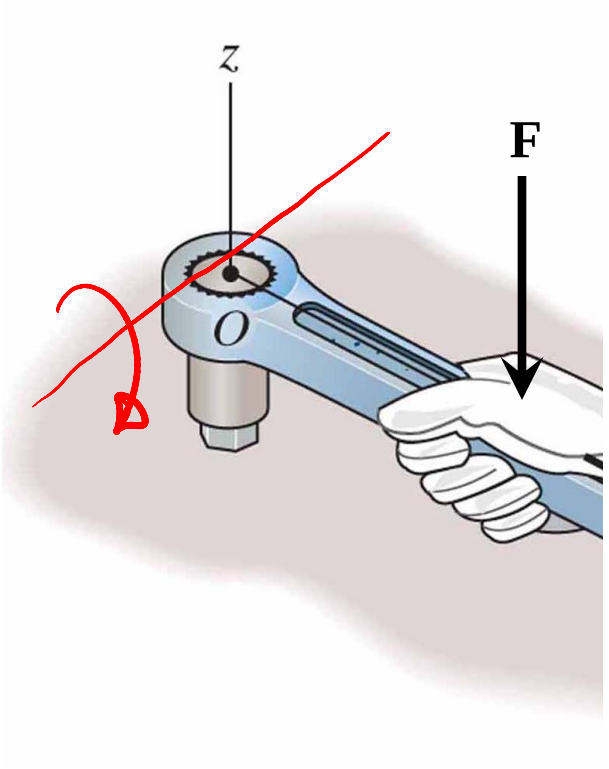


Note direction: axis of rotation is perpendicular to directions of both F and d.

Not all forces create a moment about point O.



This F creates a tendency for *translation*, but **not rotation** about point *O*.

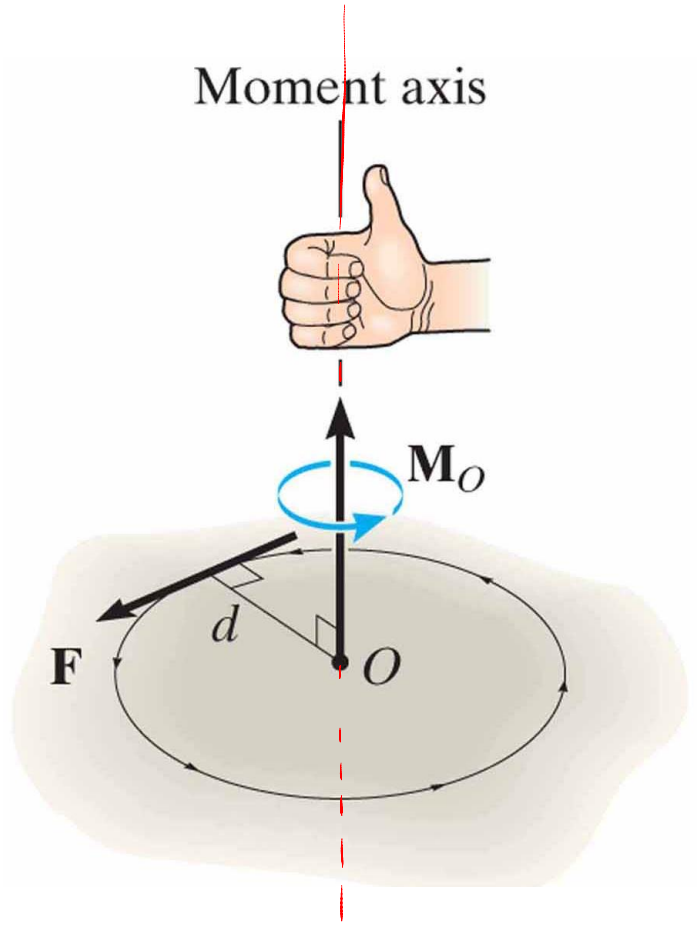


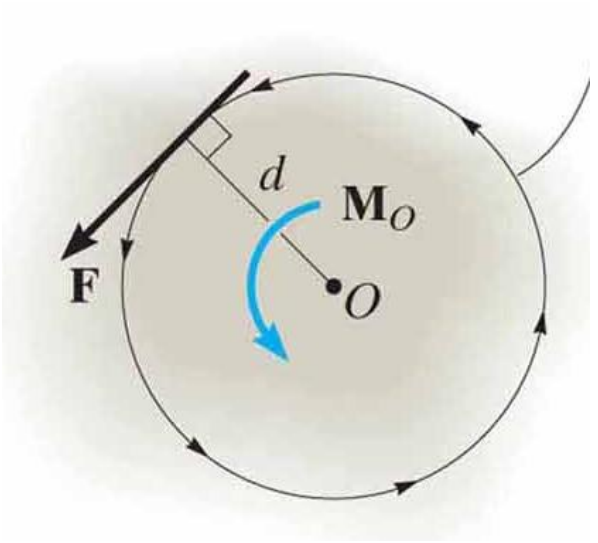
This F creates a tendency for rotation, but **not around the z-axis**.

The Right-Hand Rule

Sign Convention:

Sense of M_O vector is given by thumb of right hand.



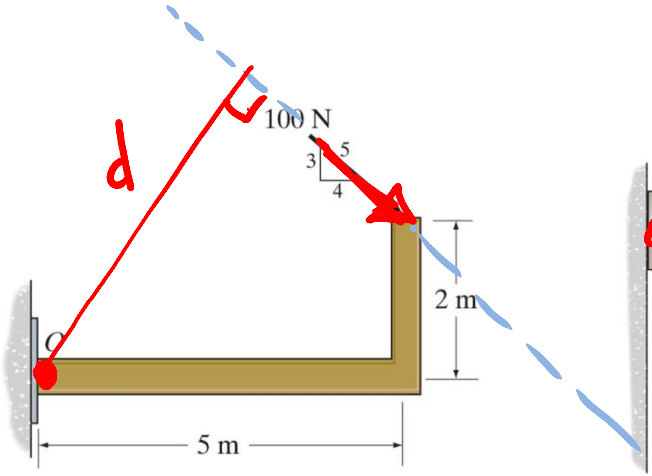


Hence in 2D, counter-clockwise is positive (thumb out of plane).

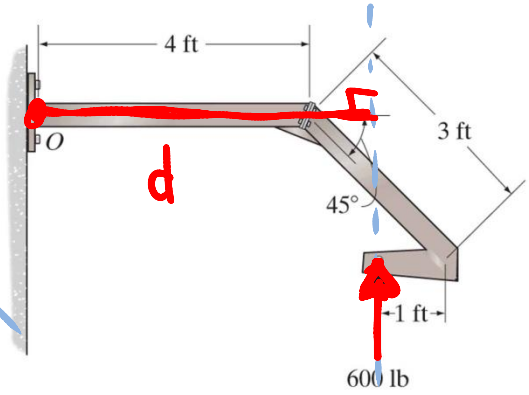
Sense of M_O

Example 1:

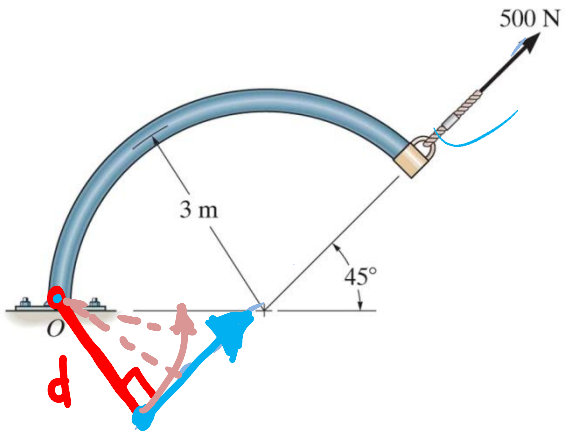
Determine the sense of M_O for each example below:



(−)



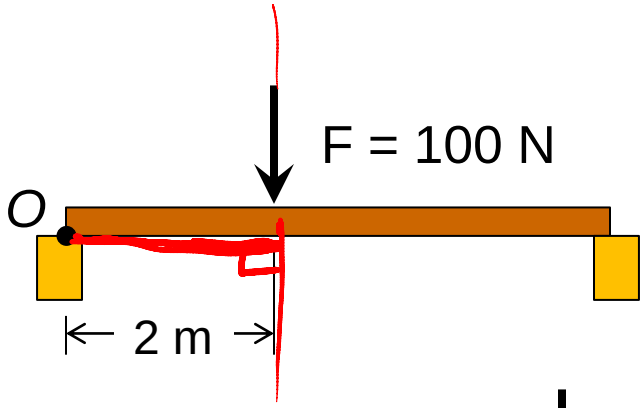
(+)



(+)

Magnitude of M_O

* Note: F doesn't have to cause actual rotation to create M, just a tendency to rotate.

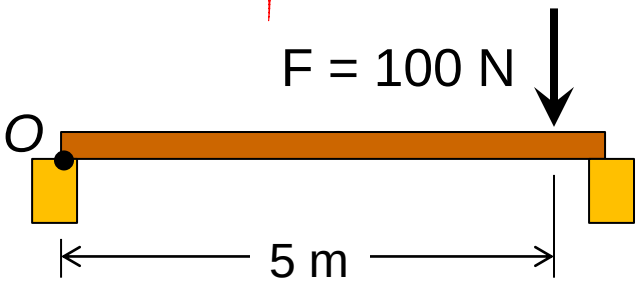


$\vec{M}_O = (-200\,\text{Nm})\,\hat{k}$

$M_O = F\,d$

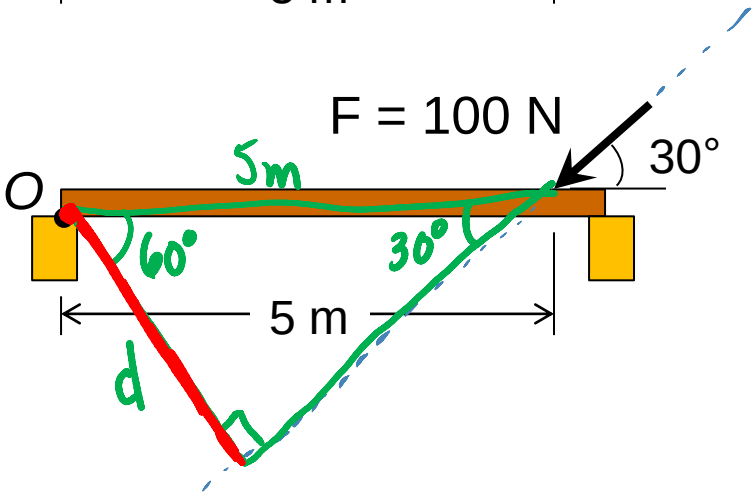
$M_O = (100\,\text{N})(2\,\text{m}) = 200\,\text{Nm}$ ↻

$= -200\,\text{Nm}$



$M_O = (100\,\text{N})(5\,\text{m}) = 500\,\text{Nm}$ ↻

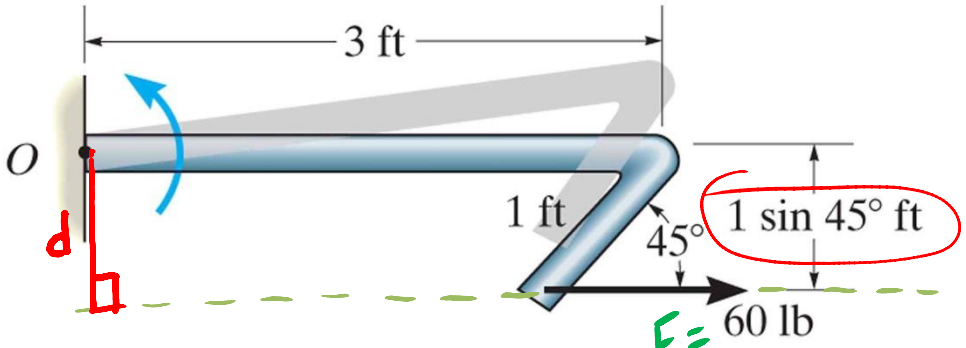
$= -500\,\text{Nm}$



$M_O = F\,d$ $\quad d = 5\,\text{m}\sin 30° = 2.5\,\text{m}$

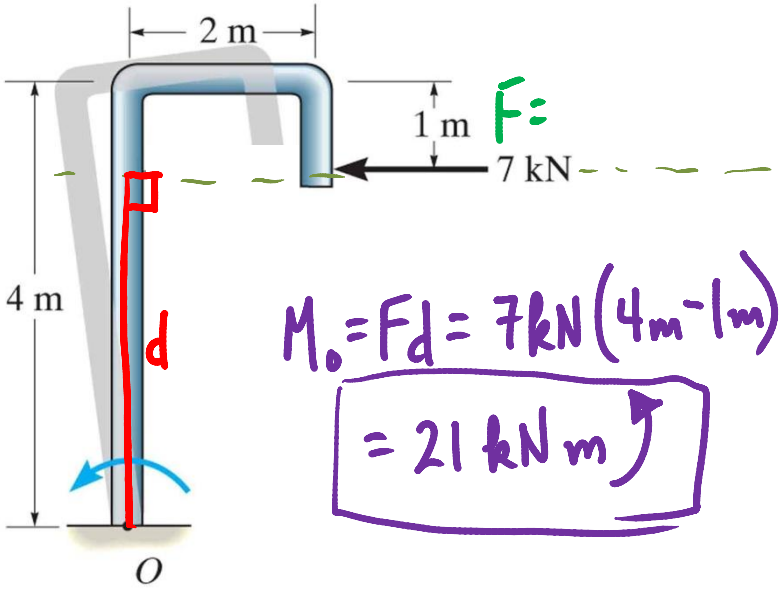
$M_O = (100\,\text{N})(2.5\,\text{m}) = 250\,\text{Nm}$ ↻

Example 2: Determine M_O for each example



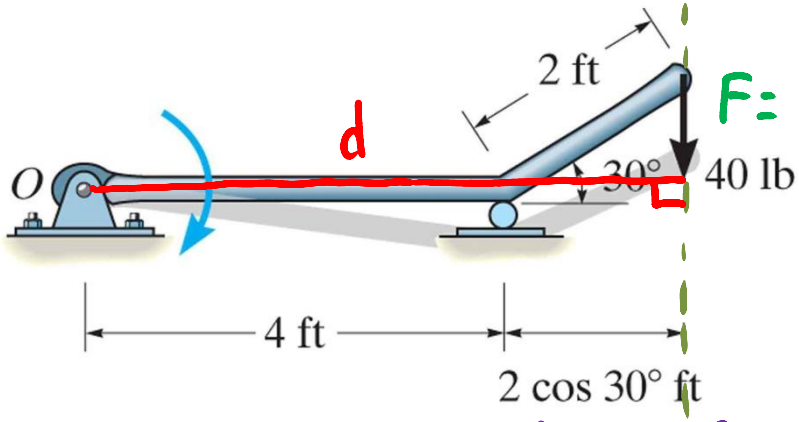
$M_O = Fd$

$= 60\,\text{lb}(1 \sin 45°\,\text{ft})$ $= 60 \sin 45°\,\text{lb ft}$

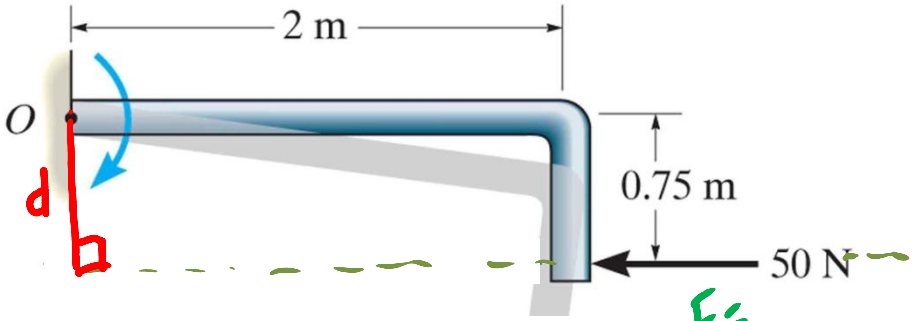


$M_O = Fd = 7\,\text{kN}(4\,\text{m} - 1\,\text{m})$

$= 21\,\text{kN m}$



$M_O = Fd = 40\,\text{lb}(4 + 2\cos 30°)\,\text{ft} = \ldots$

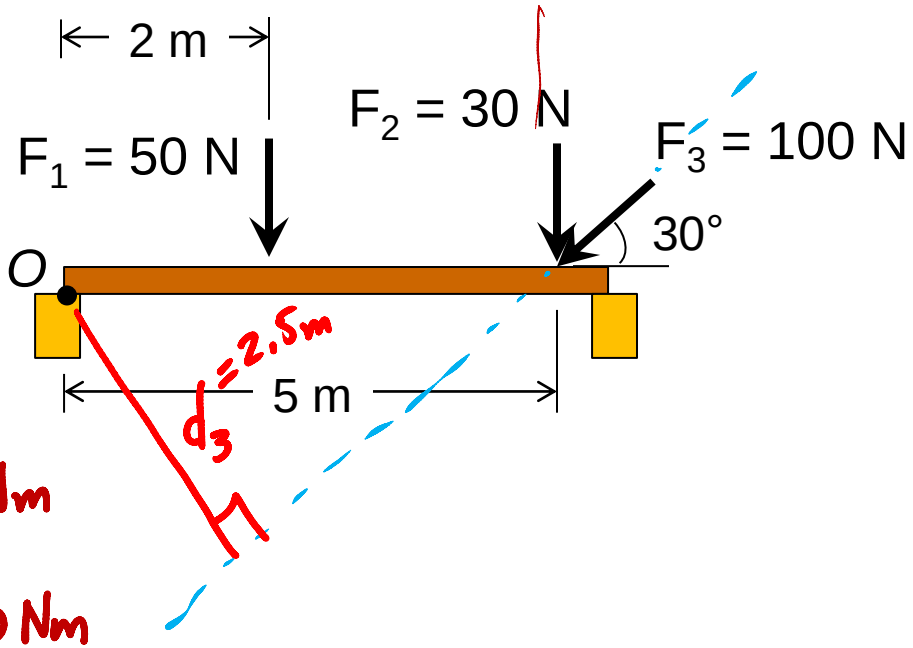


$F =$

$M_O = Fd = 50\,\text{N}(0.75\,\text{m}) = 37.5\,\text{Nm}$

Moments of Multiple Forces – Principle Of Moments

For coplanar forces, resultant moment about a point equals sum of individual moments caused by each of the forces about that point.



$$M_{R_O} = \sum M_{i_O} = \sum F_i d_i$$

$M_{1O} = F_1 d_1 = (50N)(2m) = -100Nm$

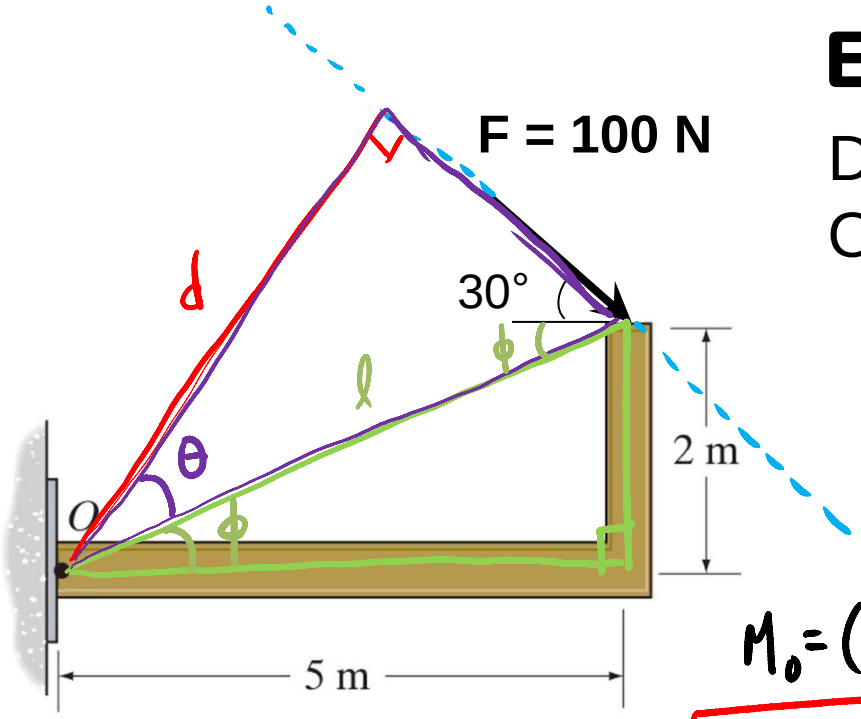
$M_{2O} = F_2 d_2 = (30N)(5m) = -150Nm$

$M_{3O} = F_3 d_3 = (100N)(2.5m) = -250Nm$

$M_{RO} = -500Nm$ or $500Nm$

Example 3:

Determine the moment about point O caused by the force shown.



Find: M_O

Given: F, geometry

$M_O = Fd$

$\ell = \sqrt{5^2+2^2} = \sqrt{29}$

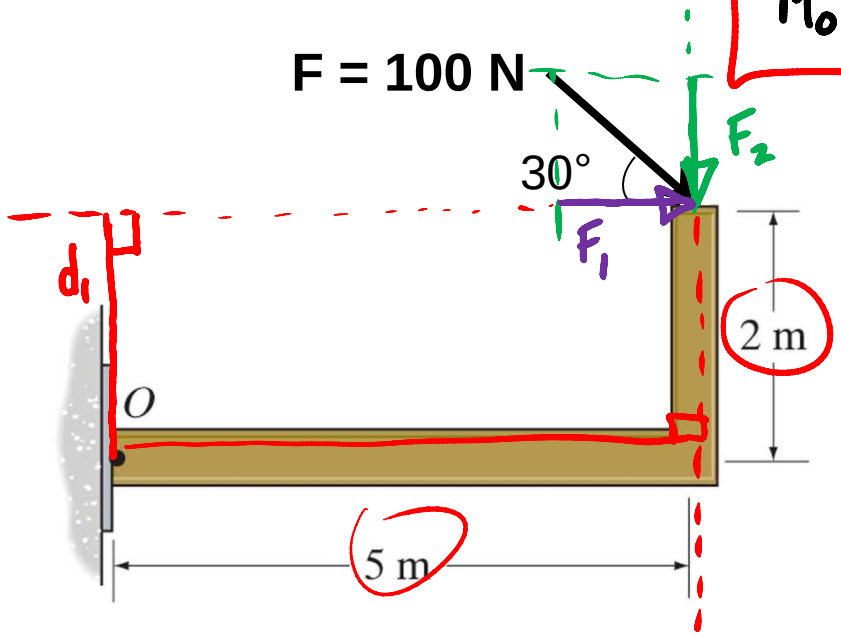
$\phi = \tan^{-1}\left(\frac{2}{5}\right) = 21.8°$

$\theta = 90° - (30° + \phi) = 38.2°$

$d = \ell \cos\theta = 4.23\,m$

$M_O = (100N)(4.23\,m)$

$M_O = 423\,Nm$ ↘



$M_{RO} = M_{1O} + M_{2O} = F_1 d_1 + F_2 d_2$

$F_1 = F\cos 30° = 86.6\,N$ $F_2 = F\sin 30° = 50N$

$d_1 = 2\,m$ $d_2 = 5\,m$

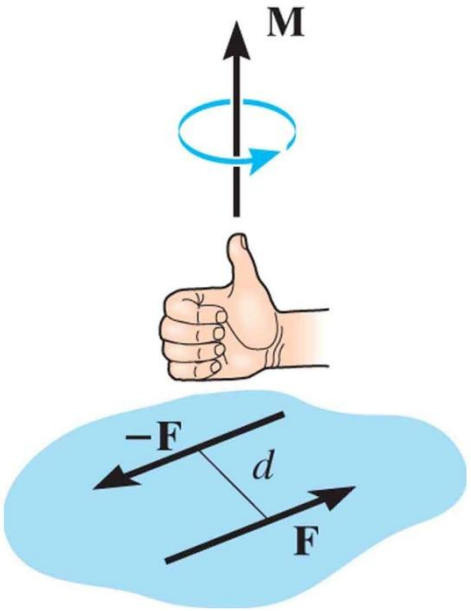
$M_O = -(86.6N)(2m) - (50N)(5m)$

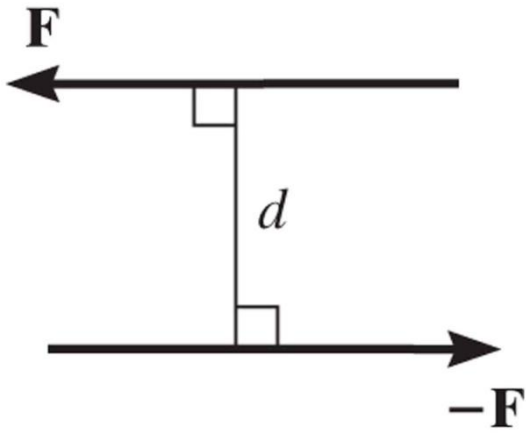
$= 423\,Nm$ ↘

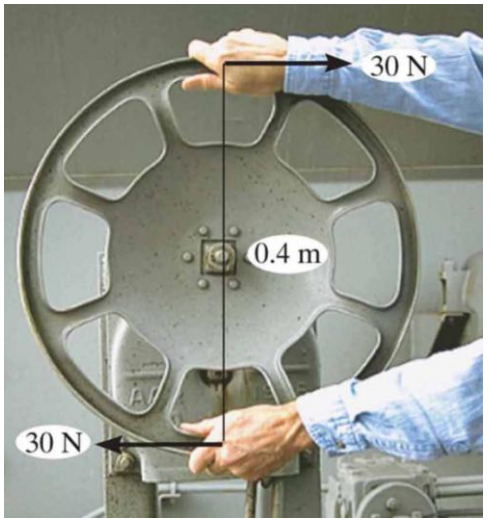
Moment of a Couple

Couple:

- Two parallel forces with equal magnitudes, opposite directions, separated by a distance.
- They generate a pure rotation (no translation).

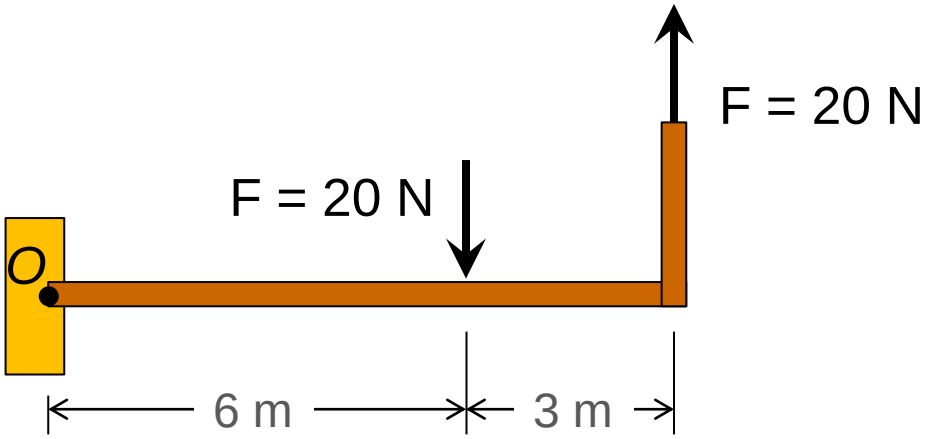






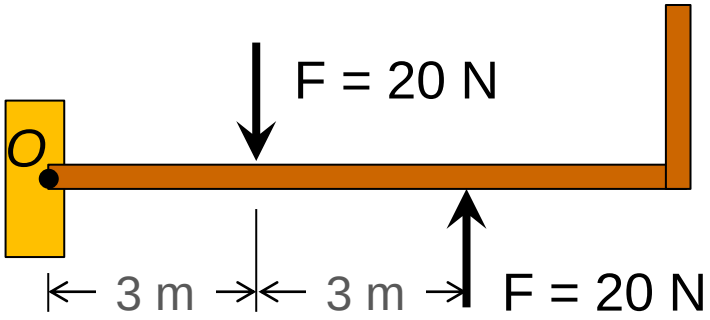
Moment of Couple $\mathbf{M = F\,d}$

Equivalent Couples

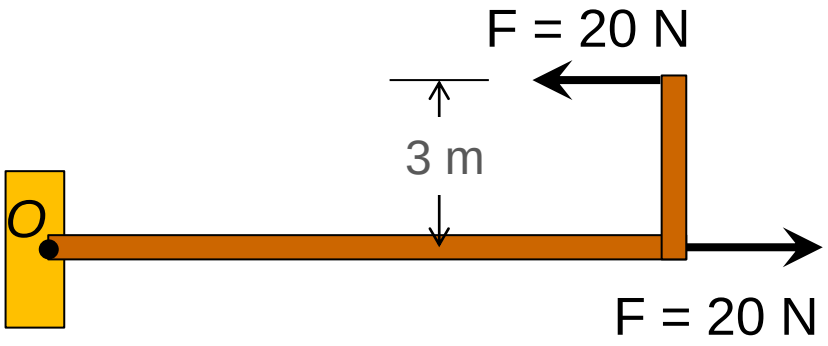


$M_O = M_{10} + M_{20} = -120\,\text{Nm} + 180\,\text{Nm}$

$= 60\,\text{Nm}\,\circlearrowright$

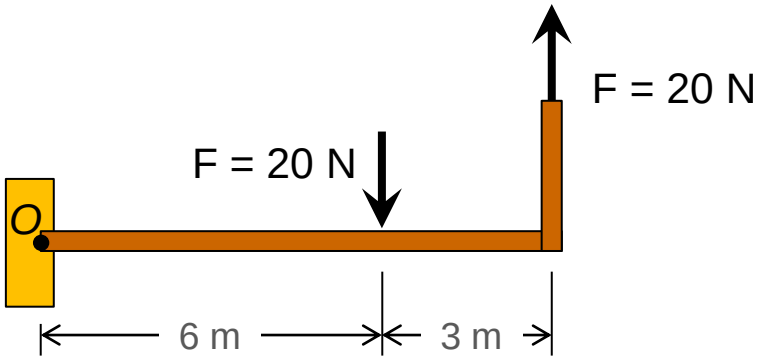


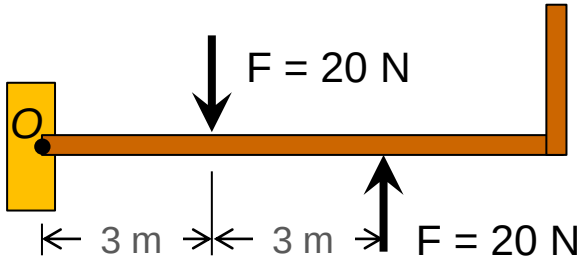
$M_O = -60\,\text{Nm} + 120\,\text{Nm} = 60\,\text{Nm}\,\circlearrowright$

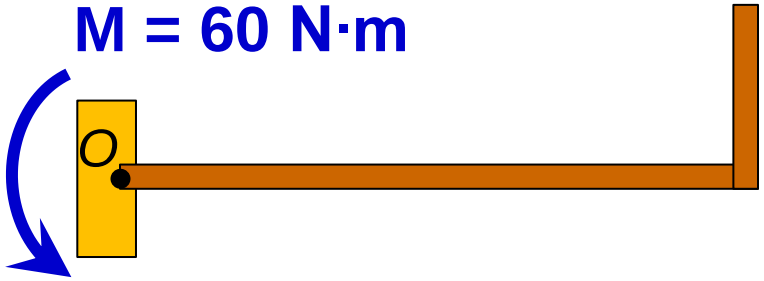


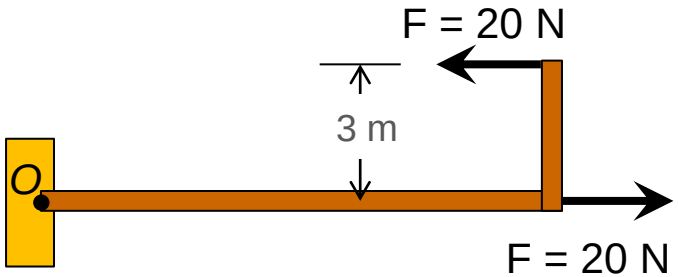
$M_O = 0 + 60\,\text{Nm} = 60\,\text{Nm}\,\circlearrowright$

A Couple Moment is a Free Vector









Example 4: Determine the couple moment created by the forces shown.