# Lab 5 <br> Nodal and Mesh Analysis 

## Objectives

- concepts

1. node voltage and mesh current
2. systems of equations
3. matrix inverse

- skills

1. simulating a circuit
2. building a circuit on a breadboard
3. analyzing circuits using nodal and mesh analysis

## Key Prerequisites

- Chapter 3: Methods of Analysis


## Required Resources

- Circuit Simulator of your choice [CircuitLab or PSpice]
- TinkerCad breadboard simulator [optional]
- MATLAB or FreeMat

The purpose of this lab is to integrate much of what we have been learning in the past few weeks. We are going to take an in-depth look at a single circuit with two voltage sources and four resistors. We will first model the circuit using a circuit simulator. Then we will construct the circuit in real life on a breadboard and verify the same performance with our voltmeter. As further documentation, we will build the same circuit on the virtual breadboard in http://circuits.io. Finally, we will solve for the principle circuit variables using first nodal analysis and then mesh analysis to confirm the power of these analysis techniques. In this way we will experience the process that engineers often go through in developing and validating their circuit designs, alternating from the use of design tools to the experimental verification of their desired circuit's performance.

This lab will also serve to prepare you for the first laboratory practical exam next week, which will require you to breadboard a circuit on circuits.io given a schematic, and measure voltage and current using virtual multimeter.

## Discussion and Procedure

At some point in the development of a circuit design, an engineer will turn to a circuit simulator to determine whether the desired performance can be obtained using the intended approach. Circuit simulators save a tremendous amount of time in flushing out hidden pitfalls, and enable designers to reach the final iteration much more quickly than they would if they had to build successive iterations of their circuit in hardware.

## Part 1 - Build a Circuit Model

1. Develop a simulation of the following circuit. You may use whichever simulator you used for Lab 4. After building the circuit, take a screenshot of your model showing the node voltages V1, V2 and V3, and the currents through resistors R1, R2, R3 and R4 and paste into your datasheet. Use a reference direction of positive current flowing to the right for all horizontal resistors, and flowing downward for all vertical resistors. As a verification, your voltage at node 2 should be 9.593 V .


## Part 2 - Build the Circuit on your Breadboard

2. Take out your circuit kit and select the resistors R1 through R4. Measure these resistors with your ohmmeter and record the readings in Table 1 of your datasheet.
3. This circuit uses two 12 V supplies. Connect two short jumper wires (of different color) to the second barrel-jack to screw-terminal adapters so that you can bring the second 12 V supply to the breadboard. Insert the jacks on the power supply plugs and connect to the wall. Measure the power supply voltages and record them also in Table 1 of your datasheet. If they differ significantly, make sure you identify which supply you plan to use for Vx , and which for Vz .
4. Modify your simulation from step 1 with the actual resistor values and power supply voltages, then record another screen capture in your datasheet showing the adjusted node voltages and resistor currents that you would expect to measure in your actual circuit.
5. Now unplug the power supplies and build the circuit on the breadboard. You must come up with your own plan for how to best layout the circuit. There are many possible solutions, but you should keep the following points in mind:
a. It will be easier to debug and trace your circuit if it bears some resemblance to the schematic.
b. It will be easier to measure the different circuit variables if you include a lot of space between components, adding extra jumper wires where appropriate.
c. You don't need to connect your power supplies to the "rails" along the top and bottom of the breadboard. You can connect your power supplies using the adapters to any point on the breadboard in such a way as to emulate the layout of the circuit in the schematic.
6. Before powering up your circuit, it's a good idea to attach a voltmeter to some known point, such as between node 2 and ground, so that when power comes on you can tell immediately whether you have made a mistake in the wiring.
7. Connect power to the circuit and debug if needed. You should have very good agreement between your node 2 voltage and the expected voltage on your second simulation screenshot. When you do, take a photo of your circuit showing the voltmeter value at node 2 , and a second photo close-up of just the breadboard wiring. Insert these into your datasheet.
8. Record the actual node voltages for nodes 1 through 3 in Table 2 of your datasheet, and compute the percentage of error between your measurements and your circuit model from step 4. If you have any errors greater than $2 \%$, make sure you debug your circuit or your measurement approach. People in lab had $3.5 \%$ errors due to wiring problems that were resolved when corrected.
9. Measure the voltage across each resistor (you can do this by connecting the voltmeter in parallel to the resistor you are tring to measure, or you could do this by subtracting the node voltages on either side of the resistor). Use the volage you measure and Ohm's law to compute the current through resistors R1 through R4 and record these in Table 3 of your datasheet, along with the percentage of error, again using an error of more than $2 \%$ as a clue that you must debug your work.

## Part 3 - Build the Circuit on a Virtual Breadboard using TinkerCAD Circuits [optional]

10. Since it is often more convenient to build a simulated circuit, and it is much easier to read than a photograph of your circuit, please log into TinkerCAD and reconstruct your circuit using two power supplies and 4 resistors. Set the voltage sources to your measured supply voltages, and set your resistors to your measured resistance values.

If you haven't used TinkerCAD yet, you may refer to this brief video which shows how to build the circuit from Lab 2, and also shows how to measure voltage and current.
11. Connect a virtual voltmeter to measure the node voltage V1 and a virtual ammeter to measure the current through R4. Take a screenshot and paste this simulated breadboard into your datasheet.

## Part 4 - Derive the Circuit Variables using Nodal and Mesh Analysis

Now that you have verified your circuit matches the behavior of your circuit model, you will derive the circuit operating voltages and currents using Nodal and Mesh analysis.
12. Use nodal analysis to calculate the node voltages V1 through V3. Use the space below to work out your equations. See the appendix for a review of solving systems of equations in MATLAB or Freemat. Record your A and b matrices as well as the final answers to Table 4 of the datasheet. If your answers differ from your prior work, then go back and check your work for errors.

Derive Nodal Equations Here

13. Now use mesh analysis to calculate the mesh currents $\mathrm{Ia}, \mathrm{Ib}$, and Ic. Use the space below to work out your equations. Record your A and b matrices as well as the final answers for the mesh currents to Table 4 of the datasheet.

# Derive Mesh Equations Here 


14. Answer the postlab questions found at the end of the datasheet.

## Part 5 - Optional Challenge

If time permits, and for 5 percent extra credit, simulate following circuit and derive the node voltages and mesh currents in a similar manner to your previous work. Note that expected values for V 2 and Ib are already entered in your datasheet as a validation check.


## Appendix - Solving Systems of Equations using MATLAB/Freemat

The application of nodal or mesh analysis will result in a set of linear equations that must be solved for the node voltages (nodal analysis) or the mesh currents (mesh analysis). MATLAB can be utilized to solve a set of linear equations. Let's suppose that the nodal equations for a given circuit are:

$$
\left[\begin{array}{ccc}
7 & -3 & -4 \\
-3 & 6 & -2 \\
-4 & -2 & 11
\end{array}\right]\left[\begin{array}{c}
\mathrm{V}_{1} \\
\mathrm{~V}_{2} \\
\mathrm{~V}_{3}
\end{array}\right]=\left[\begin{array}{c}
-11 \\
3 \\
25
\end{array}\right]
$$

This equation is of the form: $\mathrm{Ax}=\mathrm{b}$. A is an n x n matrix, where is the number of node voltages. The dimension of the both vector x and b is $\mathrm{n} x 1$. The unknown node voltages are contained in vector x . Using MATLAB to solve for the node voltages is illustrated in Figure 1 below.

The matrix A is entered first. Note that a semi-colon is utilized to indicate the end of a row. The vector $b$ is entered next. The node voltages are calculated by multiplying the inverse of $A$ by $b$. This is accomplished with the statement

$$
\mathrm{x}=\operatorname{inv}(\mathrm{A}) * \mathrm{~b} \quad \text { OR } \quad \mathrm{x}=\mathrm{Alb} \quad \text { (the latter is the preferred method) }
$$

```
--> A = [7 -3 -4; -3 6 -2; -4 -2 11]
A =
    7 -3 -4
    -3 6 -2
    -4 -2 11
--> b = [-11; 3; 25]
b =
    -11
    3
    25
--> x = inv(A) *b
x =
    1.0000
    2.0000
    3.0000
--> x = A\b
x =
    1.0000
    2.0000
    3.0000
```

Figure 1. Solution of a set of linear equations using MATLAB or FreeMat

Lab 5: Nodal and Mesh Analysis

