Second Order Circuits

Revised 5/6/15

Table 1: Natural Response of Series RLC

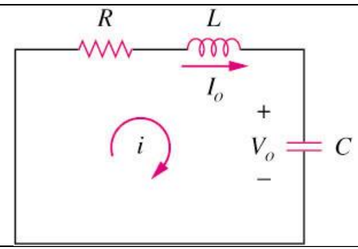
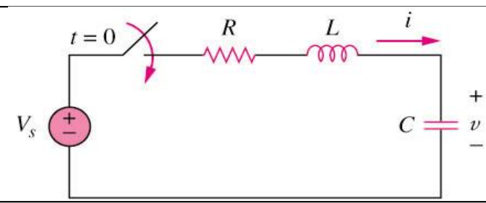
Circuit			
Diff-Eq	$\frac{d^2i}{dt^2} + 2\alpha\frac{di}{dt} + \omega_0^2 i = 0$	$\alpha = \frac{R}{2L}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$	
Damping	Over: $\omega_0^2 < \alpha^2$	Under: $\omega_0^2 > \alpha^2$	Critical: $\omega_0^2 = \alpha^2$
Form of Soln	$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$i(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$	$i(t) = A_1 t\, e^{-\alpha t} + A_2 e^{-\alpha t}$
Roots	$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$s_{1,2} = -\alpha \pm j\omega_d$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$s_{1,2} = -\alpha$
Boundary Conditions	$i(0+) = A_1 + A_2$ $\frac{di(0+)}{dt} = \frac{v_L(0+)}{L} = s_1 A_1 + s_2 A_2$	$i(0+) = B_1$ $\frac{di(0+)}{dt} = \frac{v_L(0+)}{L} = -\alpha B_1 + \omega_d B_2$	$i(0+) = A_2$ $\frac{di(0+)}{dt} = \frac{v_L(0+)}{L} = A_1 - \alpha A_2$

Table 2: Step Response of Series RLC

Circuit			
Diff-Eq	$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{v_s}{LC}$	$\alpha = \frac{R}{2L}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$	
Damping	Over: $\omega_0^2 < \alpha^2$	Under: $\omega_0^2 > \alpha^2$	Critical: $\omega_0^2 = \alpha^2$
Form of Soln	$v(t) = v(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$v(t) = v(\infty) + e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$	$v(t) = v(\infty) + A_1 t\, e^{-\alpha t} + A_2 e^{-\alpha t}$
Roots	$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$s_{1,2} = -\alpha \pm j\omega_d$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$s_{1,2} = -\alpha$
Boundary Conditions	$v(0+) = v(\infty) + A_1 + A_2$ $\frac{dv(0+)}{dt} = \frac{i_C(0+)}{C} = s_1 A_1 + s_2 A_2$	$v(0+) = v(\infty) + B_1$ $\frac{dv(0+)}{dt} = \frac{i_C(0+)}{C} = -\alpha B_1 + \omega_d B_2$	$v(0+) = v(\infty) + A_2$ $\frac{dv(0+)}{dt} = \frac{i_C(0+)}{C} = A_1 - \alpha A_2$

Second Order Circuits

Table 3: Natural Response of Parallel RLC

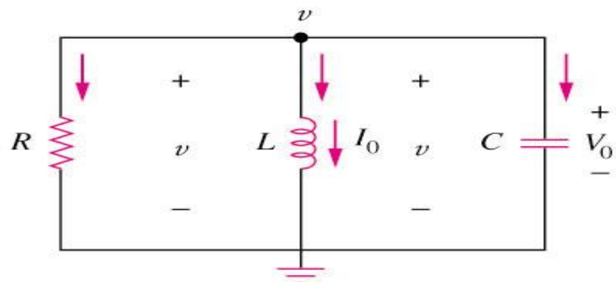
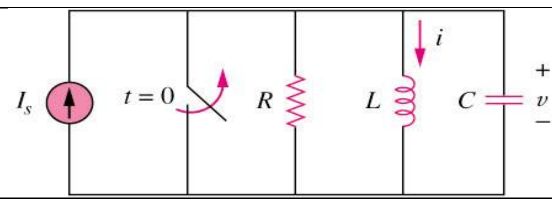
Circuit			
Diff-Eq	$\frac{d^2v}{dt^2} + 2\alpha\frac{dv}{dt} + \omega_0^2 v = 0$ where $\alpha = \frac{1}{2RC}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$		
Damping	Over: $\omega_0^2 < \alpha^2$	Under: $\omega_0^2 > \alpha^2$	Critical: $\omega_0^2 = \alpha^2$
Form of Soln	$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$v(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$	$v(t) = (A_2 + A_1 t)\, e^{-\alpha t}$
Roots	$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$s_{1,2} = -\alpha \pm j\omega_d$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$s_{1,2} = -\alpha$
Boundary Conditions	$v(0+) = A_1 + A_2$ $\frac{dv(0+)}{dt} = \frac{i_C(0+)}{C} = s_1 A_1 + s_2 A_2$	$v_c(0+) = B_1$ $\frac{dv(0+)}{dt} = \frac{i_C(0+)}{C} = -\alpha B_1 + \omega_d B_2$	$v(0+) = A_2$ $\frac{dv(0+)}{dt} = \frac{i_C(0+)}{C} = A_1 - \alpha A_2$

Table 4: Step Response of Parallel RLC

Circuit			
Diff-Eq	$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$ where $\alpha = \frac{1}{2RC}$ and $\omega_0 = \sqrt{\frac{1}{LC}}$		
Damping	Over: $\omega_0^2 < \alpha^2$	Under: $\omega_0^2 > \alpha^2$	Critical: $\omega_0^2 = \alpha^2$
Form of Soln	$i(t) = i(\infty) + A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$i(t) = i(\infty) + e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t)$	$i(t) = i(\infty) + A_1 t\, e^{-\alpha t} + A_2 e^{-\alpha t}$
Roots	$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$	$s_{1,2} = -\alpha \pm j\omega_d$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$	$s_{1,2} = -\alpha$
Boundary Conditions	$i(0+) = i(\infty) + A_1 + A_2$ $\frac{di(0+)}{dt} = \frac{v_L(0+)}{L} = s_1 A_1 + s_2 A_2$	$i(0+) = i(\infty) + B_1$ $\frac{di(0+)}{dt} = \frac{v_L(0+)}{L} = -\alpha B_1 + \omega_d B_2$	$i(0+) = i(\infty) + A_2$ $\frac{di(0+)}{dt} = \frac{v_L(0+)}{L} = A_1 - \alpha A_2$