ENGR12
Circuit Theory

Chapter 6

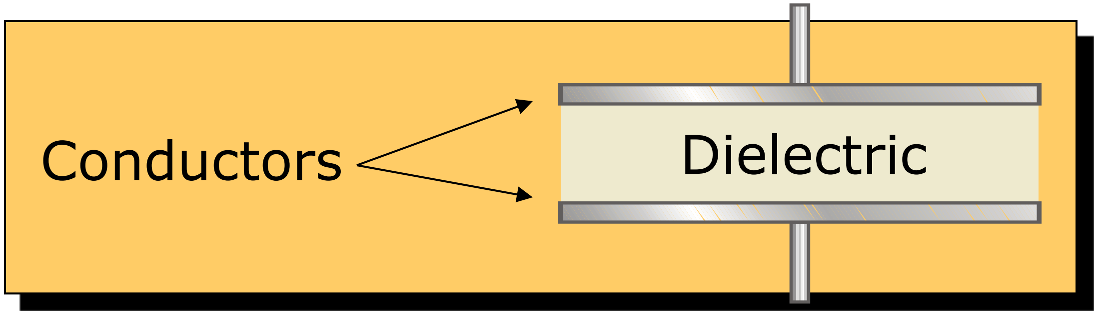
Capacitors and Inductors

Capacitors and Inductors
Chapter 6

The Capacitor

Capacitors are one of the fundamental passive components. In its most basic form, it is composed of two plates separated by a dielectric.

The ability to store charge is the definition of **capacitance**.

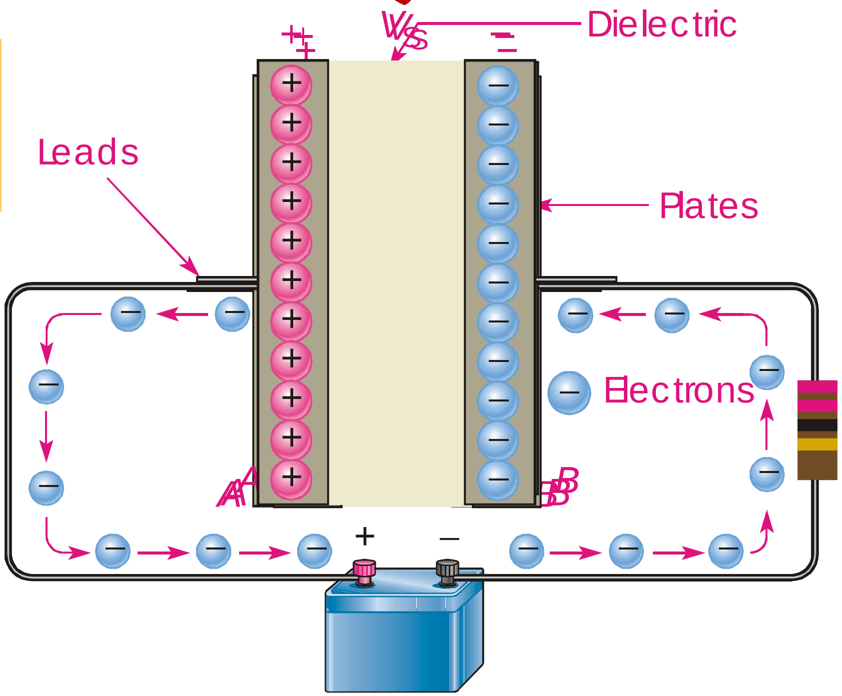


The Capacitor

The charging process…

Initially uncharged / Charging / Fully charged

No current crosses dielectric



A capacitor with stored charge can act as a temporary battery.

The Capacitor

Capacitance is the ratio of charge to voltage

$$C = \frac{Q}{V}$$

Rearranging, the amount of charge on a capacitor is determined by the size of the capacitor (C) and the voltage (V).

$$Q = CV$$

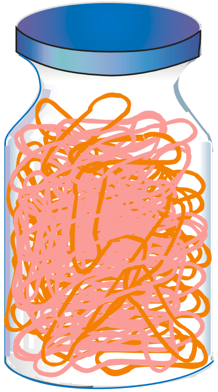
Example

If a 22 μF capacitor is connected to a 10 V source, the charge is 220 μC

The Capacitor

An analogy:

Imagine you store rubber bands in a bottle that is nearly full.



You could store more rubber bands (like charge or Q) in a bigger bottle (capacitance or C) *or* if you push them in more (voltage or V). Thus,

$$Q = CV$$

The Capacitor

A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two plates. The energy of a charged capacitor is given by the equation

$$W = \frac{1}{2}CV^2$$

where

W = the energy in joules
C = the capacitance in farads
V = the voltage in volts

The Capacitor

The capacitance of a capacitor depends on three physical characteristics.

$$C = 8.85 \times 10^{-12}\,\text{F/m}\left(\frac{\varepsilon_r A}{d}\right)$$

C is directly proportional to

the **relative dielectric constant**

and the **plate area**.

C is inversely proportional to

the **distance** between the plates

Capacitance

Example

Find the capacitance of a 4.0 cm diameter sensor immersed in oil if the plates are separated by 0.25 mm. $\left(\varepsilon_r = 4.0 \text{ for oil}\right)$

$$C = 8.85\times10^{-12}\,\text{F/m}\left(\frac{\varepsilon_r A}{d}\right)$$

The plate area is $A = \pi r^2 = \pi\left(0.02\text{ m}^2\right) = 1.26\times10^{-3}\text{ m}^2$

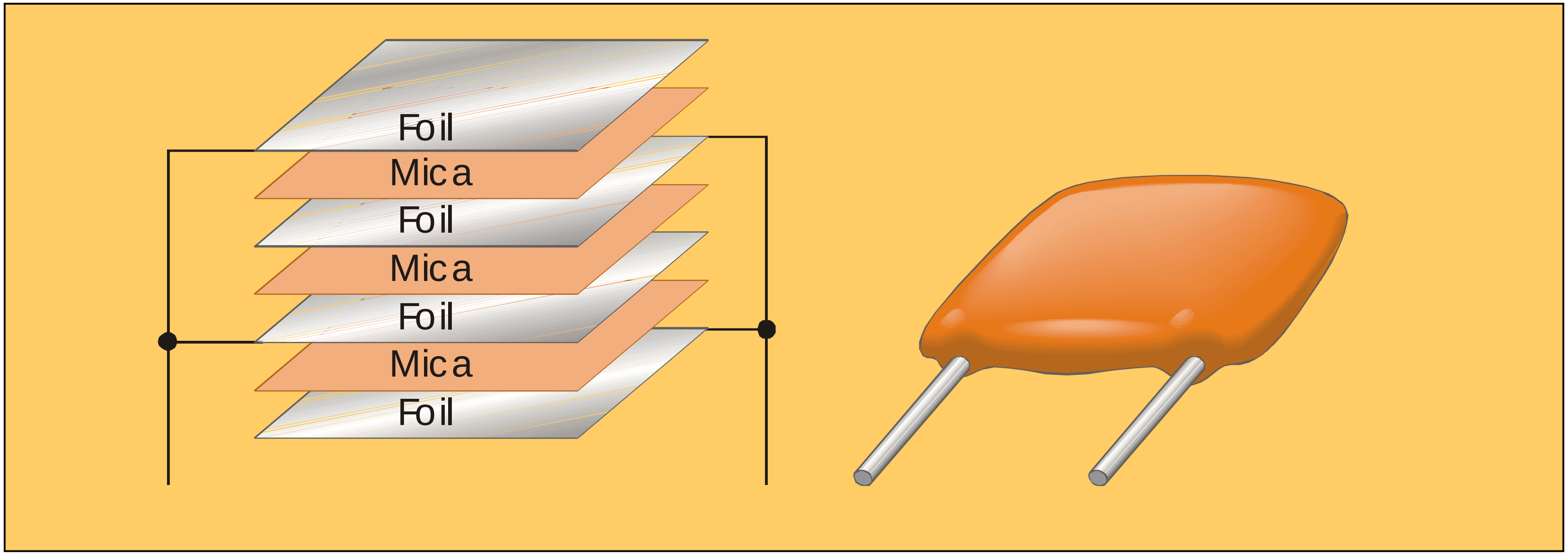
The distance between the plates is 0.25×10^{-3} m

$$C = 8.85\times10^{-12}\,\text{F/m}\left(\frac{(4.0)\left(1.26\times10^{-3}\text{ m}^2\right)}{0.25\times10^{-3}\text{ m}}\right) = 178\text{ pF}$$

Capacitor types

Mica

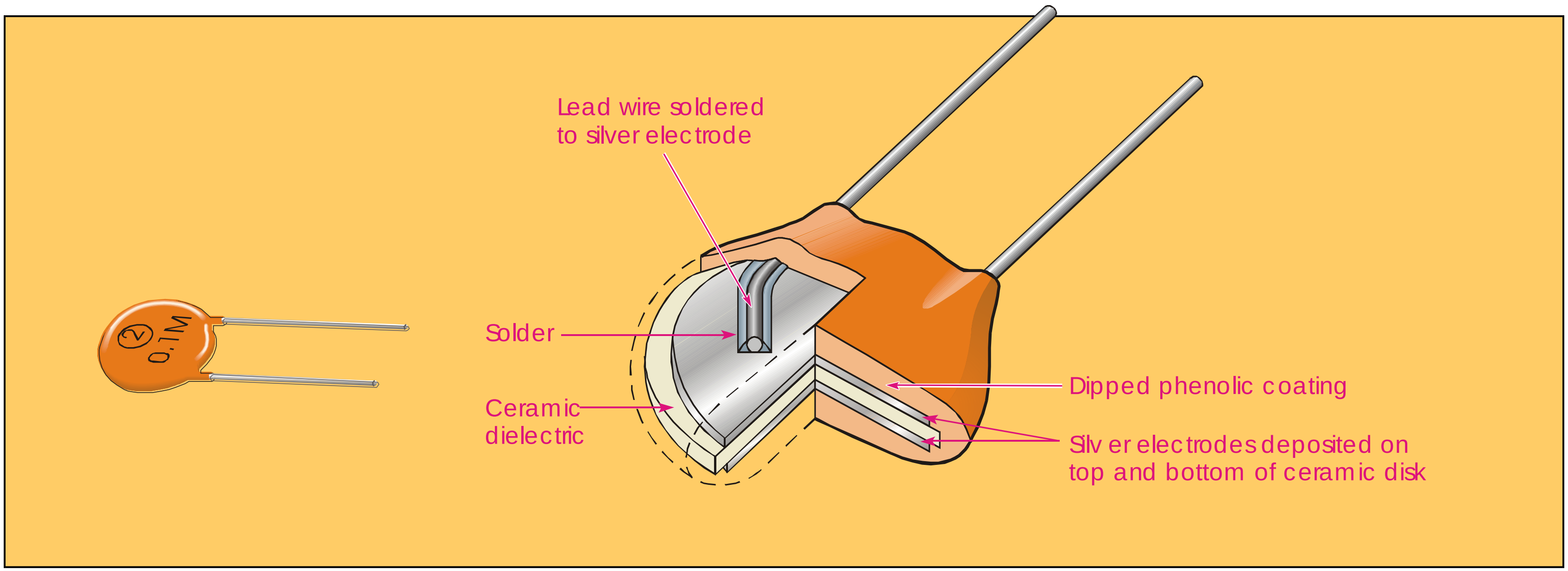
Mica capacitors are small with high working voltage. The **working voltage** is the voltage limit that cannot be exceeded.



Capacitor types

Ceramic disk

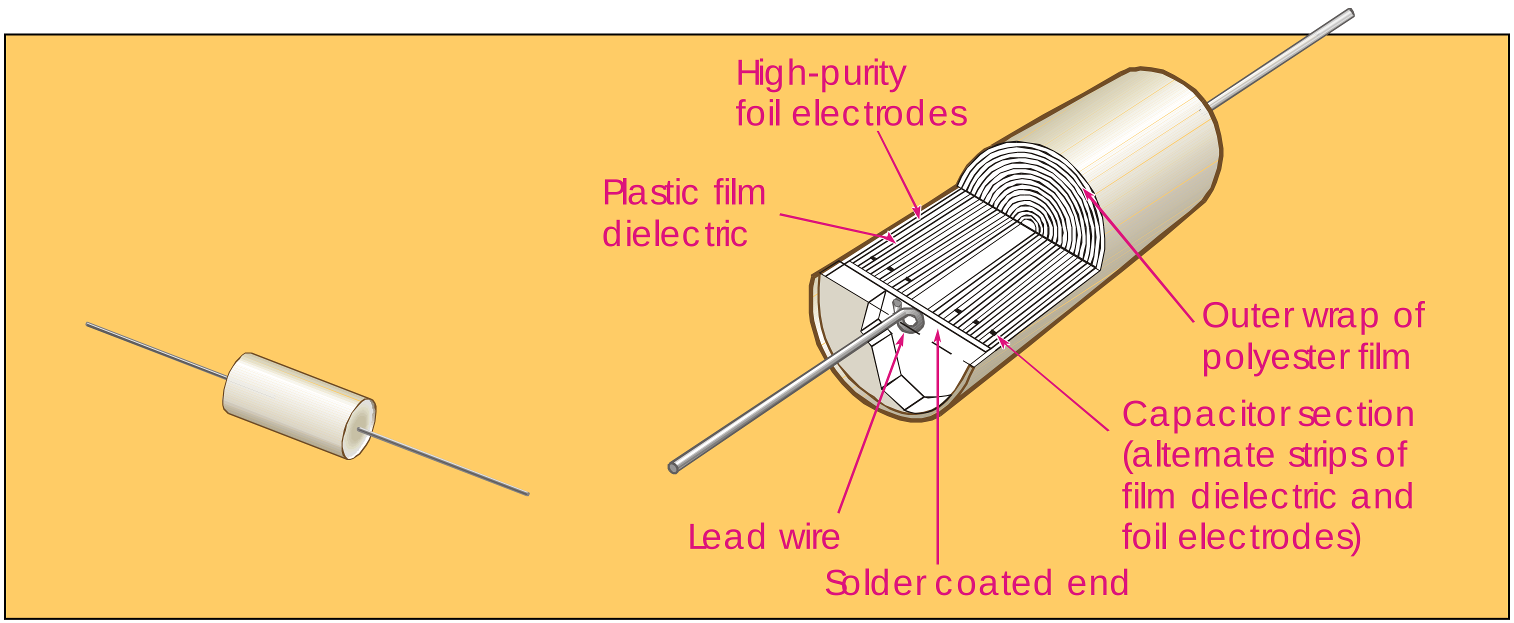
Ceramic disks are small nonpolarized capacitors They have relatively high capacitance due to high ε_{r}.



Capacitor types

Plastic Film

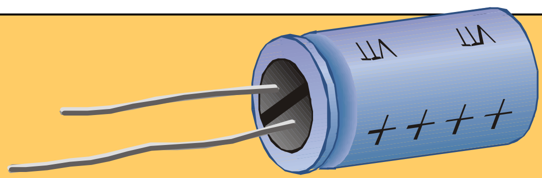
Plastic film capacitors are small and nonpolarized. They have relatively high capacitance due to larger plate area.



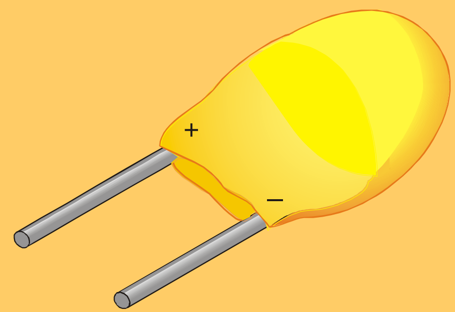
Capacitor types

Electrolytic (two types)

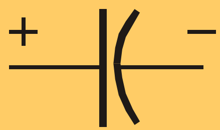
Electrolytic capacitors have very high capacitance but they are not as precise as other types and tend to have more leakage current. Electrolytic types are polarized.



Al electrolytic



Ta electrolytic



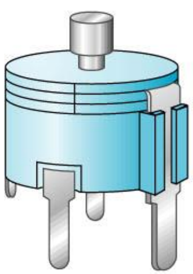
Symbol for any electrolytic capacitor

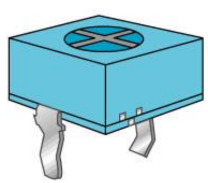
Capacitor types

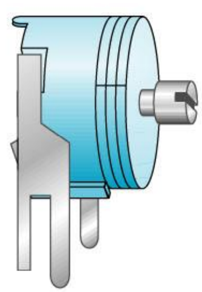
Variable

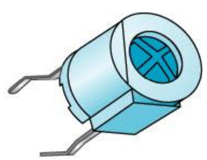
Variable capacitors typically have small capacitance values and are usually adjusted manually.

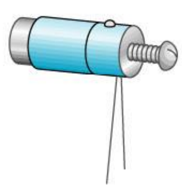
A solid-state device that is used as a variable capacitor is the varactor diode; it is adjusted with an electrical signal.







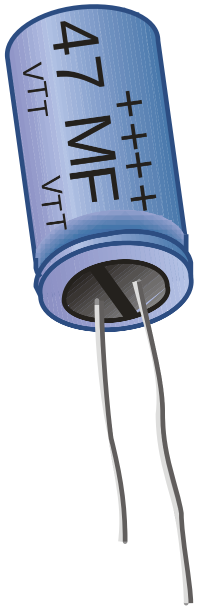




Capacitor labeling

Capacitors use several labeling methods. Small capacitors values are frequently stamped on them such as .001 or .01, which have units of microfarads.

Electrolytic capacitors have larger values, so are read as μF. The unit is usually stamped as μF, but some older ones may be shown as MF or MMF).



Capacitor labeling

A label such as 103 or 104 is read as 10×10^3 (10,000 pF) or 10×10^4 (100,000 pF) respectively. (Third digit is the multiplier.)

When values are marked as 330 or 6800, the units are picofarads.

Example

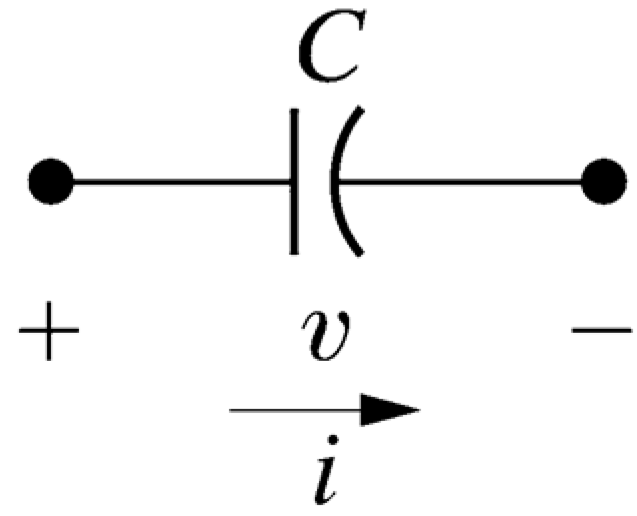
What is the value of each capacitor? Both are 2200 pF.

Capacitance, a Recap

- Capacitance occurs whenever electrical conductors are separated by a dielectric, or insulating material.
- Applying a voltage to the conductors displaces the charge within the dielectric.
- Current does not actually flow through the dielectric.

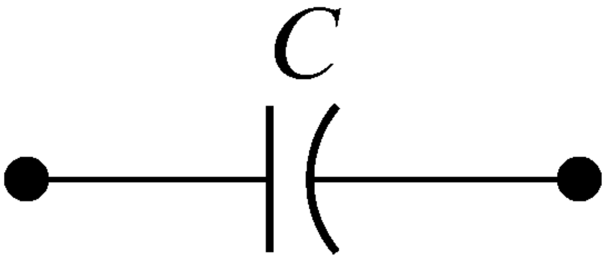
Capacitor

- Circuit Symbol
- Component designation (C)
- Units – Farads
  - Usually μF or pF

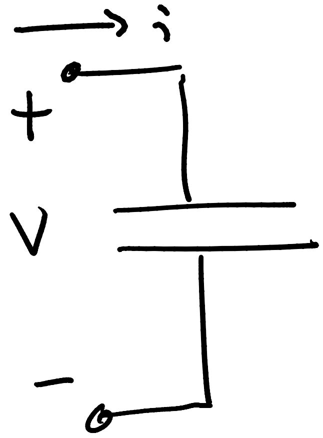


- Reference directions for voltage and current

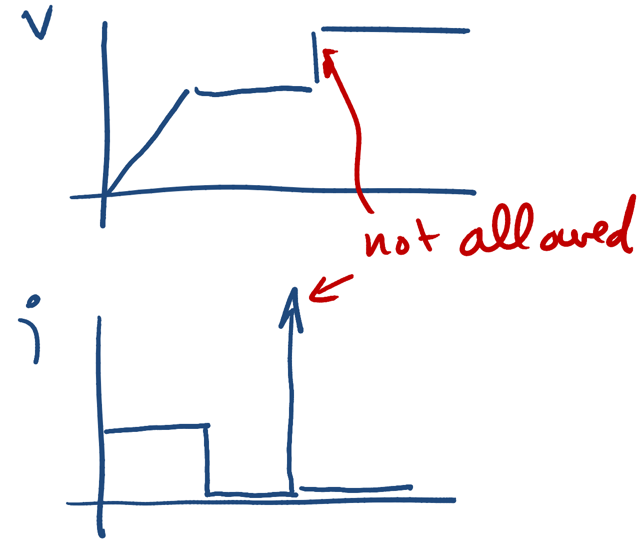
Voltage-Current Relationship



$$i = C \frac{dv}{dt}$$



Observations



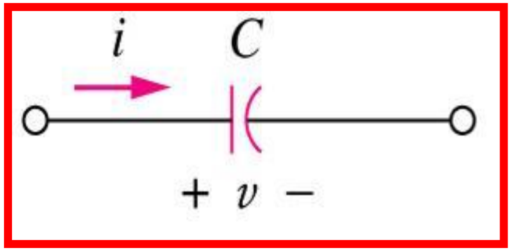
$$i = C\frac{dv}{dt}$$

The **voltage across a capacitor cannot change instantaneously** (the current would be infinite).

If the voltage across the terminals is constant, the current will be zero.
(**looks like an open circuit at DC**).

Only a time-varying voltage can produce a displacement current.

Example 1



The voltage across a 100-μF capacitor is

$$v(t) = 50\sin(120\,\pi t)\ \mathrm{V}$$

Find the displacement current i flowing into capacitor as a function of time

$$i = C\frac{dV}{dt} = (100\times10^{-6}\ F)\,50\,(120\pi)\cos(120\pi t)\ V/s$$

$$= 1.89\cos(120\pi t)\ A$$

Answer:

i(t) = 1.8850 cos(120 π t)

Express the voltage across the capacitor as a function of the current

$$i = C\frac{dv}{dt}$$

$$idt = Cdv \rightarrow dv = \frac{1}{C}idt$$

$$\int_{v(t_0)}^{v(t)} dx = \frac{1}{C}\int_{t_0}^{t} id\tau$$

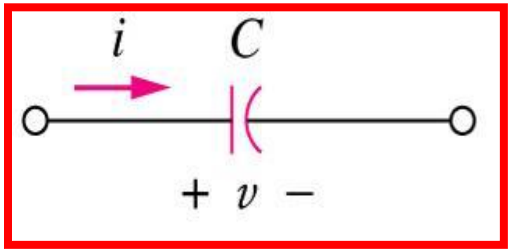
$$v(t) = \frac{1}{C}\int_{t_0}^{t} id\tau + v(t_0)$$

$$v(t) = \frac{1}{C}\int_{0}^{t} id\tau + v(0)$$

or $v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\,d\tau$

In English, please?

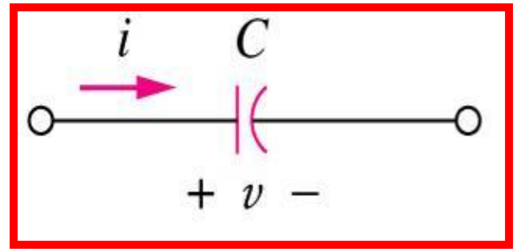
$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\,d\tau$$



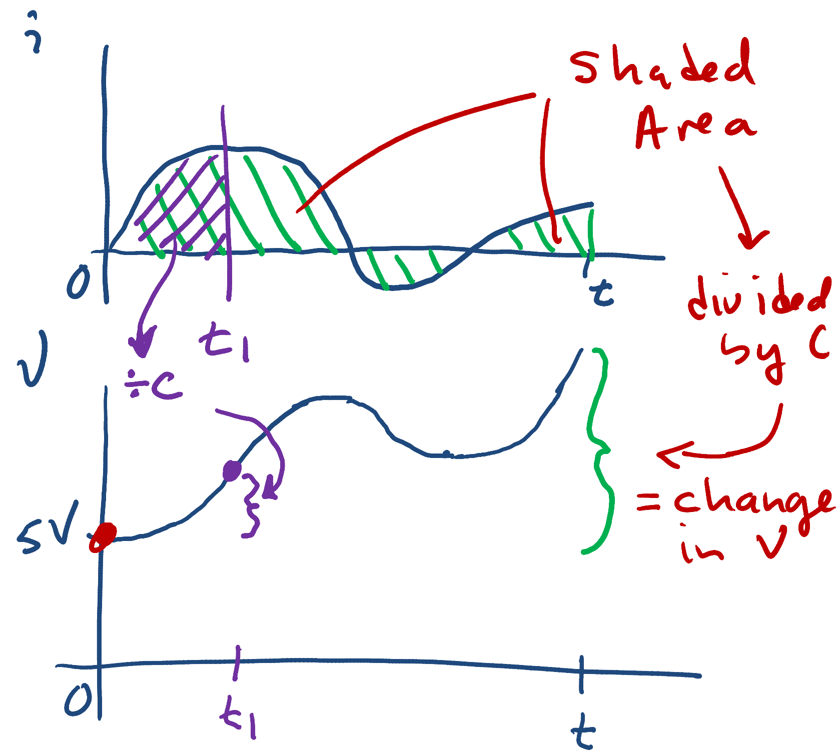
- Translation: to find the voltage at time t:
  - take the known voltage at some start time t0
  - add to it the integral from t0 to t of the current, divided by C

What this looks like graphically

$$v(t) = v(t_0) + \frac{1}{C}\int_{t_0}^{t} i\,d\tau$$



- At time 0 we measure capacitor v = 5 V.
- A time varying current is then applied to the capacitor
- Voltage at any time t is then found as



v(t) =v(0)+ [area under i from 0 to t]/C

works for any t

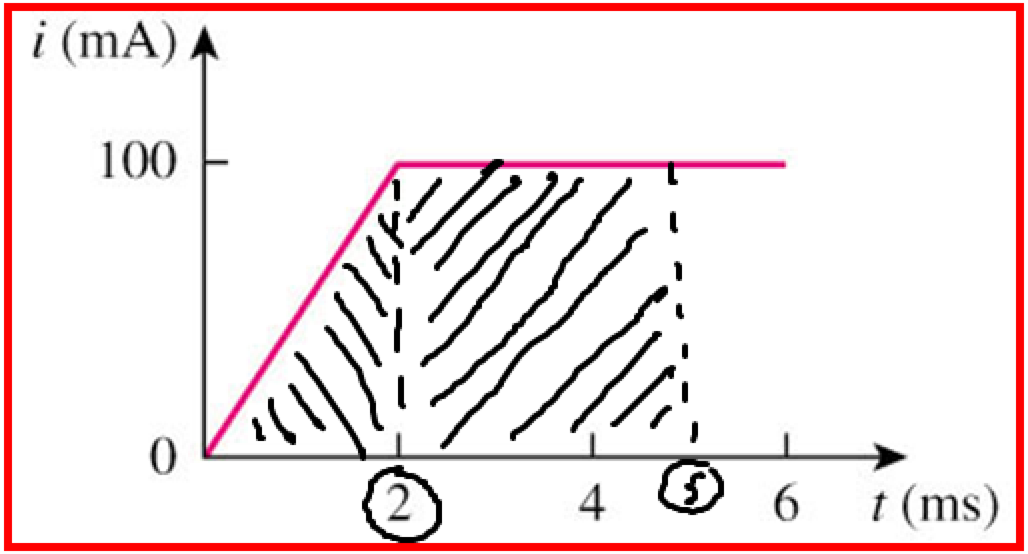
Example 2

An initially uncharged 1-mF capacitor has the current shown below across it.

Calculate the voltage across it at *t* = 2 ms and *t* = 5 ms.

V(2) = 0 + [area of Triangle]/.001F

$$= \frac{\frac{1}{2}(.002s)(.1A)}{.001F} = 0.1\ V$$



V(5) = v(2) + [area of square]/.001F

$$= 0.1 + \frac{(.003s)(.1A)}{.001F} = 0.4V$$

Answer:

v(2ms) = 100 mV

v(5ms) = 400 mV

Power and Energy for the Capacitor

$$p = vi = Cv\frac{dv}{dt}$$

$$p = i\left[\frac{1}{C}\int_{t}^{t} i\,d\tau + v(t_0)\right]$$

p =

$$\frac{dw}{dt} = Cv\frac{dv}{dt}$$

$$dw = Cv\,dv$$

integrate v to whatever voltage is

gives us energy

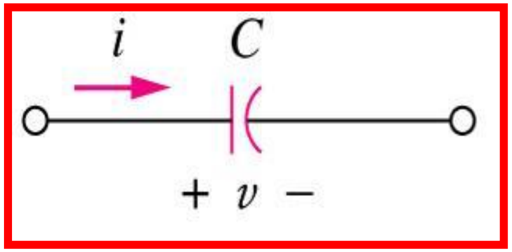
$$\int_0^w dx = C\int_0^v y\,dy$$

not time dependant

$$w = \frac{1}{2}Cv^2$$

Example 6.4

- A voltage pulse described as follows is applied across the terminals of a 0.5μF capacitor:



$$v(t) = 0\ Volts, \qquad t \le 0s$$

$$v(t) = 4t\ Volts, \qquad 0s \le t \le 1s$$

$$v(t) = 4e^{-(t-1)}\ Volts, \qquad t \ge 1s$$

- Derive the expressions for the capacitor a) current,

  b) power, and c) energy.

A) Derive the expressions for the capacitor current

$i = C \frac{dV}{dt}$ $C = .5\mu F$

$$v = \begin{cases} v(t) = 0 \ Volts, & t \le 0s \\ v(t) = 4t \ Volts, & 0s \le t \le 1s \\ v(t) = 4e^{-(t-1)} \ Volts, & t \ge 1s \end{cases}$$

$t < 0$

$i = (.5 \times 10^{-6})(0) = 0$

$0 \le t \le 1s$

$i = (.5 \times 10^{-6})(4) = 2\mu A$

$t \ge 1s$

$i = (.5 \times 10^{-6})(-4e^{-(t-1)}) = -2e^{-(t-1)} \mu A$

B) Derive the expressions for the capacitor power

$p = vi$

$t \leq 0$

$p = 0$

$0 \leq t \leq 1s$

$p = (4t)(2\mu A) = 8t \ \mu W$

$t \geq 1s$

$p = (4e^{-(t-1)})(-2e^{-(t-1)}) = -8e^{-2(t-1)} \ \mu W$

C) Derive the expressions for the capacitor energy

$$w = \frac{1}{2} C v^2$$

$\underline{t \le 0}$

$$w = 0$$

$\underline{0 \le t \le 1s}$

v^2 ↓

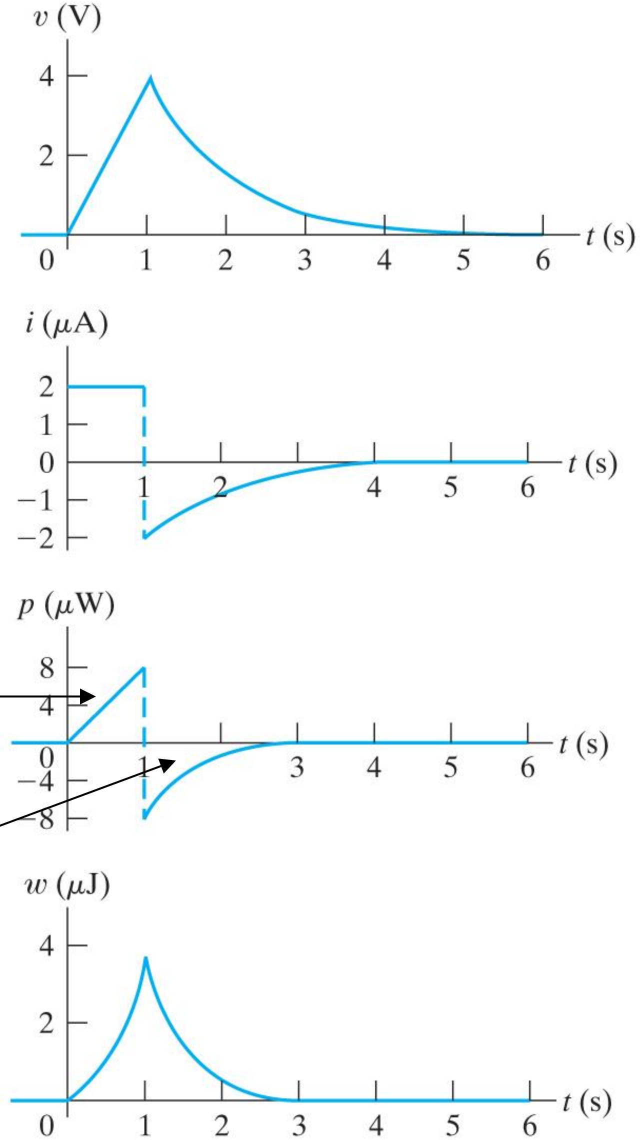
$$w = \frac{1}{2}(.5 \times 10^{-6})(16t^2) = 4t^2 \mu J$$

$\underline{t \ge 1s}$

$$w = \frac{1}{2}(.5 \times 10^{-6})(16 e^{-2(t-1)}) = 4e^{-2(t-1)} \mu J$$

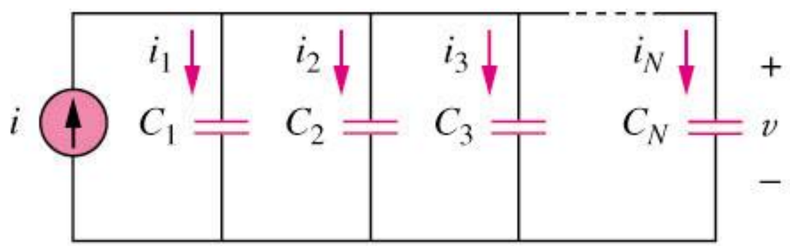
Energy is being stored whenever the power is positive.

Energy is being delivered by the capacitor whenever the power is negative.

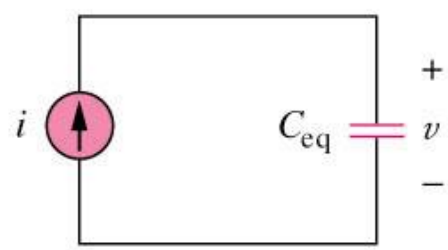


6.2 Series and Parallel Capacitors (1)

- The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



(a)

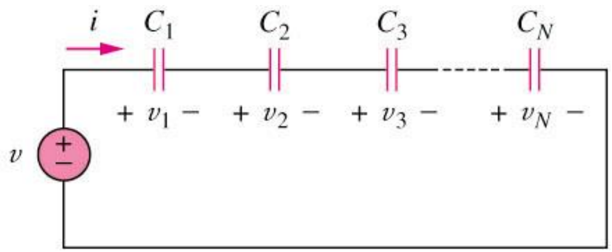


(b)

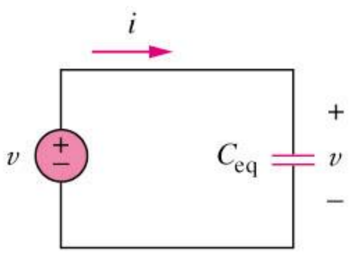
$$C_{eq} = C_1 + C_2 + ... + C_N$$

6.2 Series and Parallel Capacitors (2)

- The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)

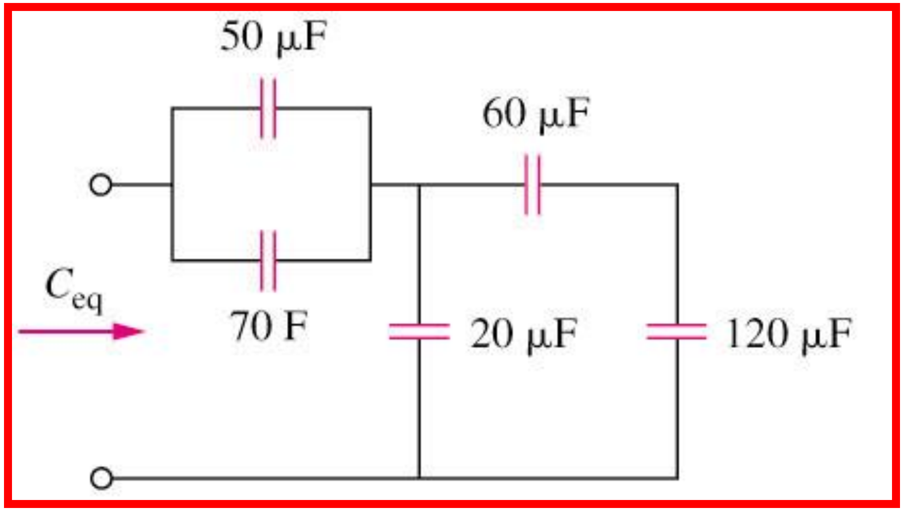


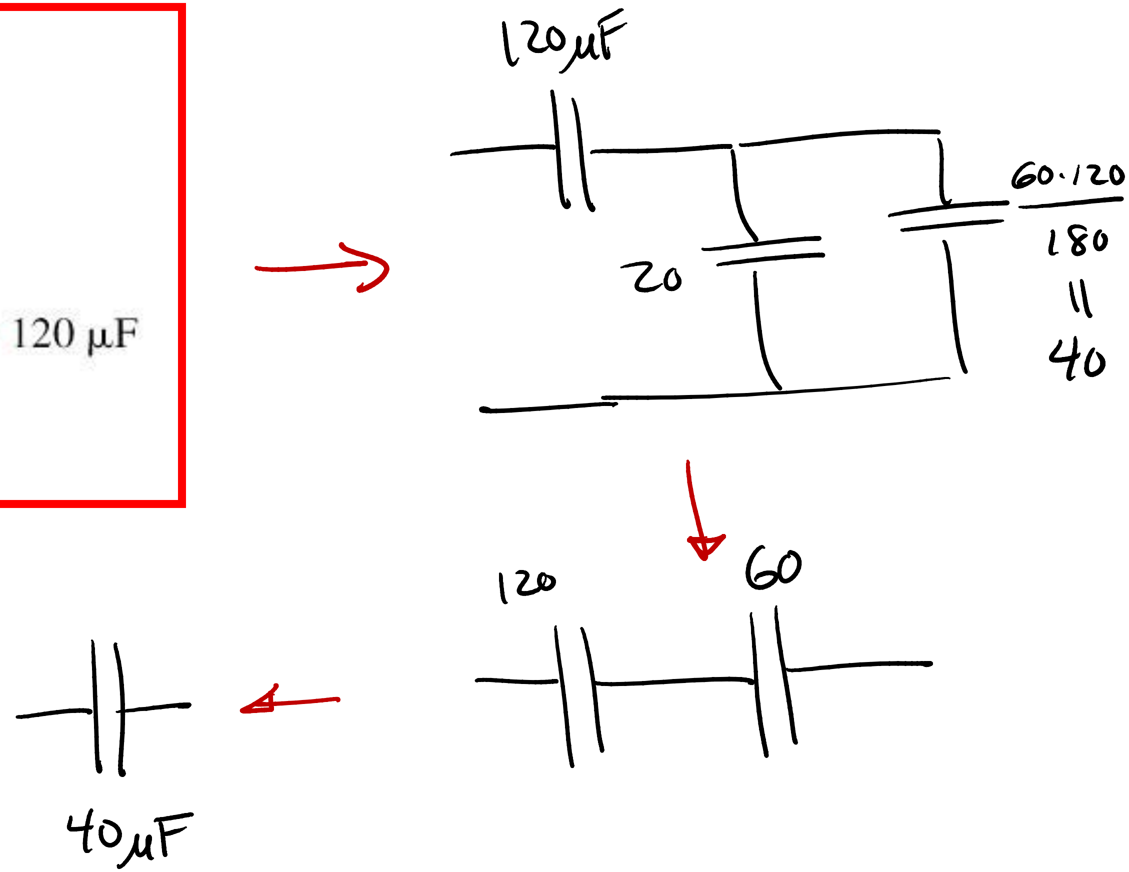
(b)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



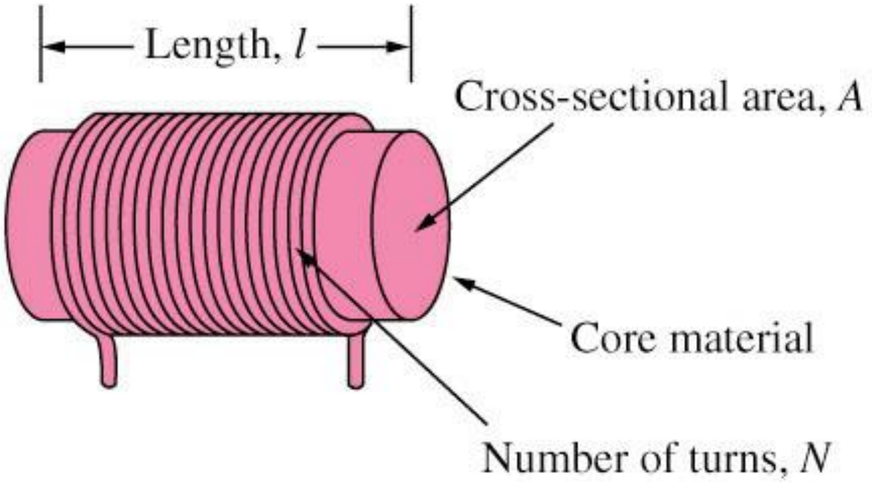


Answer:

C_{eq} = 40μF

6.3 Inductors (1)

- An inductor is a passive element designed to store energy in its magnetic field.

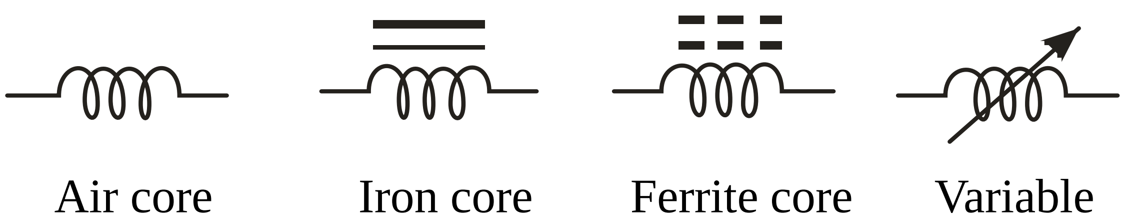


- An inductor consists of a coil of conducting wire.

Inductance

Inductance is the property of a conductor to oppose a change in current. The effect of inductance is greatly magnified by winding a coil on a magnetic material.

Common symbols for inductors (coils) are



Self Inductance

Self-inductance is usually just called inductance, symbolized by *L*. Self-inductance is a measure of a coil’s ability to establish an induced voltage as a result of a change in its current. The induced voltage always opposes the change in current, which is basically a statement of Lenz’s law.

The unit of inductance is the **henry** (**H**). One henry is the inductance of a coil when a current, changing at a rate of one ampere per second, induces one volt across the coil.

Self Inductance

The induced voltage is given by the formula $v_{\text{ind}} = L\dfrac{di}{dt}$

Example

What is the inductance if 37 mV is induced across a coil if the current is changing at a rate of 680 mA/s?

$$v_{\text{ind}} = L\frac{di}{dt}$$

Rearranging,

$$L = \frac{v_{\text{ind}}}{di/dt} = \frac{0.037\ \text{V}}{0.68\ \text{A/s}} = 54\ \text{mH}$$

Factors affecting inductance

Four factors affect the amount of inductance for a coil. The equation for the inductance of a coil is

$$L = \frac{N^2 \mu A}{l}$$

where

L = inductance in henries

N = number of turns of wire

μ = permeability in H/m (same as Wb/At-m)

l = coil length on meters

Example What is the inductance of a 2.0 cm long, 150 turn coil wrapped on an low carbon steel core that is 0.50 cm diameter? The permeability of low carbon steel is 2.5 x10^{-4} H/m (Wb/At-m).

$$A = \pi r^2 = \pi\left(0.0025\text{ m}\right)^2 = 7.85\times10^{-5}\text{ m}^2$$

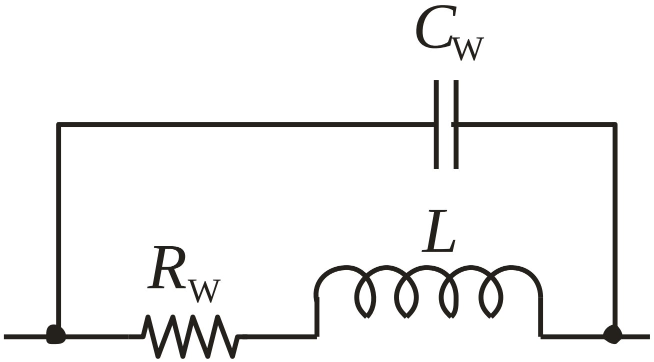
$$L = \frac{N^2 \mu A}{l}$$

$$= \frac{\left(150\text{ t}\right)^2\left(2.5\times10^{-4}\text{ Wb/At-m}\right)\left(7.85\times10^{-5}\text{ m}^2\right)}{0.020\text{ m}}$$

$$= 22\text{ mH}$$

Physical parameters affecting inductance

The inductance given by the equation in the previous slide is for the ideal case. In practice, inductors have winding resistance (R_W) and winding capacitance (C_W). An equivalent circuit for a practical inductor including these effects is:



Lenz's law

Recall Lenz's law states,

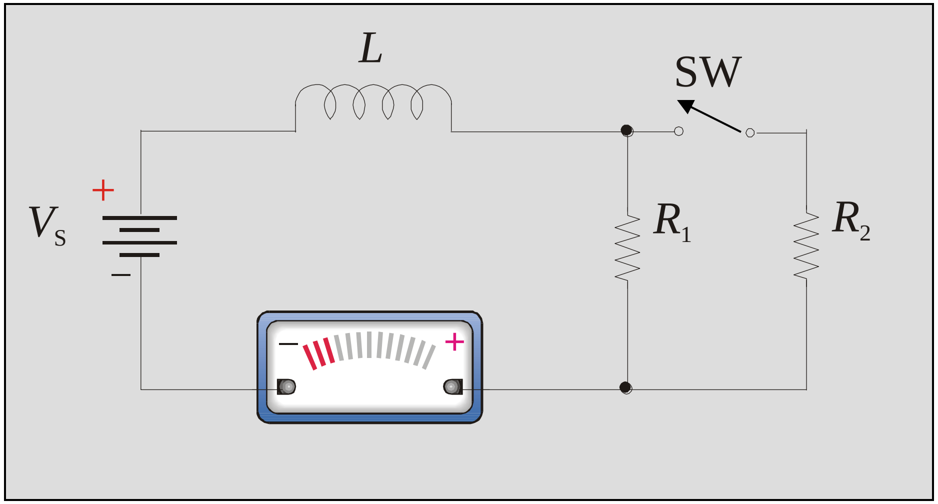
When the current through a coil *changes*, an induced voltage is created across the coil that always opposes the change in current.

In a practical circuit, the current can change because of a change in the load as shown in the following circuit example...

Lenz's law

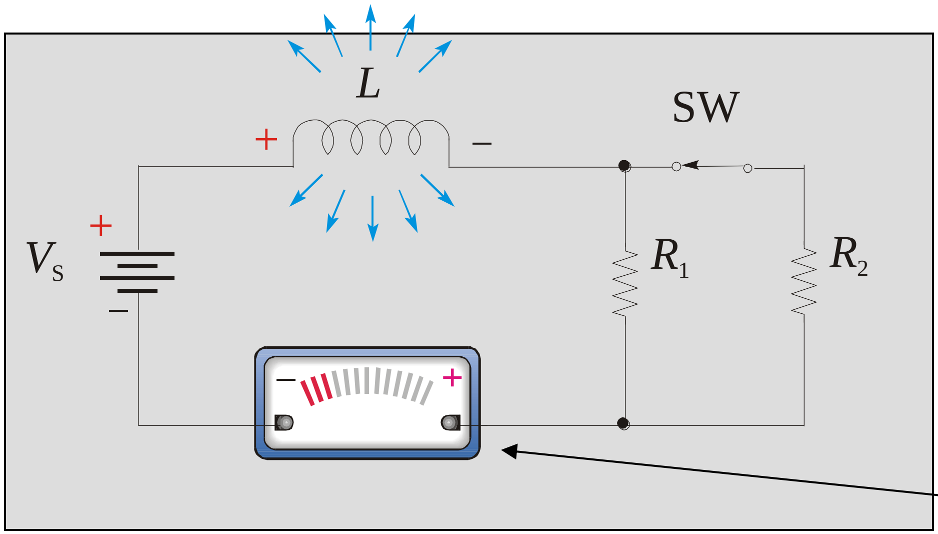
A basic circuit to demonstrate Lenz's law is shown.

Initially, the SW is open and there is a small current in the circuit through L and R_1.



Lenz's law

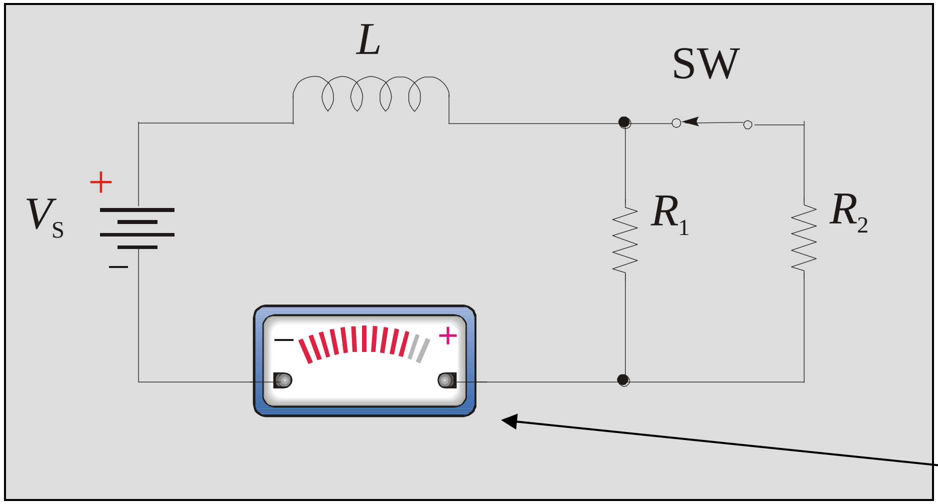
SW closes and immediately a voltage appears across *L* that tends to oppose any *change* in current.



Initially, the meter reads same current as before the switch was closed.

Lenz's law

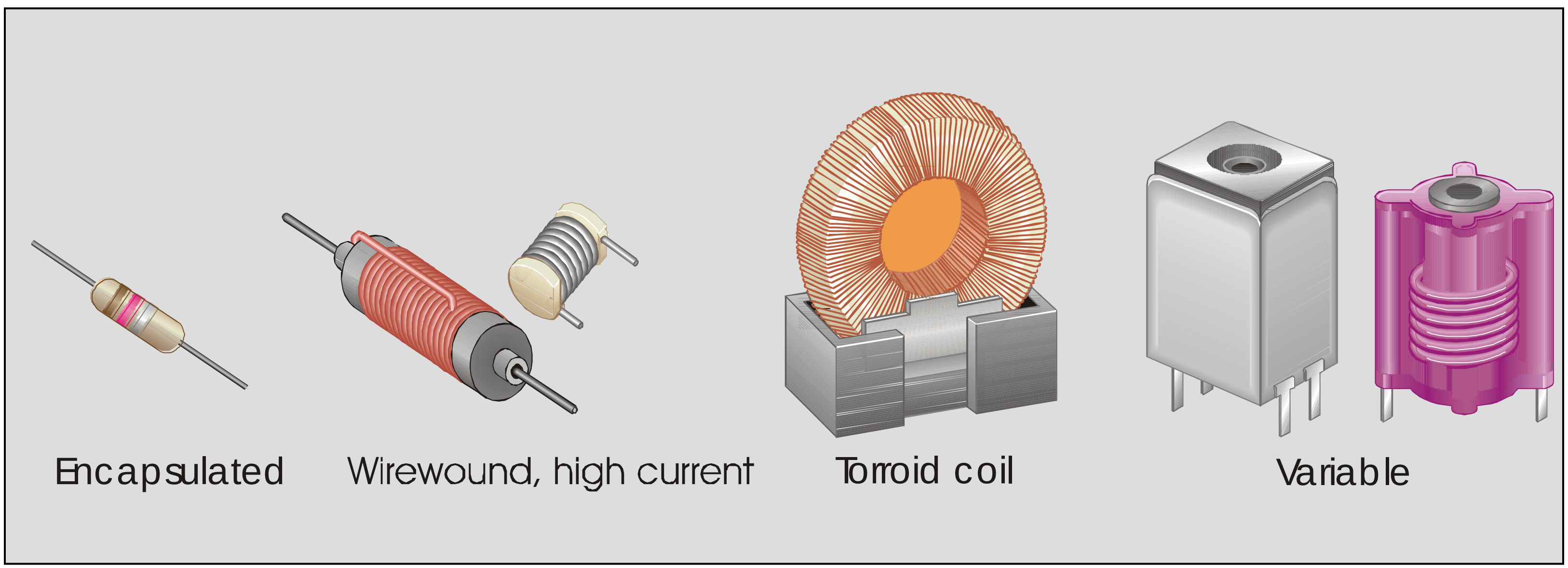
After a time, the current stabilizes at a higher level (due to R_2) as the voltage decays across the coil.



Later, the meter reads a higher current because of the load change.

Practical inductors

Inductors come in a variety of sizes. A few common ones are shown here.

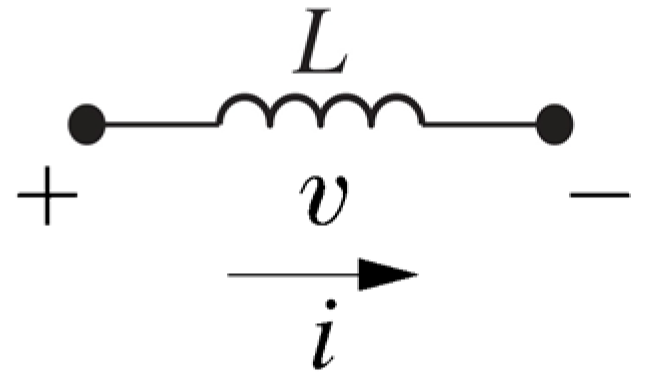


Inductance

- Inductor
  - A coil of wire wrapped around a supporting core (magnetic or non-magnetic)
  - The time-varying current in the wire produces a time-varying magnetic field around the wire
  - A voltage is induced in any conductor linked by the magnetic field
  - Inductance relates the induced voltage to the current

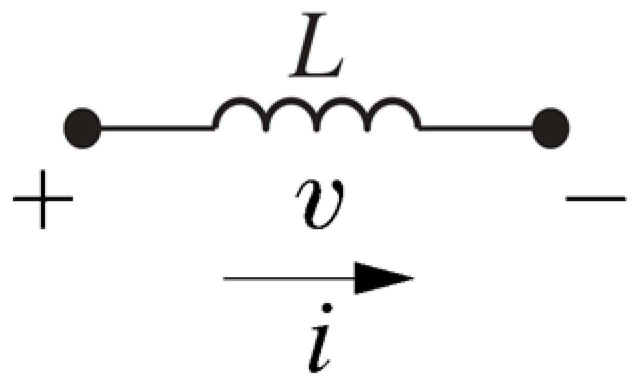
Inductor

- Circuit Symbol
- Component designation (L)
- Units -- Henry(s)
  - Usually mH or µH

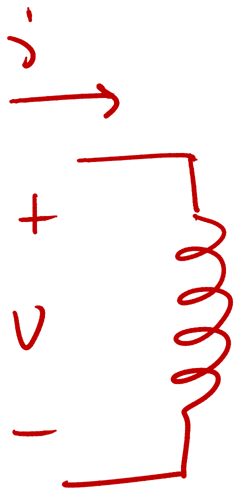


- Reference directions for voltage and current

Voltage-Current Relationship

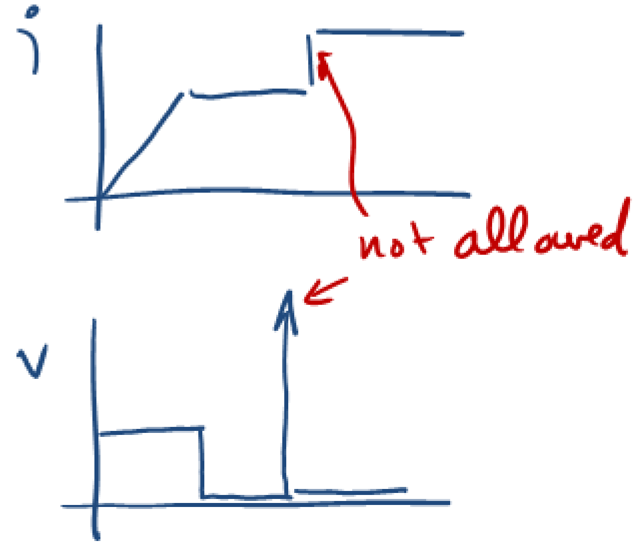


$$v = L\frac{di}{dt}$$



Observations

$$v = L\frac{di}{dt}$$



The **current across an inductor cannot change instantaneously** (the voltage would be infinite).

If the current through the inductor is constant, the voltage will be zero.
(**looks like a short circuit at DC**).

Only a time-varying current can produce an induced voltage.

Current in an inductor in terms of the Voltage across the inductor

$$v = L\frac{di}{dt}$$

$$vdt = Ldi = L\left(\frac{di}{dt}\right)dt$$

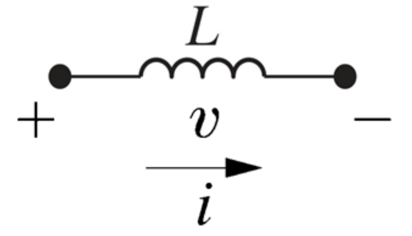
$$vdt = Ldi$$

$$\int_{t_0}^{t} v d\tau = L \int_{i(t_0)}^{i(t)} dx$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v d\tau + i(t_0) \rightarrow \frac{1}{L}\int_{t_0}^{t} v d\tau + i(0)$$

Again, in English

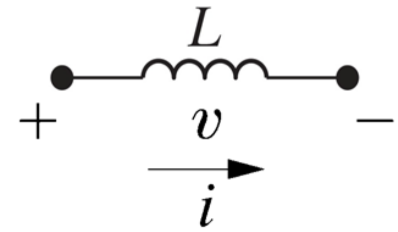
$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\,d\tau$$



- Translation: to find the current at time t:
  - take the known current at some start time t0
  - add to it the integral from t0 to t of the voltage, divided by L

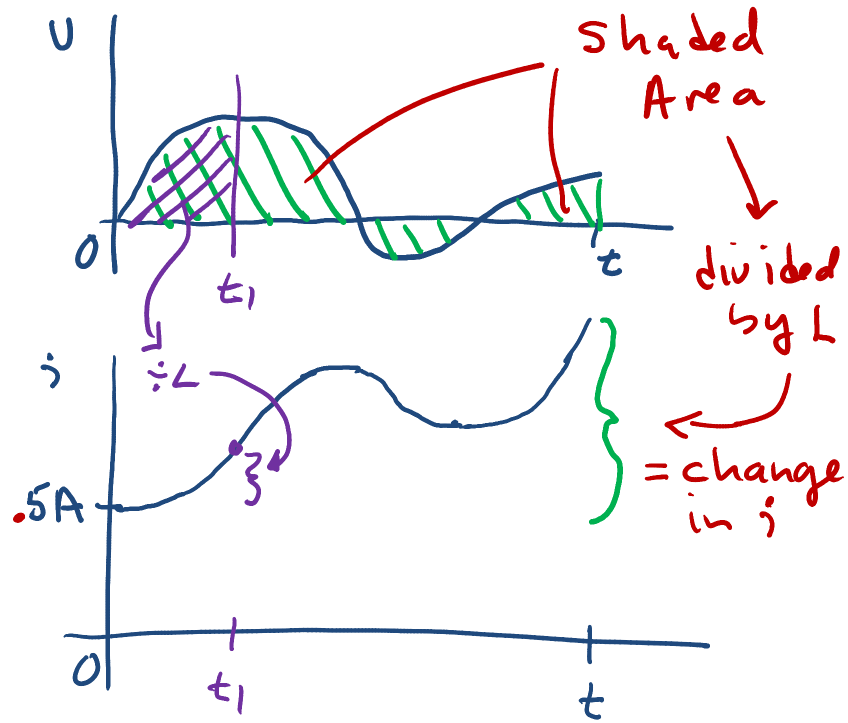
What this looks like graphically

$$i(t) = i(t_0) + \frac{1}{L}\int_{t_0}^{t} v\,d\tau$$



- At time 0 we measure inductor i = .5 A.
- A time varying voltage is then applied to the inductor
- Current at any time t is then found as

i(t) =i(0)+ [area under v from 0 to t]/L



Power and Energy in an Inductor

$$p = vi = Li\frac{di}{dt}$$

$$p = vi = v\left[\frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)\right]$$

$$p = \frac{dw}{dt} = Li\frac{di}{dt}$$

$$dw = Li\,di$$

$$\int_0^w dx = L\int_0^i y\,dy$$

$$w = \frac{1}{2}Li^2$$

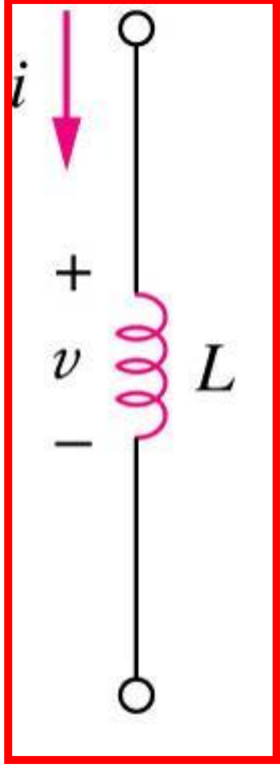
Example 5

The terminal voltage of a 2-H inductor is

$$v = 10(1-t)\ \text{V}$$

Find the current flowing through it at t = 4 s and the energy stored in it at t = 4 s.

Assume i(0) = 2 A.



$$i(t) = i(0) + \frac{1}{L}\int_{t_0}^{t} v\, d\tau = 2 + \frac{1}{2}\int_0^4 10(1-x)\,dx$$

$$= 2 + \frac{1}{2}\left[10x - \frac{10x^2}{2}\right]_0^4$$

$$= 2 + \frac{1}{2}\left[40 - \frac{160}{2}\right] = 2 - 20 = -18\text{A}$$

$$w = \frac{1}{2}Li^2 = \frac{1}{2}(2)(-18)^2 = 324\text{J}$$

Answer:

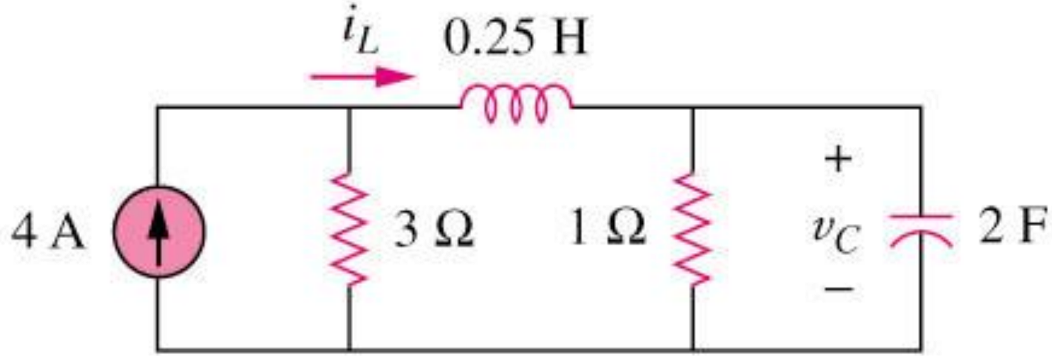
i(4s) = -18A

w(4s) = 320J

$$2 + 0.5\int_0^4 10\,(1-x)\,dx$$

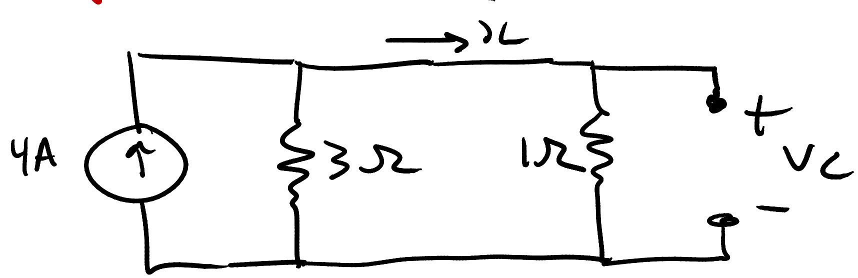
Example 6

Determine v_C, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



At DC steady state C → open, L → short

Redraw



$i_L = 4\left(\frac{3}{3+1}\right) = \underline{3A}$, $w_L = \frac{1}{2}Li^2 = \frac{1}{2}\left(\frac{1}{4}\right)(3)^2 = \underline{1.125\,J}$

$V_C = i_L(1\Omega) = \underline{3V}$, $w_C = \frac{1}{2}CV^2 = \frac{1}{2}(2)(3)^2$

$= \underline{9\,J}$

Answer:

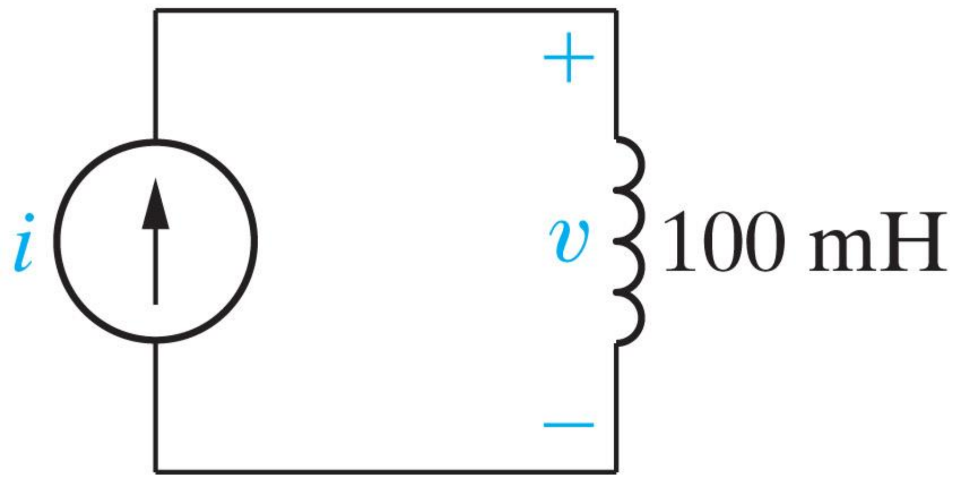
$i_L = 3A$

$v_C = 3V$

$w_L = 1.125J$

$w_C = 9J$

Example 6.1 – Sketch current, find max using Wolfram Alpha



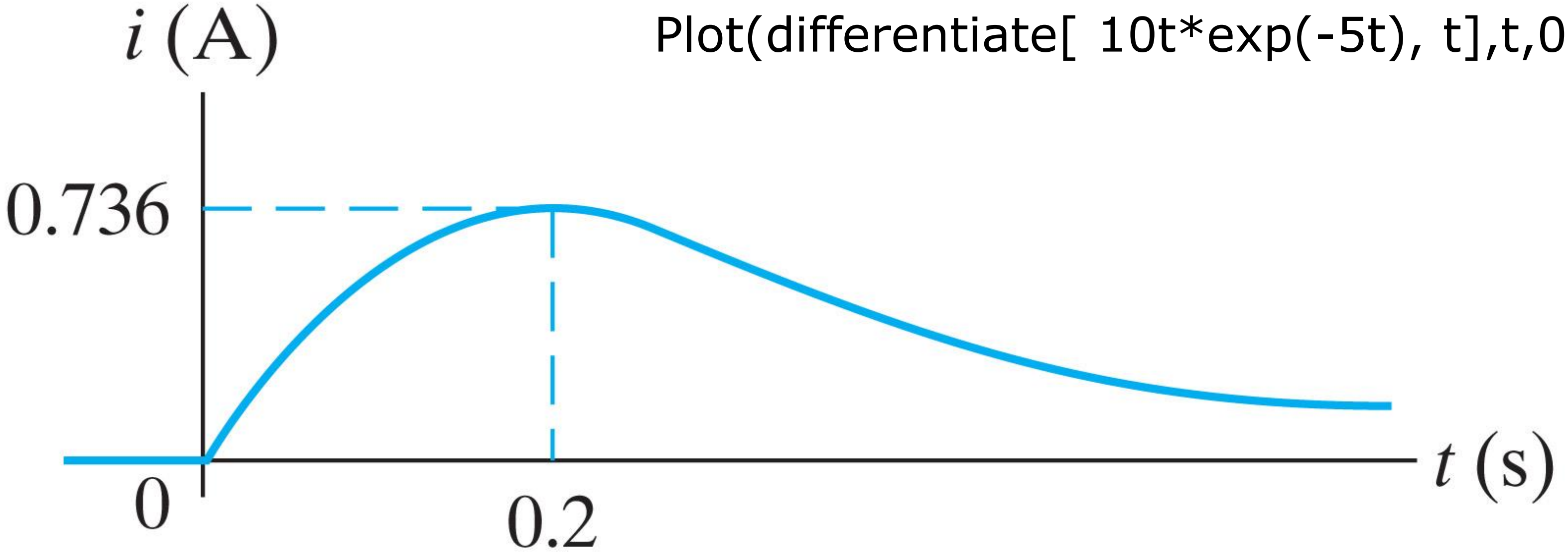
$$i = 0, \qquad t < 0$$

$$i = 10te^{-5t}\,\mathrm{A}, \qquad t > 0$$

- First, sketch the current waveform.
- At what instant of time is the current maximum?
- Wolfram Alpha: Plot[10t*exp(-5t), t, 0, 1]

differentiate[10t*exp(-5t),t]

Plot(differentiate[10t*exp(-5t), t],t,0,1)



$$i = 10te^{-5t}$$

$$\frac{di}{dt} = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1-5t)$$

$$\frac{di}{dt} = 0 \rightarrow 1-5t = 0 \rightarrow t = \frac{1}{5} = 0.2s$$

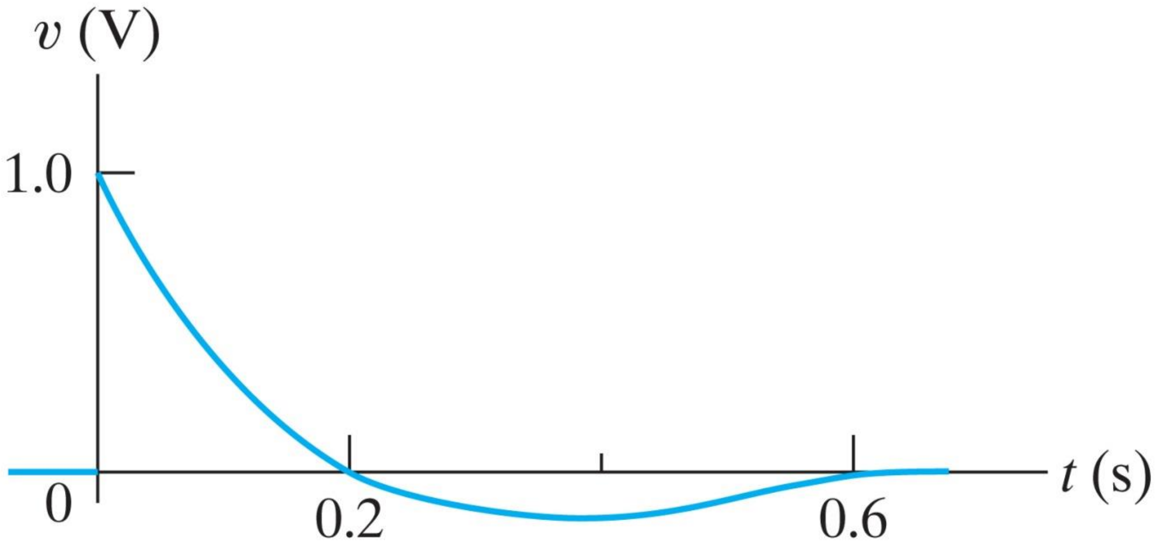
Example 6.2 Using Derivative formula

Given: current thru inductor

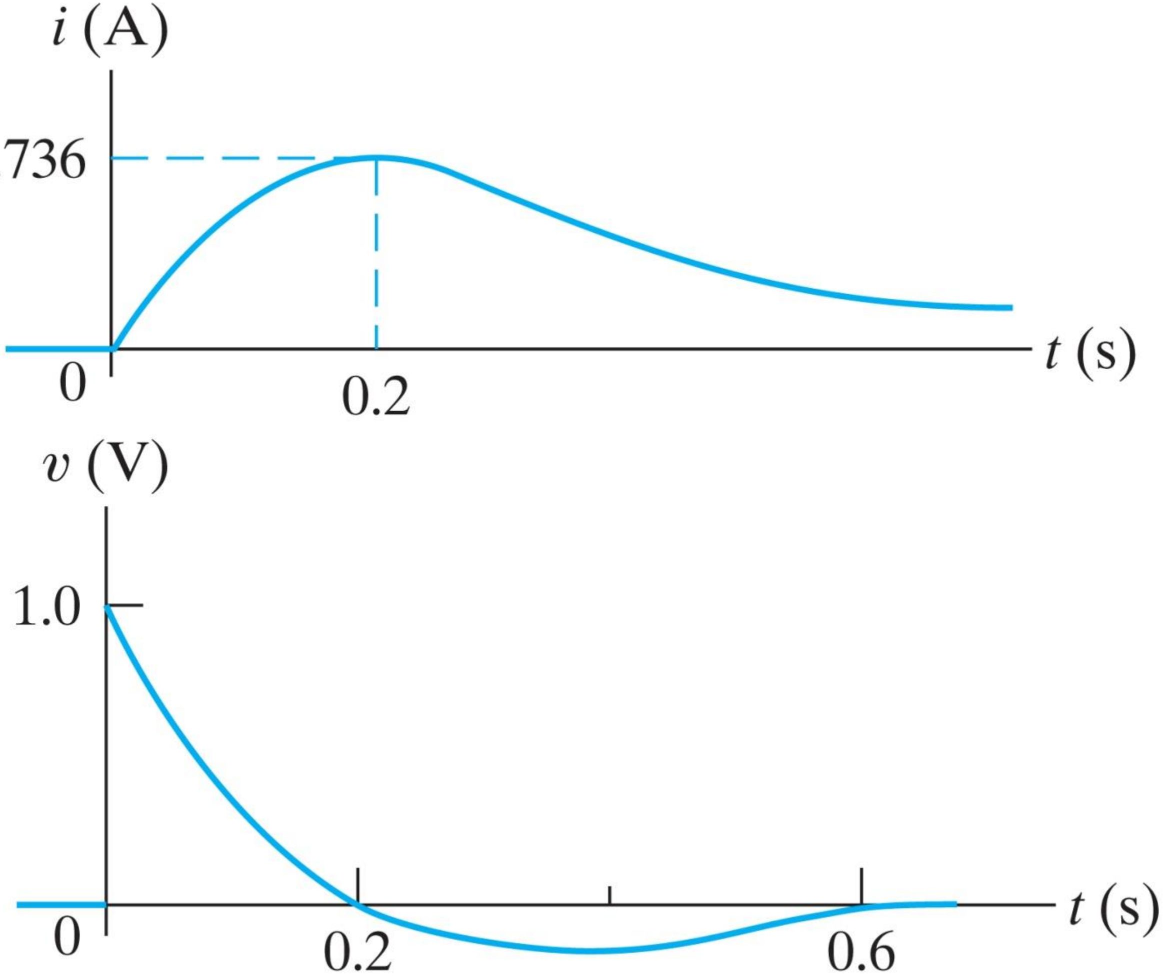
Find: voltage across inductor

$$i = 10te^{-5t}$$

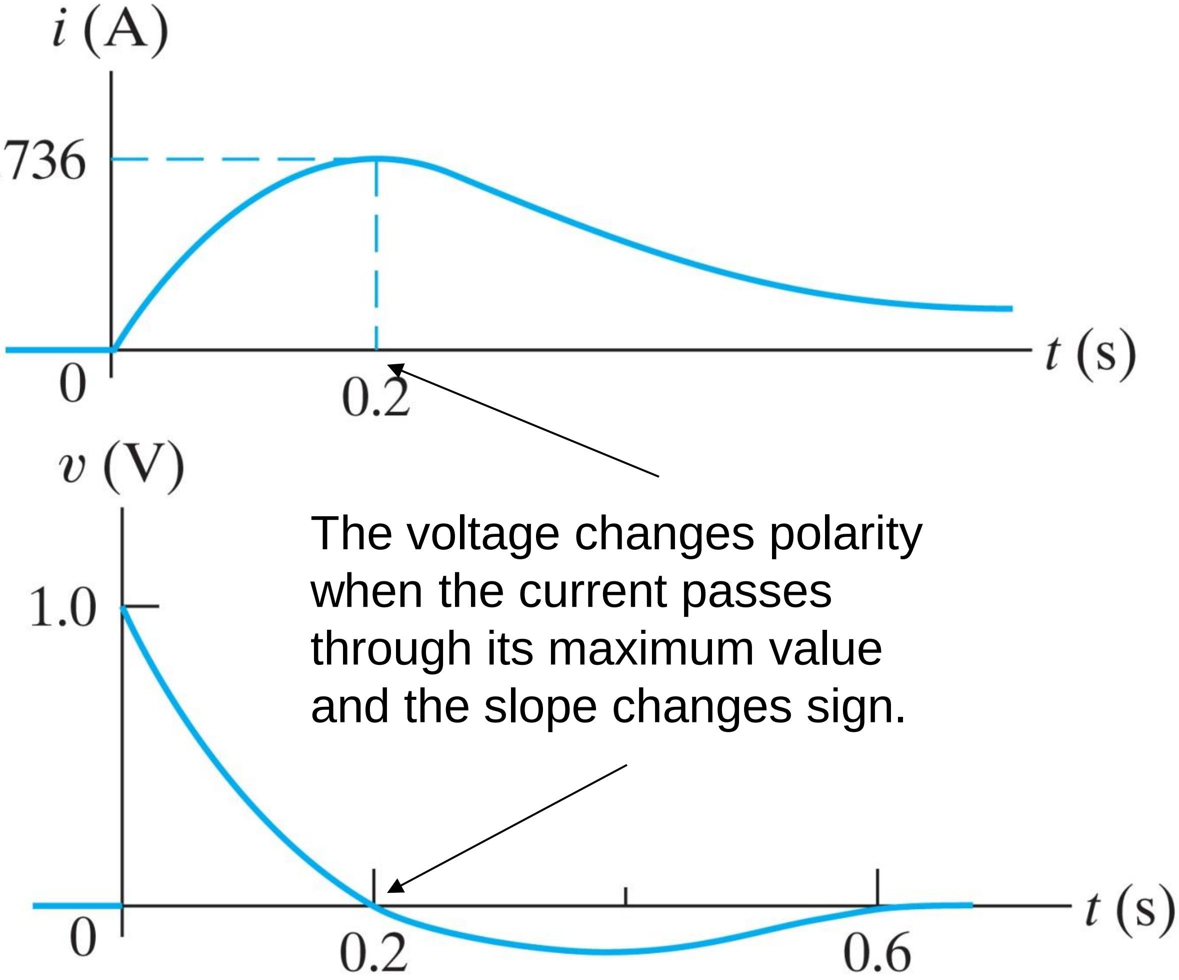
$$v = L\frac{di}{dt} = (0.1)10e^{-5t}(1-5t) = e^{-5t}(1-5t)$$



Are the voltage and current at maximum at the same time?

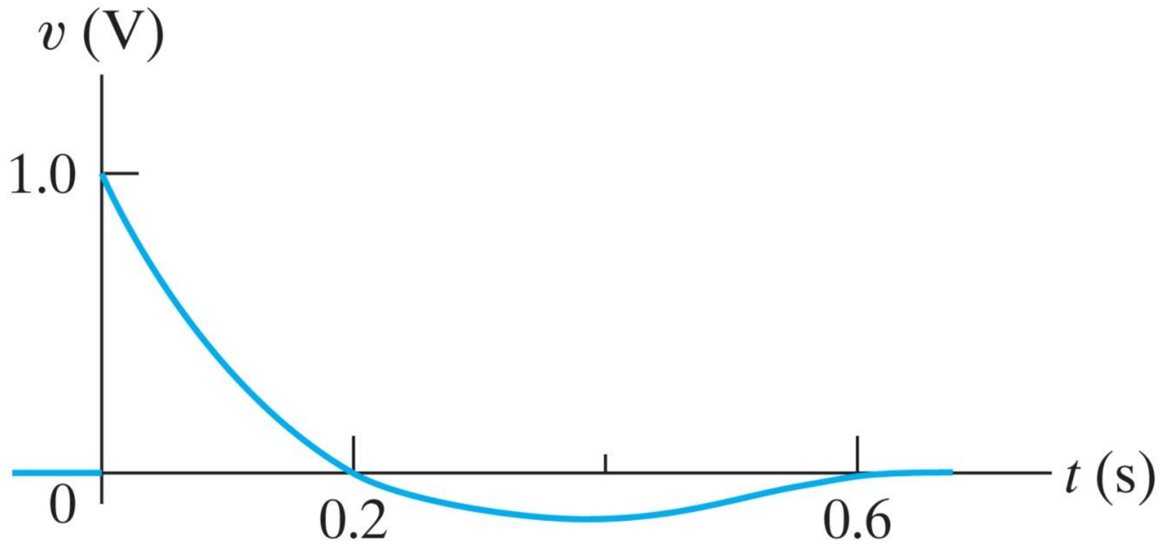


At what instant of time does the voltage change polarity?



The voltage changes polarity when the current passes through its maximum value and the slope changes sign.

Is there ever an instantaneous change in voltage across the inductor? If so, when?



Yes.

The voltage across the inductor changes instantaneously at t=0.

Using wolframalpha.com to differentiate

- diff[*f(x)*, x]   takes derivative of some f(x) wrt x

e^{-5t} E^(-5t)

- diff[10t*exp(-5t),t]   → works in wA
  - Use * when needed to clarify
  - Use exp( ) to raise e to some exponent

WolframAlpha computational knowledge engine

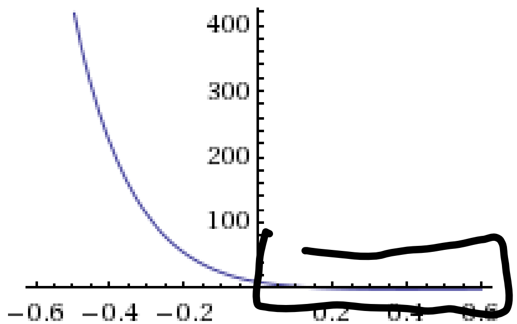
diff[10t*exp(-5t),t]

Derivative:

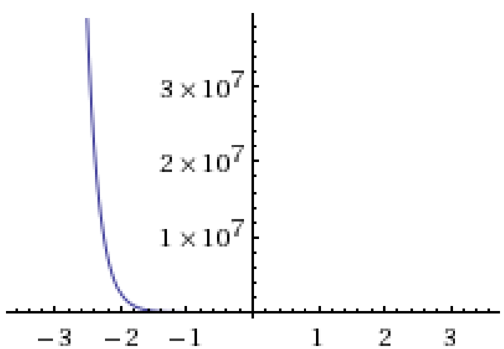
Show steps

$$\frac{d}{dt}(10\,t\,\exp(-5\,t)) = e^{-5\,t}\,(10 - 50\,t)$$

Plots:



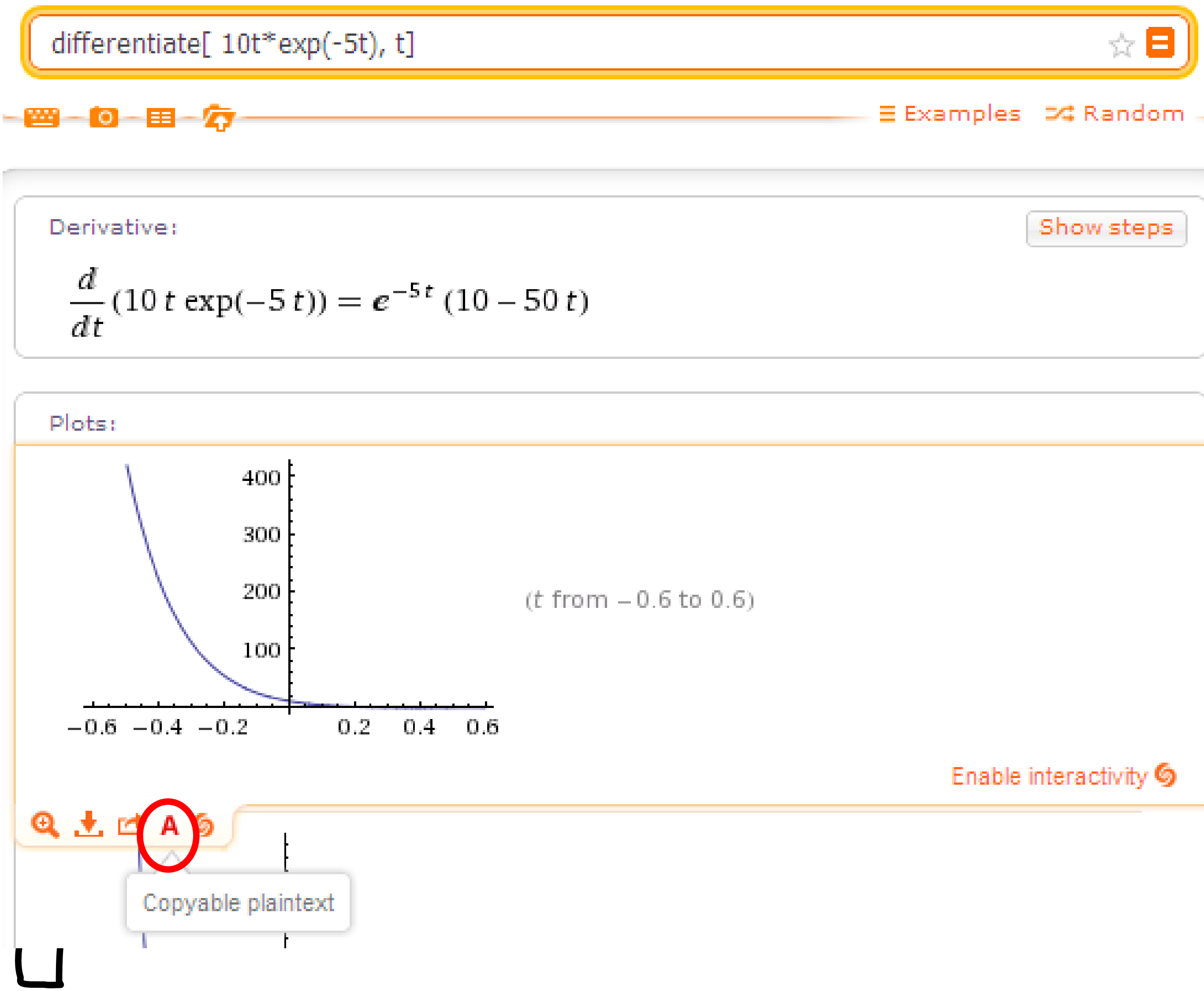
(t from -0.6 to 0.6)



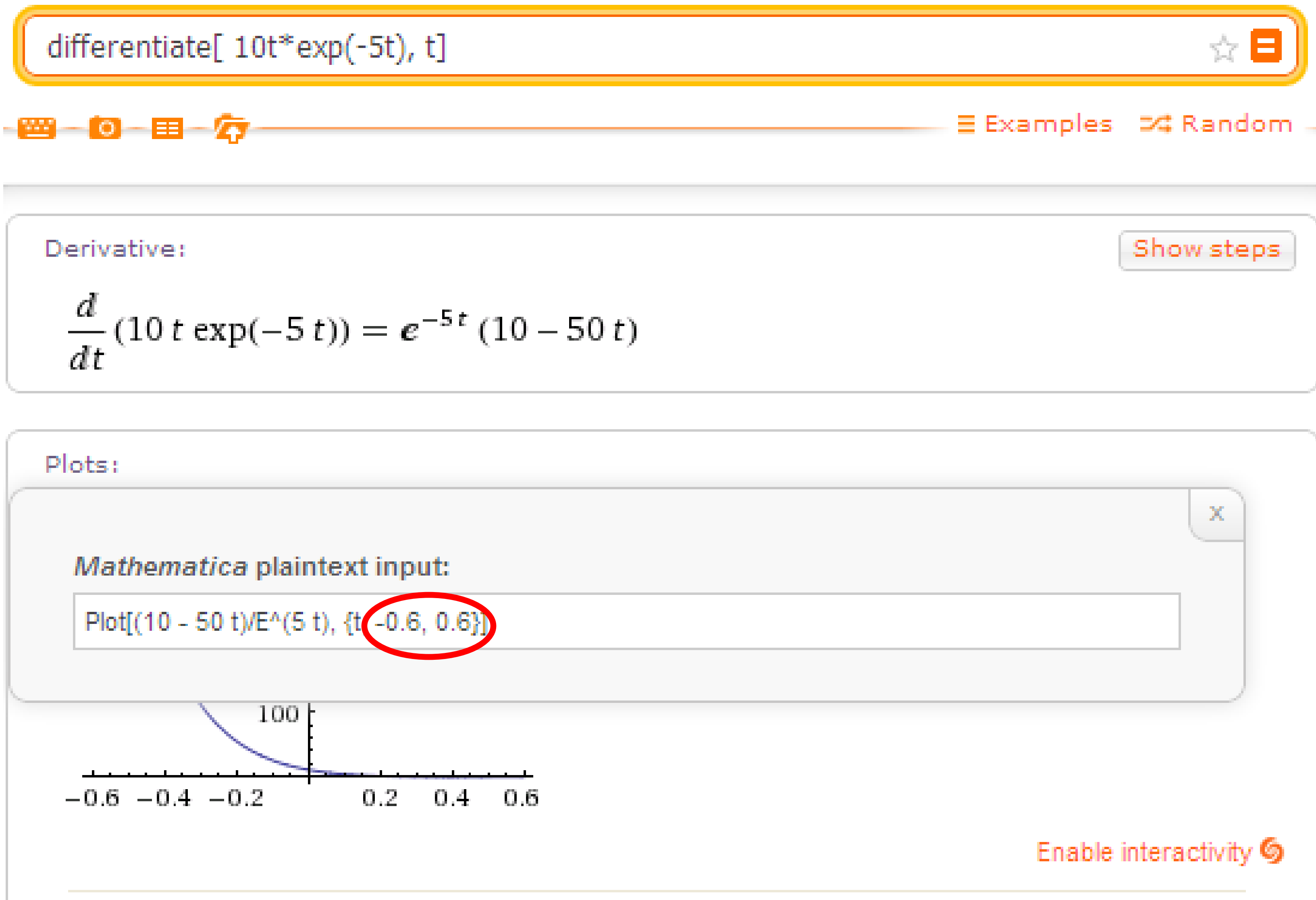
(t from -3 to 3)

Alternate forms:

If you don't like the plot range, hover mouse over letter A below plot,



Copy paste back into WA with new limits

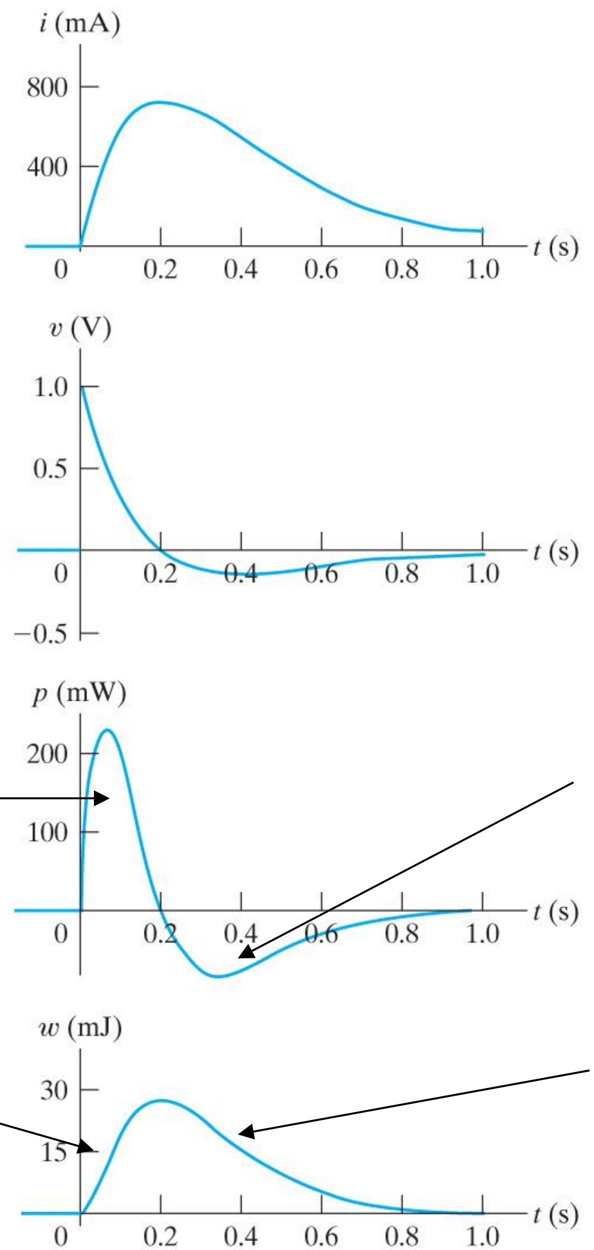


Example 6.3

- Plot i, v, p, and w for Example 6.3. Line up the plots vertically to allow easy assessment of each variable's behavior.
- In what time interval is energy being stored in the inductor?
- In what time interval is energy being extracted from the inductor?

Energy is being stored when power is >0.

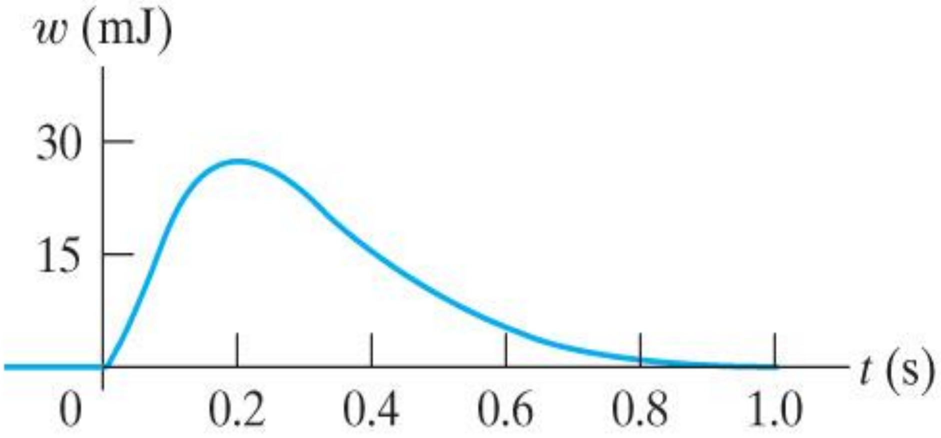
Energy is being extracted when power is <0.



An increasing energy curve indicates that energy is being stored in the inductor.

A decreasing energy curve indicates that energy is being extracted from the inductor

What is the maximum energy stored in the inductor?



$$w = \frac{1}{2}Li^2 = \frac{1}{2}(0.1)(0.736)^2$$

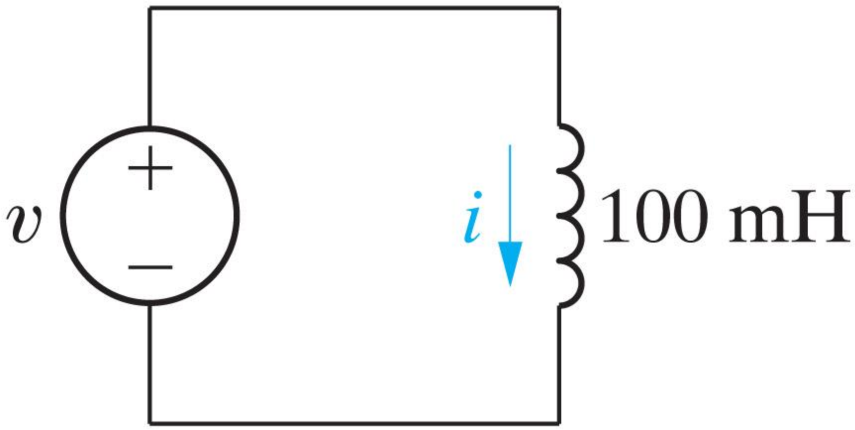
← peak i

$$w_{\max} = 27.07mJ$$

Example 6.4 Using the integral formula

Given: voltage across inductor

Find: current through inductor



$$v = 0, \qquad t < 0$$

$$v = 20te^{-10t}\ \text{V}, \quad t > 0$$

$i = i(0) + \frac{1}{L}\int v\,dt$

- Sketch the voltage as a function of time.
- Find the inductor current as a function of time.
- Sketch the current as a function of time.

Integrating with Wolfram Alpha

- Basic indefinite integral:
  - Integrate[x^2, {x}]

$$\int x^2 \, dx = \frac{x^3}{3} + \text{constant}$$

- Basic definite integral
  - Integrate[x^2,{x,0,5}]

$$\int_0^5 x^2 \, dx = \frac{125}{3}$$

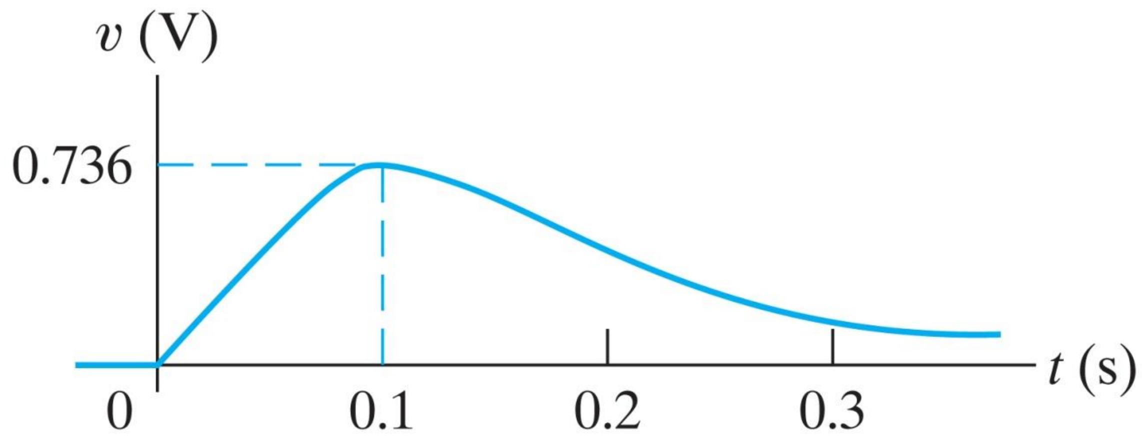
Integrating with Wolfram Alpha

- Our integrals must be done using "dummy variable substitution" which is a definite integral from t0 to t. In the last example, t0=0

Integrate wrt x from 0 to t

- In this example
  - 0 + (1/0.1)*Integrate[20x*E^(-10x),{x,0,t}]

$$0 + \frac{1}{0.1}\int_0^t 20\,x\,e^{-10\,x}\,dx \Rightarrow 2.\left(1 - e^{-10\,t}\,(10\,t + 1)\right)$$



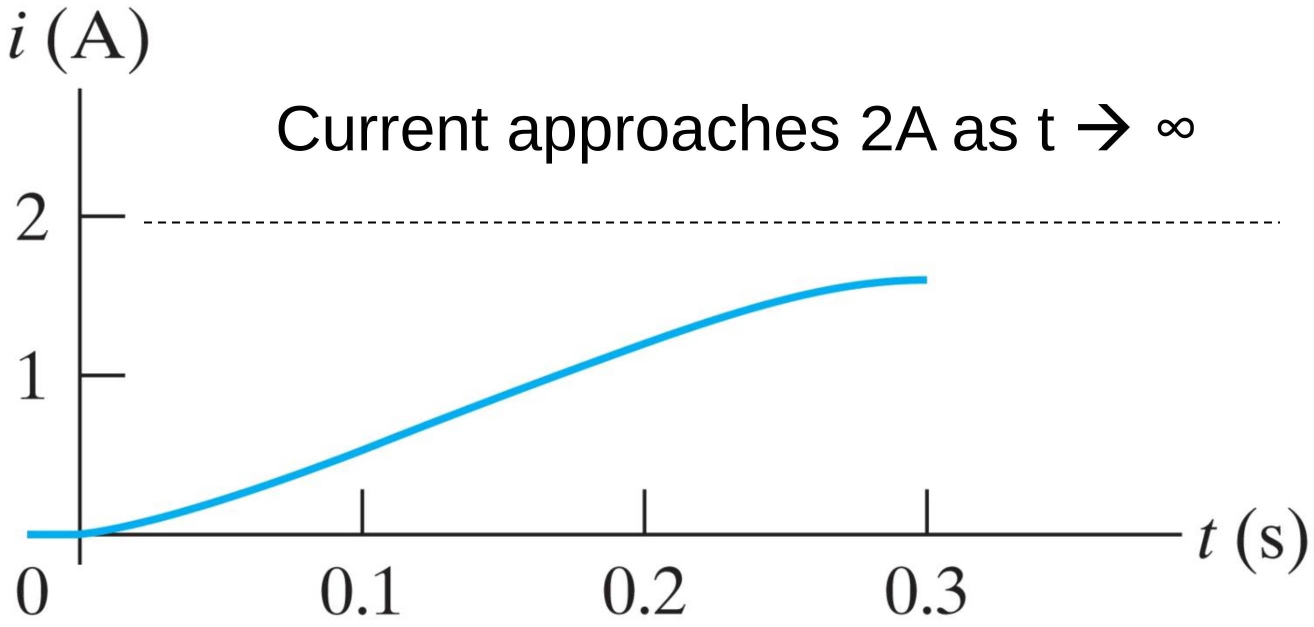
$$i(t) = \frac{1}{L}\int_0^t v\,d\tau + i(0)$$

$$i(t) = \frac{1}{0.1}\int_0^t 20\tau e^{-10\tau}\,d\tau + 0$$

$$i(t) = 200\left[\frac{-e^{-10\tau}}{100}(10\tau - 1)\right]_0^t$$

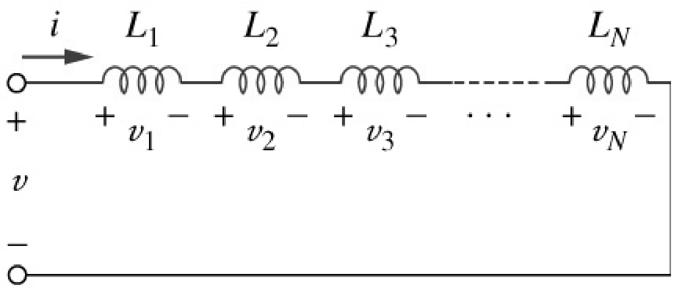
$$i(t) = 2(1 - 10te^{-10t} - e^{-10t})$$

$$i(t) = 2(1 - 10te^{-10t} - e^{-10t})$$

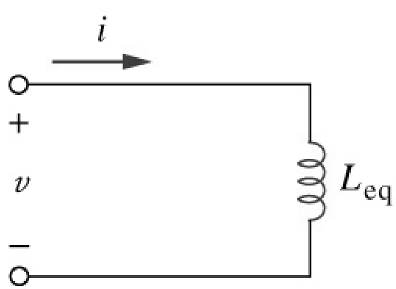


6.4 Series and Parallel Inductors (1)

- The equivalent inductance of **series-connected** inductors is the sum of the individual inductances. just like resistors



(a)

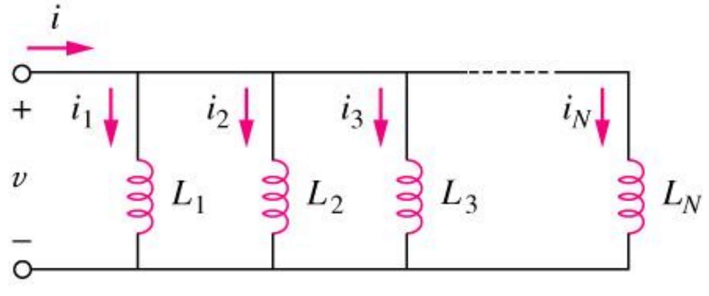


(b)

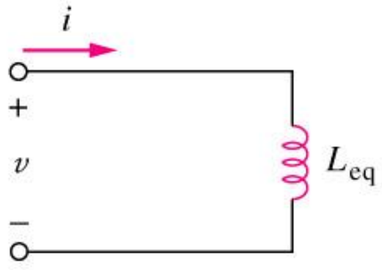
$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

6.4 Series and Parallel Inductors (2)

- The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.



(a)

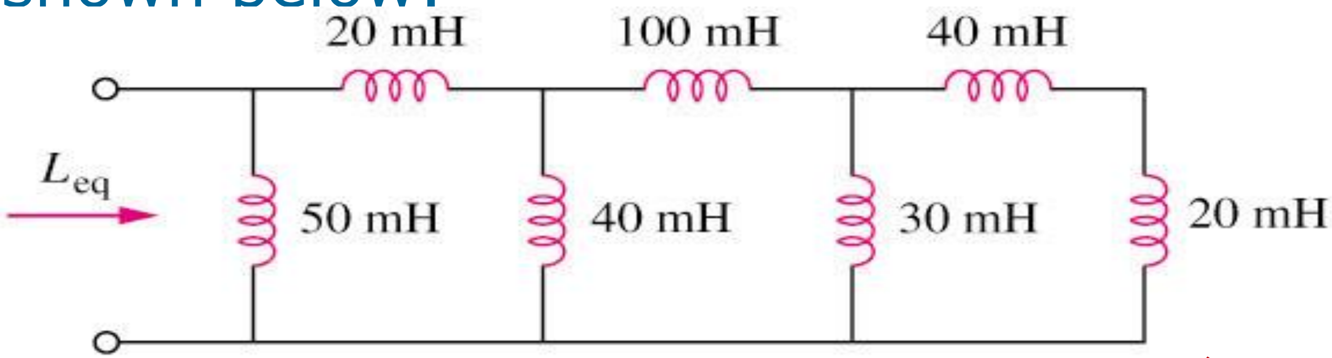


(b)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:



$30 \| 60 = 20$ mH

$40 \| 120 = 30$ mH

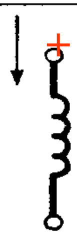
$50 \| 50 = \underline{25 \text{ mH}}$

Answer:

L_{eq} = 25mH

6.4 Series and Parallel Capacitors (4)

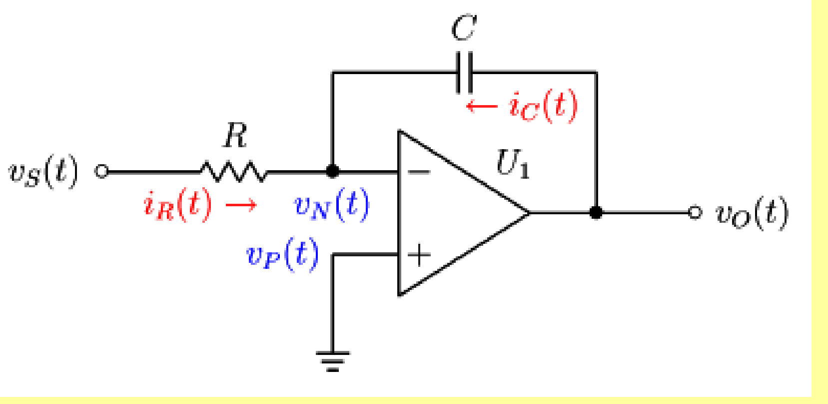
- Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
 Resistance	ohms (Ω)	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2 R$
 Inductance	henries (H)	$v = L\,\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\,\frac{di}{dt}$
 Capacitance	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\,\frac{dv}{dt}$	$p = vi = Cv\,\frac{dv}{dt}$

Analog Integration

http://ecee.colorado.edu/~mathys/ecen1400/labs/lab06/index.html

Inverting Integrator. The figure below shows the circuit of a inverting integrator.



In this case the capacitor provides some negative feedback, but since a capacitor acts like an open circuit for dc, the dc gai
feedback the output of the OpAmp eventually goes into positive or negative saturation in practical implementations. To analy
of the OpAmp to obtain

$$i_R(t) + i_C(t) = 0 \quad \Longrightarrow \quad \frac{v_S(t) - v_N(t)}{R} + C\,\frac{d[v_O(t) - v_N(t)]}{dt} = 0\,.$$

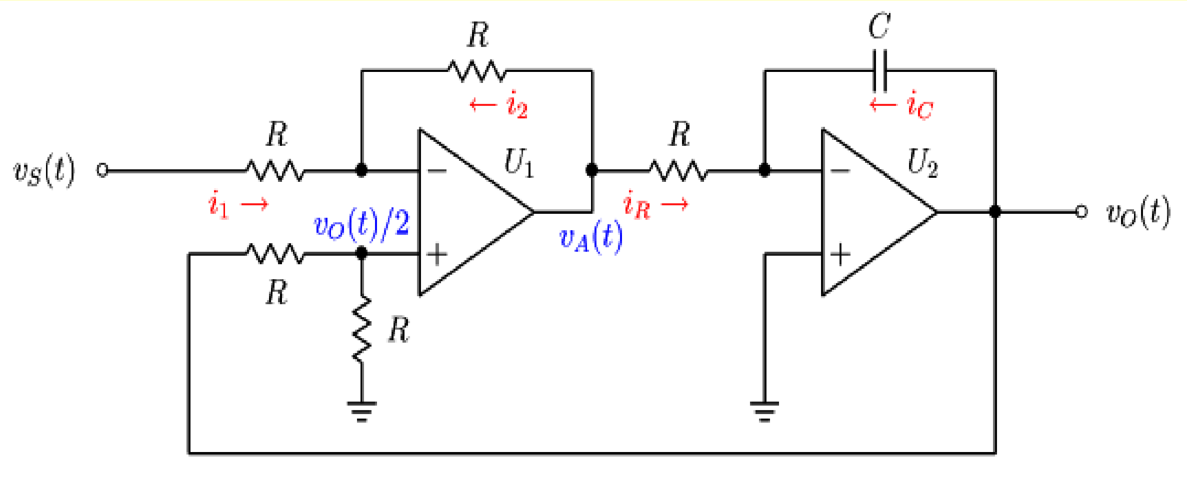
With $v_N(t) = v_P(t) = 0$ this becomes

$$\frac{v_S(t)}{R} + C\,\frac{dv_O(t)}{dt} = 0 \quad \Longrightarrow \quad \frac{dv_O(t)}{dt} = -\frac{1}{RC}\,v_S(t)\,.$$

Integrating both sides of the last expression then yields

$$v_O(t) = -\frac{1}{RC}\int_{-\infty}^{t} v_S(\tau)\,d\tau = -\frac{1}{RC}\int_{t_0}^{t} v_S(\tau)\,d\tau + v_O(t_0)\,.$$

A differential equation solver



The second OpAmp U_2 is a inverting integrator. Assuming ideal OpAmps and using KCL at the inverting input node yields

$$i_R(t) + i_C(t) = 0 \quad \Longrightarrow \quad \frac{v_A(t)}{R} + C\,\frac{dv_O(t)}{dt} = 0 \quad \Longrightarrow \quad v_A(t) = -RC\,\frac{dv_O(t)}{dt}\,.$$

Using voltage division, the voltage at the non-inverting node of OpAmp U_1 is found to be $v_O(t)/2$. Next, use KCL at the inverting input node of U_1 to obtain

$$i_1(t) + i_2(t) = 0 \quad \Longrightarrow \quad \frac{2v_S(t) - v_O(t)}{2R} + \frac{2v_A(t) - v_O(t)}{2R} = 0 \quad \Longrightarrow \quad v_S(t) = v_O(t) - v_A(t)\,.$$

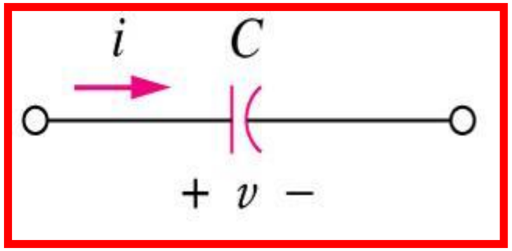
Combining the two results to eliminate the intermediate quantity $v_A(t)$ finally gives the differential equation

$$v_S(t) = v_O(t) + RC\,\frac{dv_O(t)}{dt}\,.$$

Thus, the output $v_O(t)$ of the analog computer shown above is the solution of a first order differential equation with time constant *RC*.

HANDOUTS

Example 1



The voltage across a 100-μF capacitor is

$$v(t) = 50 \sin(120\, \pi t)\ \text{V}$$

Find the displacement current i flowing into capacitor as a function of time

Answer:

$i(t) = 1.8850 \cos(120\, \pi t)$

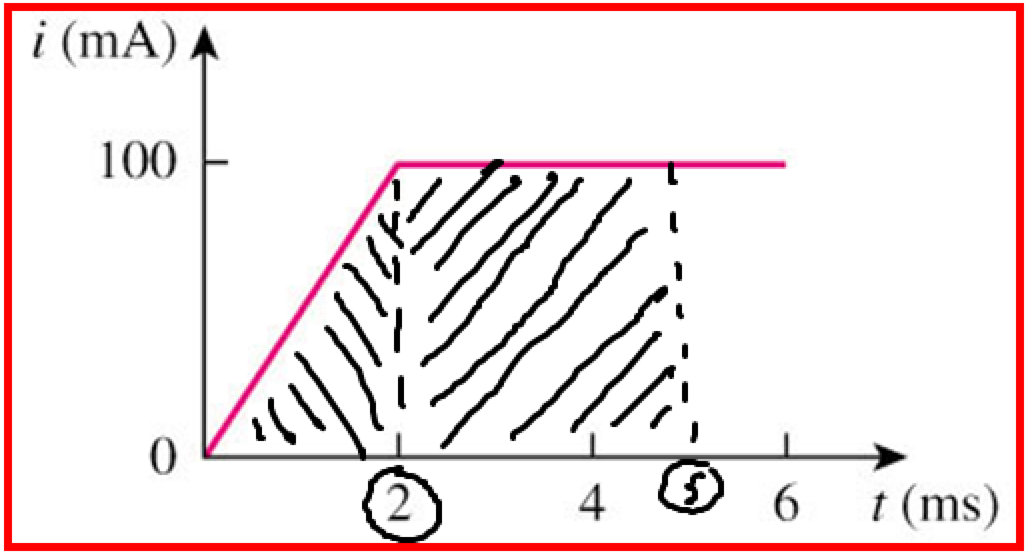
Example 2

An initially uncharged 1-mF capacitor has the current shown below across it.

Find the voltage across it at t = 2 ms and t = 5 ms.

V(2) = 0 + [area of Triangle]/.001F

V(5) = v(2) + [area of square]/.001F



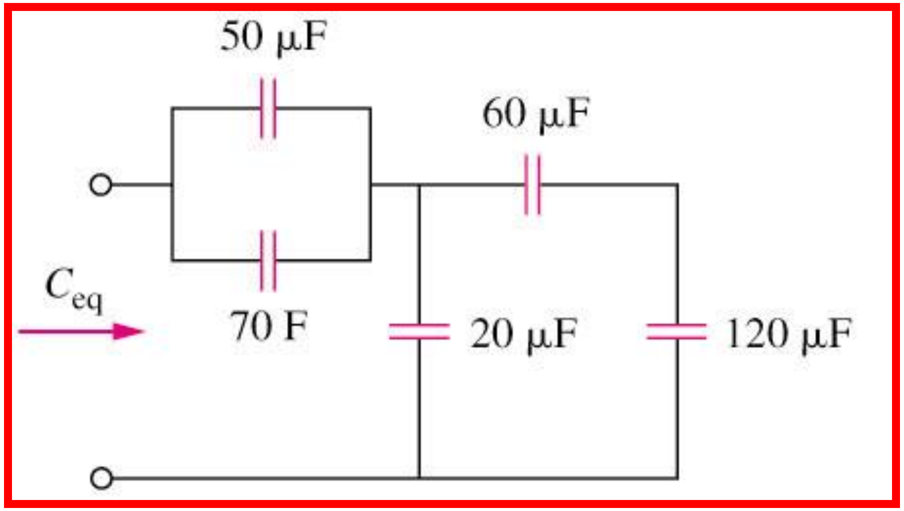
Answer:

v(2ms) = 100 mV

v(5ms) = 400 mV

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



Answer:

C_{eq} = 40μF

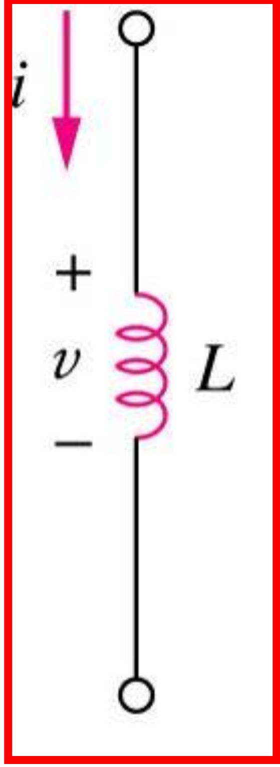
Example 5

The terminal voltage of a 2-H inductor is

$$v = 10(1-t) \text{ V}$$

Find the current flowing through it at $t = 4$ s and the energy stored in it at $t = 4$ s.

Assume $i(0) = 2$ A.



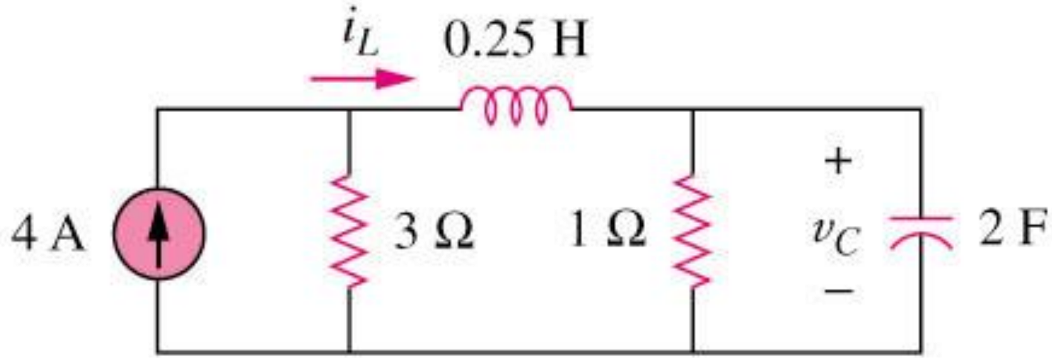
Answer:

i(4s) = -18A

w(4s) = 320J

Example 6

Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below unde



Answer:

$i_L = 3A$

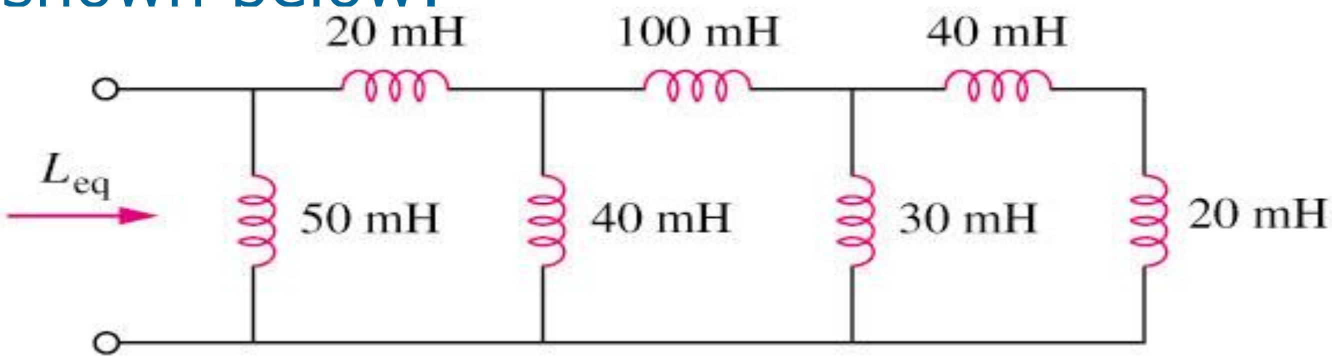
$v_C = 3V$

$w_L = 1.125J$

$w_C = 9J$

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:



Answer:

L_{eq} = **25mH**