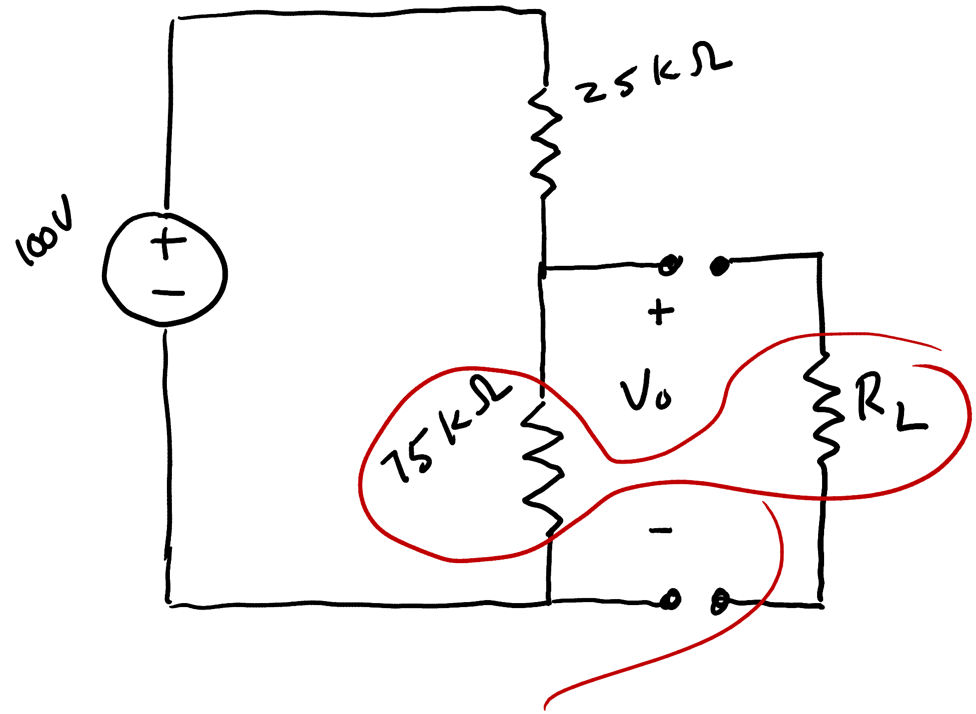
Chapter 5
The Operational Amplifier

This all came about in the 1930's when Bell Telephone had problems sending telephone signals over long distances….

amplifiers sensitive to temp and humidity → variable phone reception

Recall Voltage divider with Load RL

- "no-load" vo = 75V



- attach RL = 150k,

  vo drops to 66.6 V

Resistors in || are smaller

- The load "pulls down" the output voltage

not good for consumer electronics!

Amplifiers

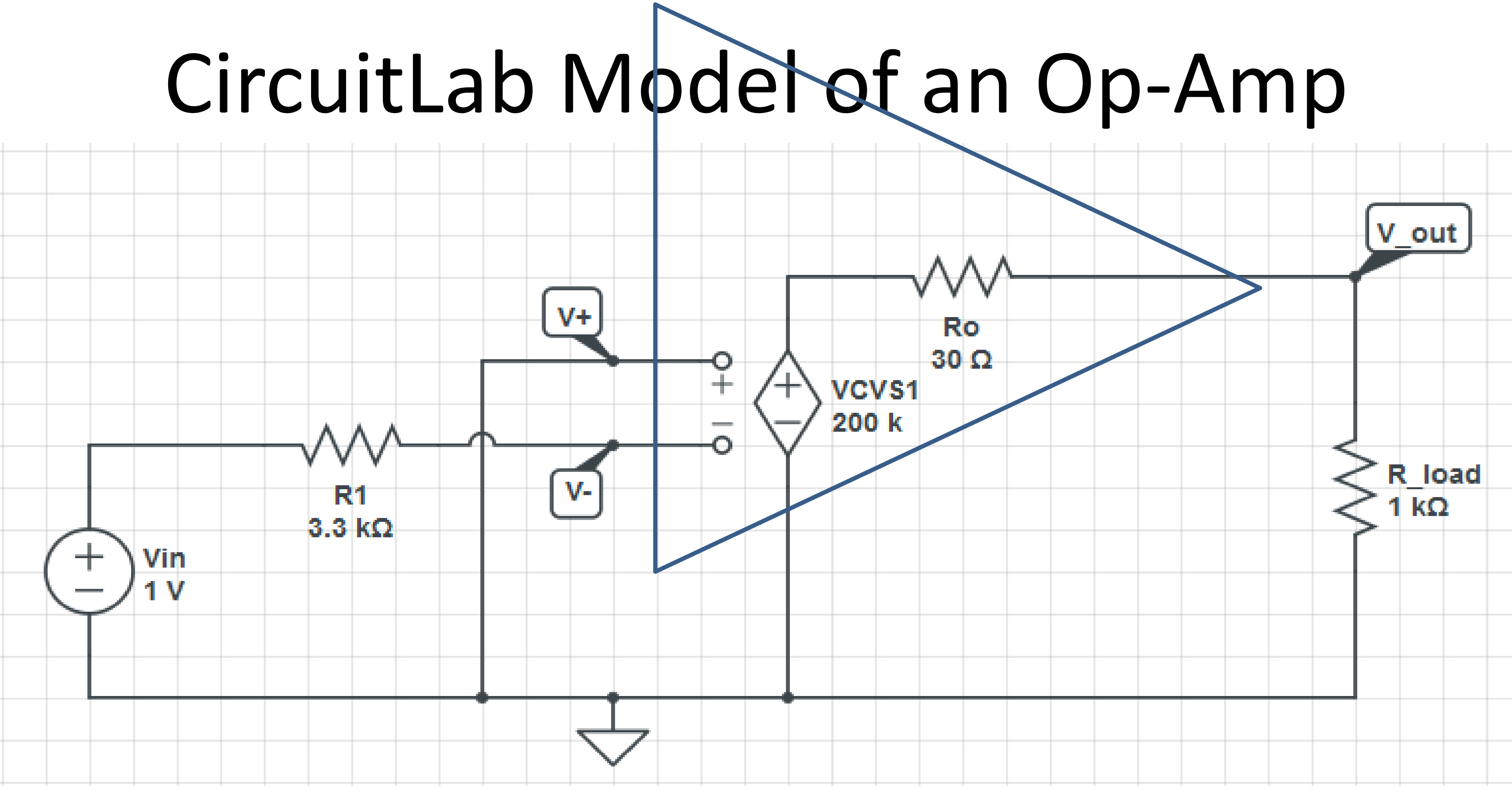
- Amplifiers are devices that magnify signals, and also remain mostly unaffected by changing load resistance. constant output

- Amplifiers are used in many instruments and electronic devices (iPod, cell phone, EEG) to boost signals (music, brainwaves) and buffer (isolate) them from loads.

Agenda

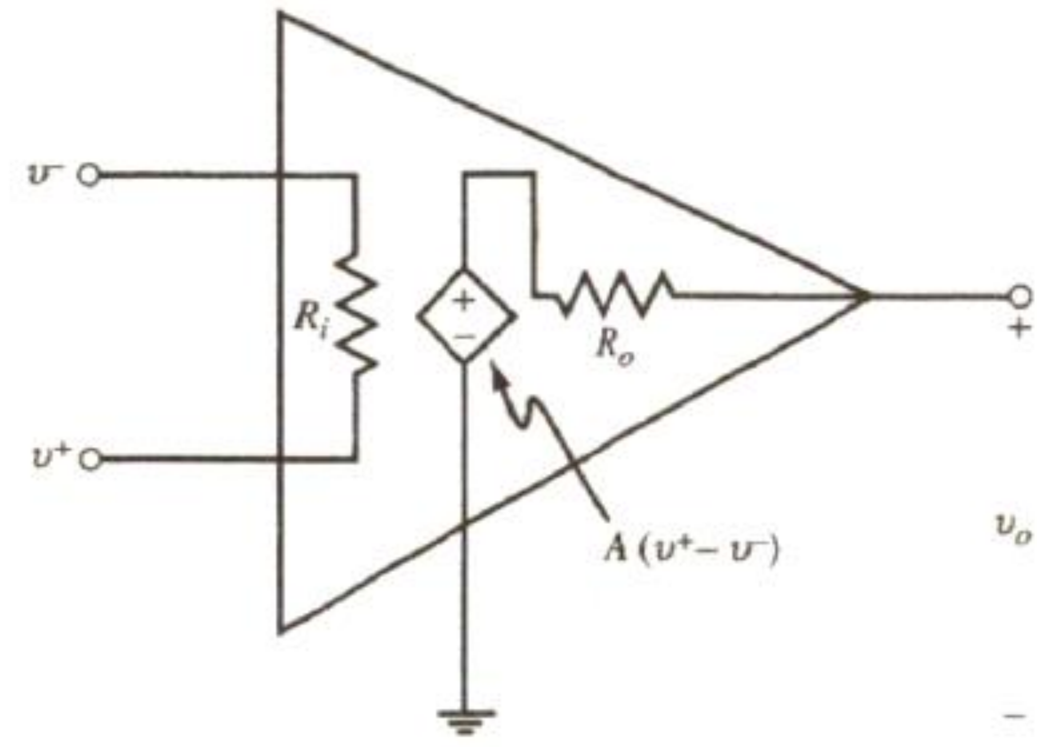
- Basic Amplifier Concepts
- The Op-Amp Model
- How to solve using KCL/KVL
- Standard Op-Amp Circuit Patterns
- Cascaded Op-Amp Circuits

CircuitLab Model of an Op-Amp

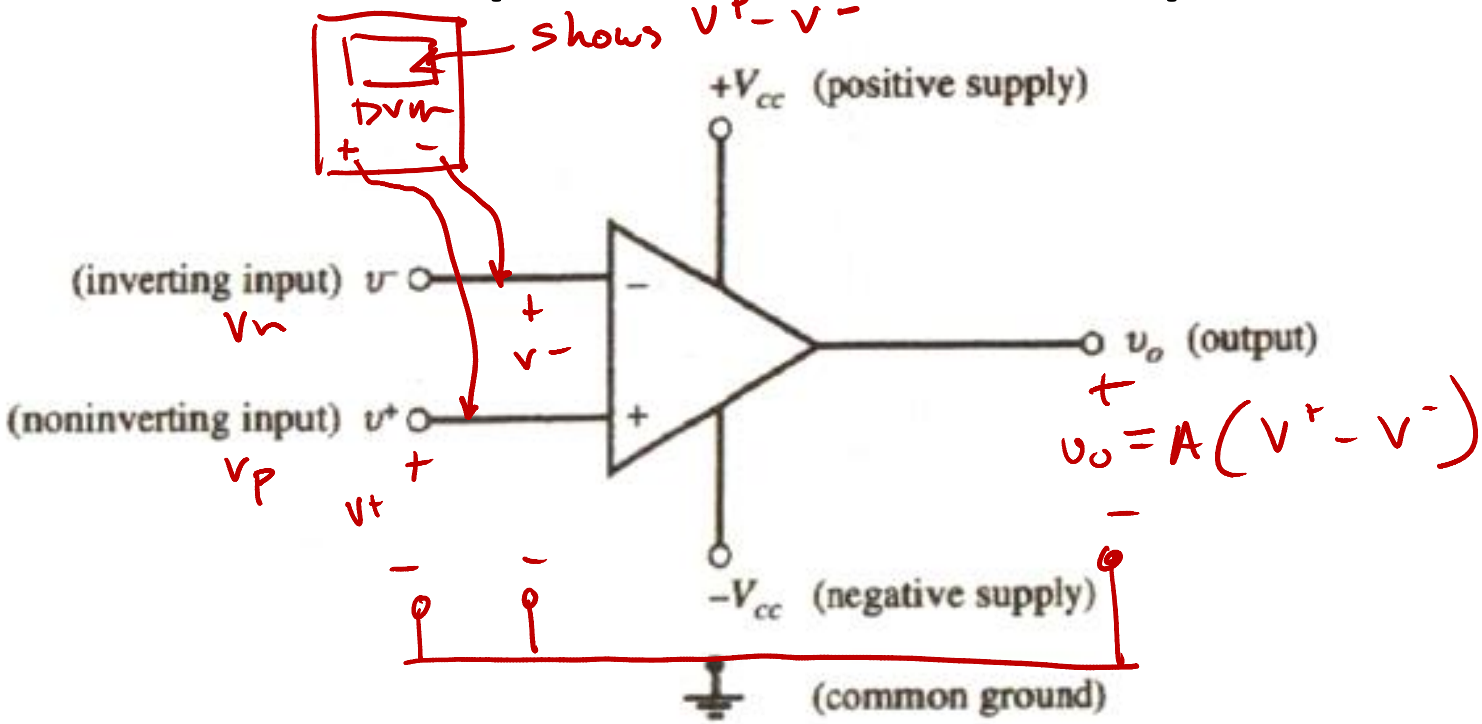


Non-ideal (realistic) Op-Amp model – use this on Assignment 6 prob5

- Typically:
  - Ri is very large 1M-ohm
  - Ro is small
  - A is $10^5 - 10^6$
  - model applies to linear range only



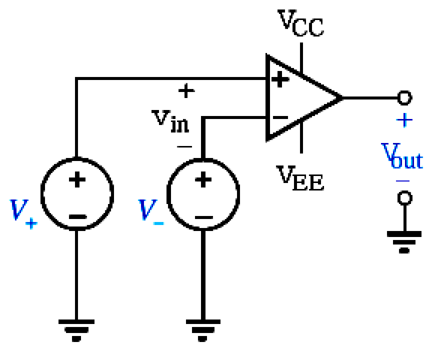
The Operational Amplifier

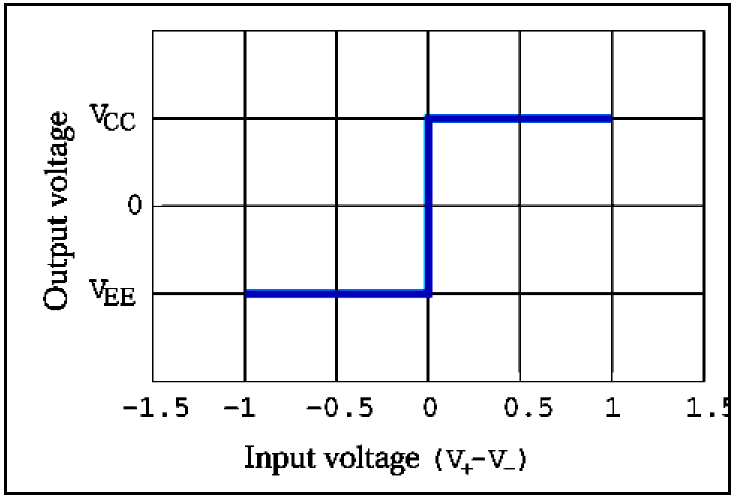


- v+ and v- are node voltages relative to ground
  sometimes we use vp and vn
- vo = A(v+ - v-) , ie. A times voltage across the input
- Vcc, -Vcc are power supply inputs, usually +/-15V

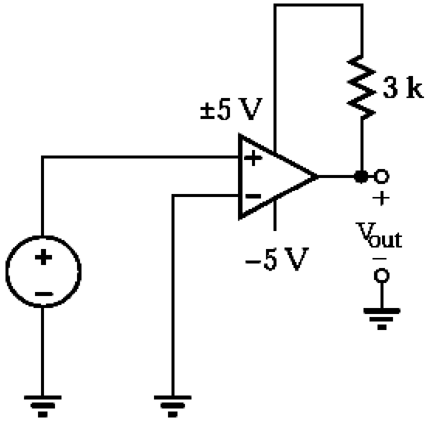
Op Amps can be used "open loop" outside linear range, v+ ≠ v-

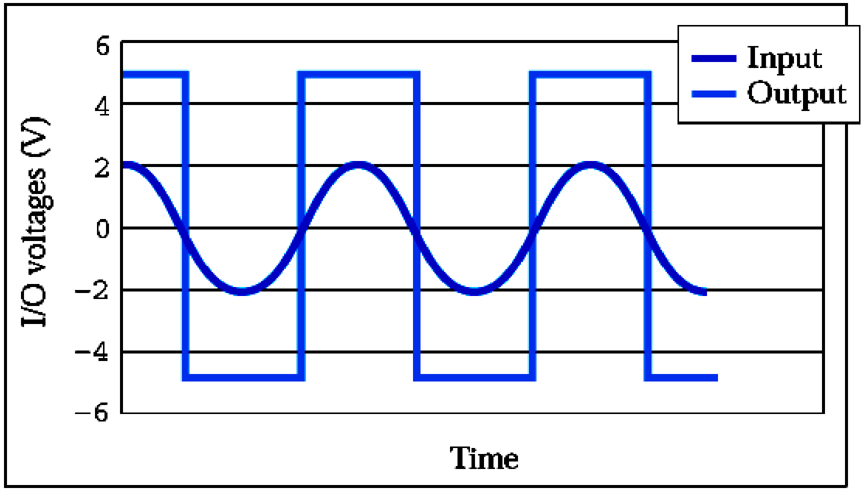
- Ideal Comparator and Transfer Characteristic





- "Zero-Cross" Detector – not so good for telephone use!

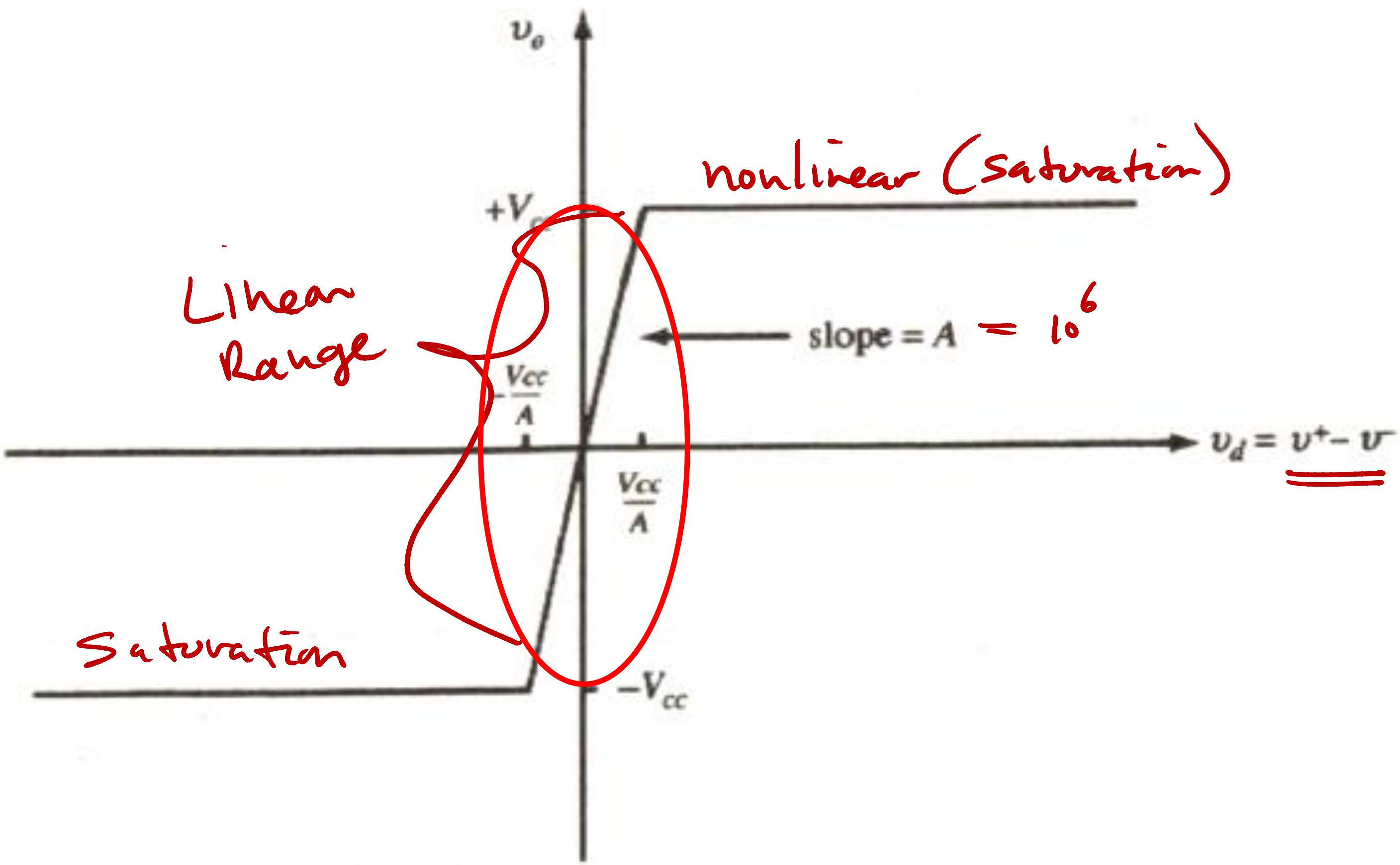




2 Ways of Using Op-Amps

- "Open Loop": very high gain amplifier
  - Useful for comparing 2 voltages
  - (nonlinear) Fixed gain, always at MAX OUTPUT!!
- "Closed Loop" with negative feedback ← ENGR 12
  - (linear) Useful for amplifying, adding, subtracting, differentiation and integration (using capacitors)
  - Variable gain, controlled by resistor selection

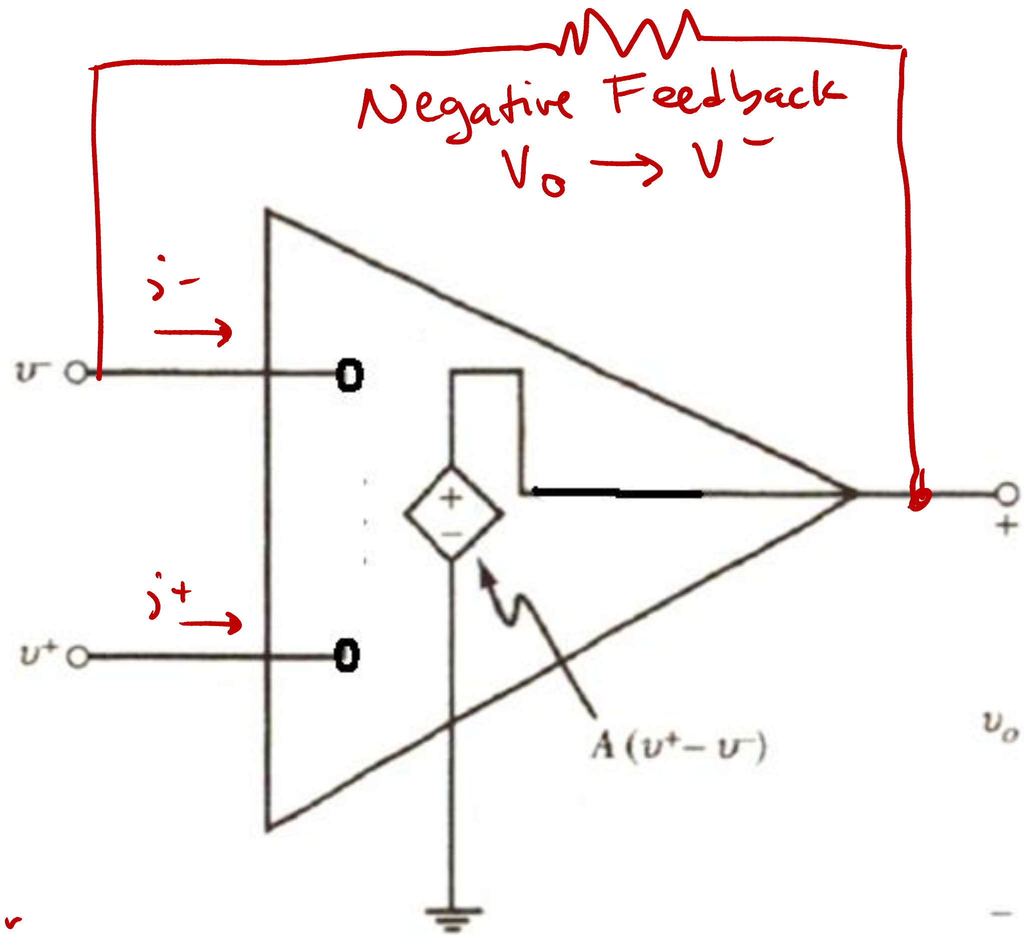
Linear Operation –
vd = Vp-Vn is from -Vcc/A to +Vcc/A



Ideal Op Amp Model -- Closed Loop

- Ri = infinity
- Ro = 0
- A = infinity
- i+ = i- = 0
- v+ = v-

IF negative feedback AND vo is within linear output range

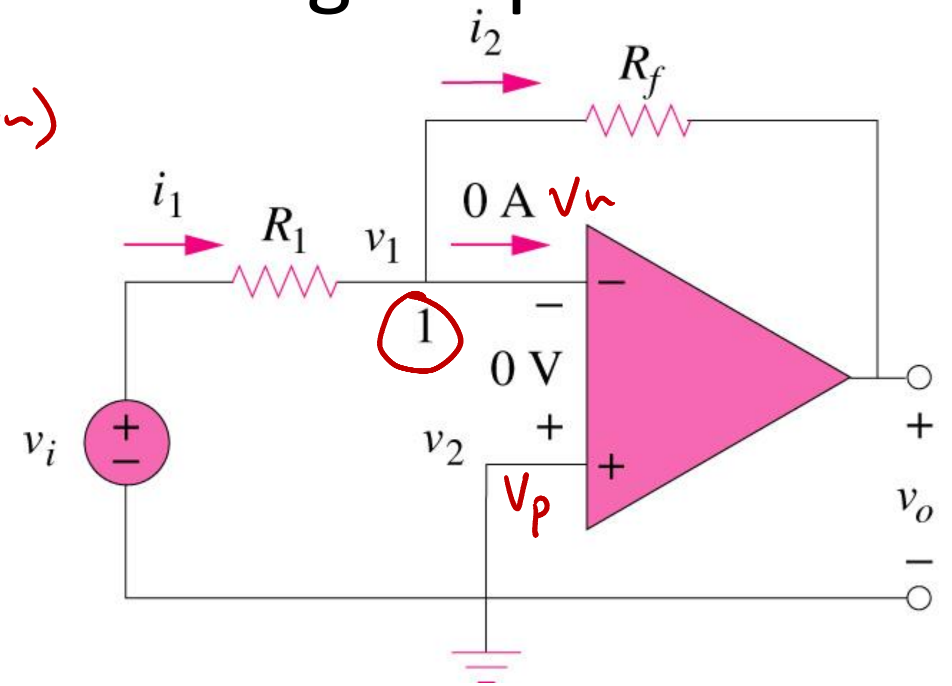


Standard Form 1: Inverting Amplifier

Neg feedback ✓

Linear vo ✓ (limit not given)

$V_1 = V_2 = 0$



KCL at Node 1

$$\frac{v_i - v_1}{R_1} + \frac{v_o - v_1}{R_f} = 0$$

$$v_o = -\frac{R_f}{R_1} v_i$$

$$\text{Gain} = \frac{v_o}{v_i} = -\frac{R_f}{R_1}$$

$$v_o = -\frac{R_f}{R_1} v_i$$

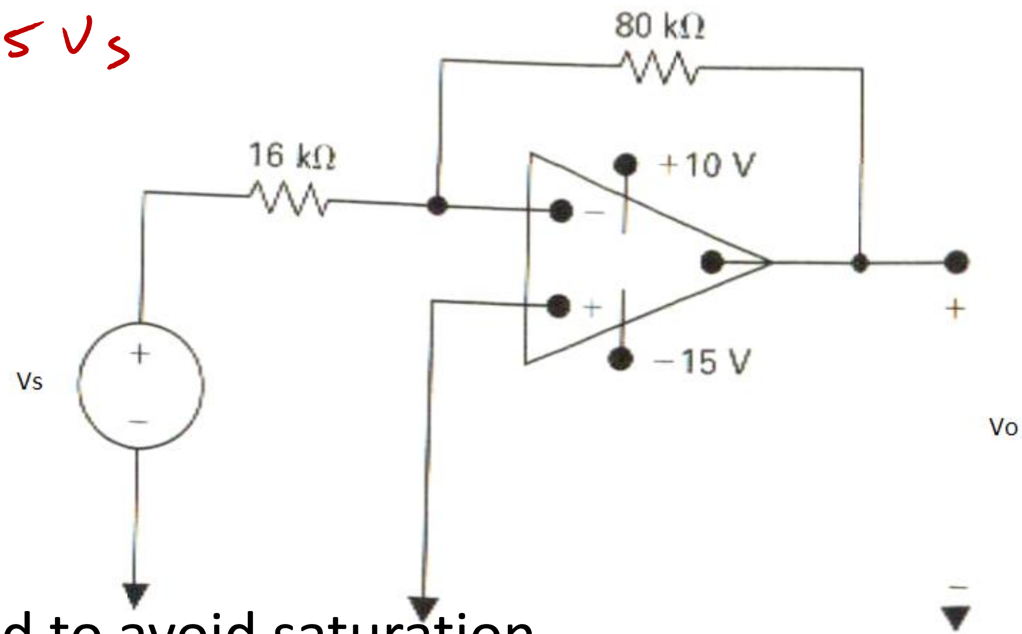
Example Inverting Op-Amp Problem

a) Calculate Vo for

Vs=0.4, 2, 3.5, -0.6, -1.6, -2.4

$$V_o = -\frac{R_f}{R_1} V_s = -\frac{80}{16} V_s = -5 V_s$$

V_s	V_o
0.4	−2
2	−10
3.5	~~−17.5~~ −15 limit
−.6	+3
−1.6	+8
−2.4	~~+12~~ +10 limit



b) Specify the range of Vs required to avoid saturation

$+10 = -5 V_s$, $V_s = -2V$

$-15 = -5 V_s$, $V_s = +3V$

↑ limits

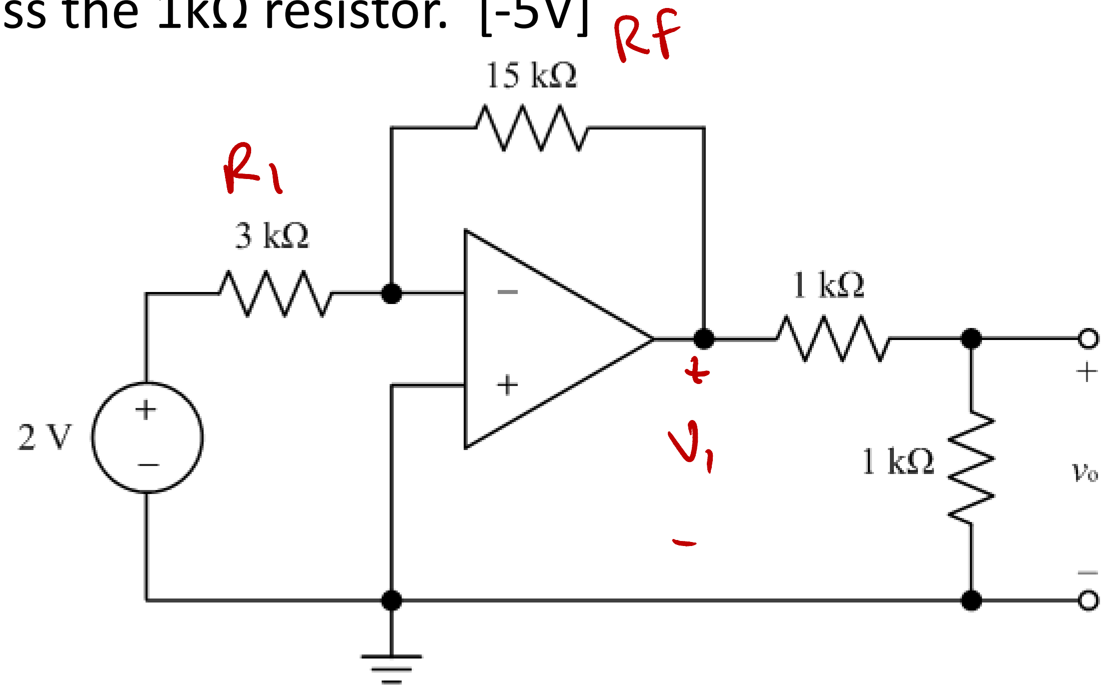
$$-2 < V_s < 3$$

1) Problem 1 INVERTING OP AMP

find the voltage vo across the 1kΩ resistor. [-5V]

$V_1 = -\frac{15}{3}(2) = -10V$

$V_o = V_1\left(\frac{1}{2}\right) = -5V$



How to Approach Op-Amp probs

1. Check for negative feedback

   All of our Op-Amp ccts will be "Closed Loop" with negative feedback
2. Assume current flowing into Vp/Vn terminals of op-amp = 0
3. Assume Op-Amp in linear range

   This means Vp must **=** Vn

   otherwise A(Vp-Vn) takes us to saturation
4. Determine value of Vp
5. Set Vn = Vp
6. Set up nodal equation at Vn node and solve for Vo
7. Check that Vo does not exceed power supply voltages +/- Vcc (if given)

   If so, then assumptions 3 and 5 do not hold

   Set Vo to the power supply voltage and recalculate

If you recognize common forms you can use formulas related to them

- Very helpful in cascaded Op-Amp problems
- Best not to depend too much on these
- You should always be able to go back to KCL/KVL

Standard Form 2: Summing Amplifier

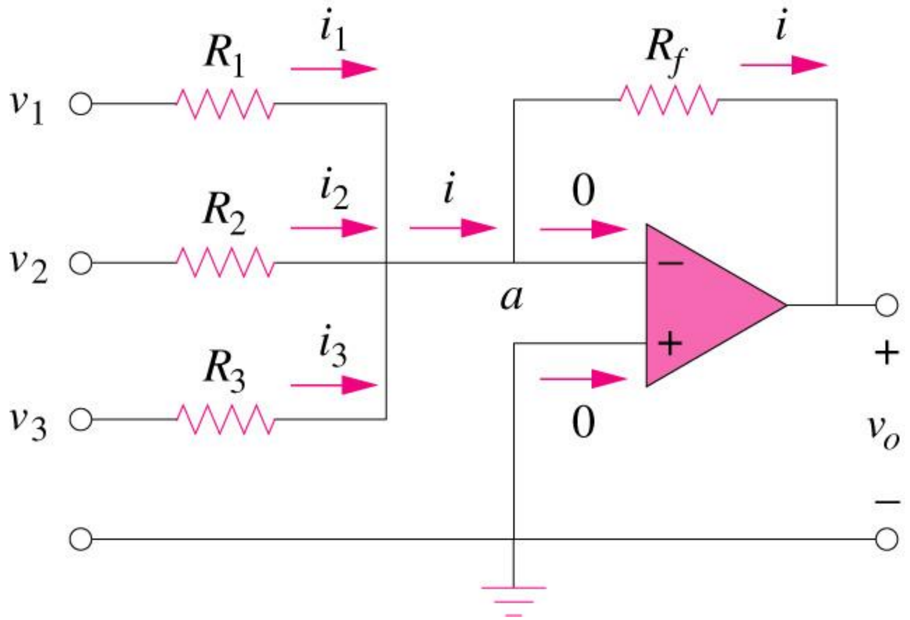
$v_n = v_p = 0$

KCL node a

$$\frac{v_1 - 0}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} - \frac{0 - v_o}{R_f} = 0$$

$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$

Sum of inputs



$$v_o = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3\right)$$

Summing Amplifier Example

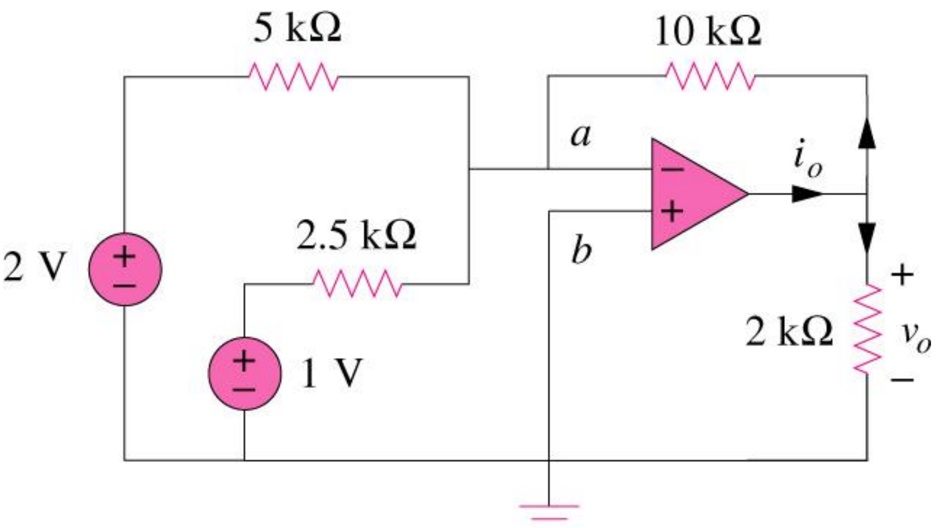
Calculate v_o and i_o in the op amp circuit shown below.

Use formula, or KCL

$$\frac{(2-0)}{5} + \frac{(1-0)}{2.5} - \frac{(0-v_o)}{10} = 0$$

$$v_o = -10\left(\frac{2}{5} + \frac{1}{2.5}\right) = -8V$$

$$i_o = \frac{v_o}{10} + \frac{v_o}{2} = -4.8 \text{ mA}$$



Ans: -8V, -4.8mA

2) Problem 2) Summing Op-Amp

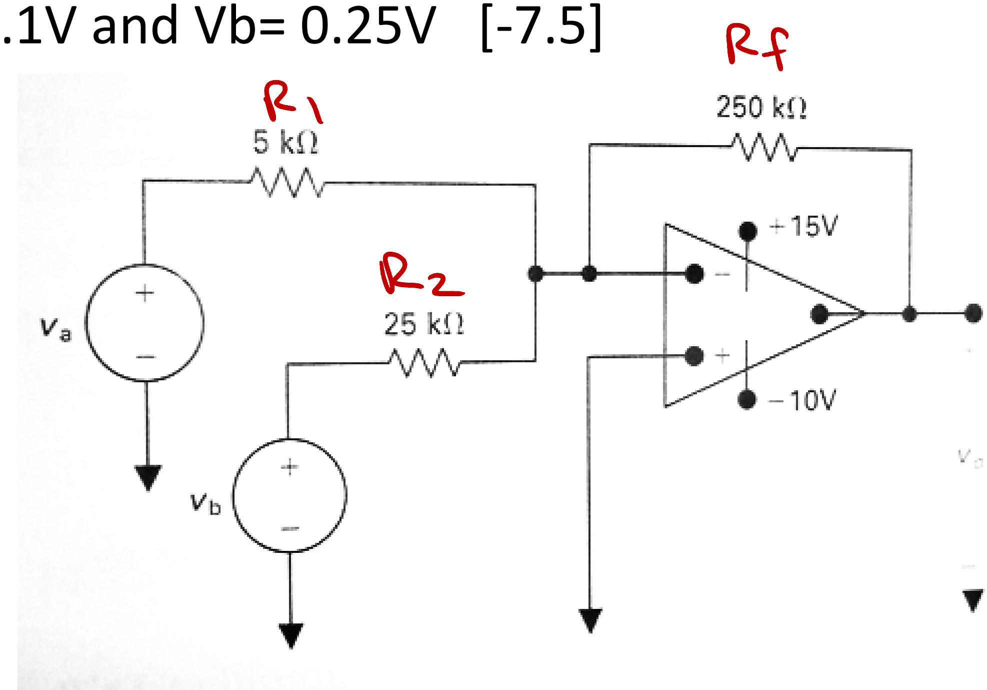
Find Vo in the circuit shown if Va=0.1V and Vb= 0.25V [-7.5]

$$V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2\right)$$

$$V_o = -\left(\frac{250}{5} V_a + \frac{250}{25} V_b\right)$$

$$= -(5 + 2.5)$$

$$= -7.5\ V$$



Standard Form 3: Non-Inverting Amplifier

$$V_1 \equiv V_2 = V_i$$

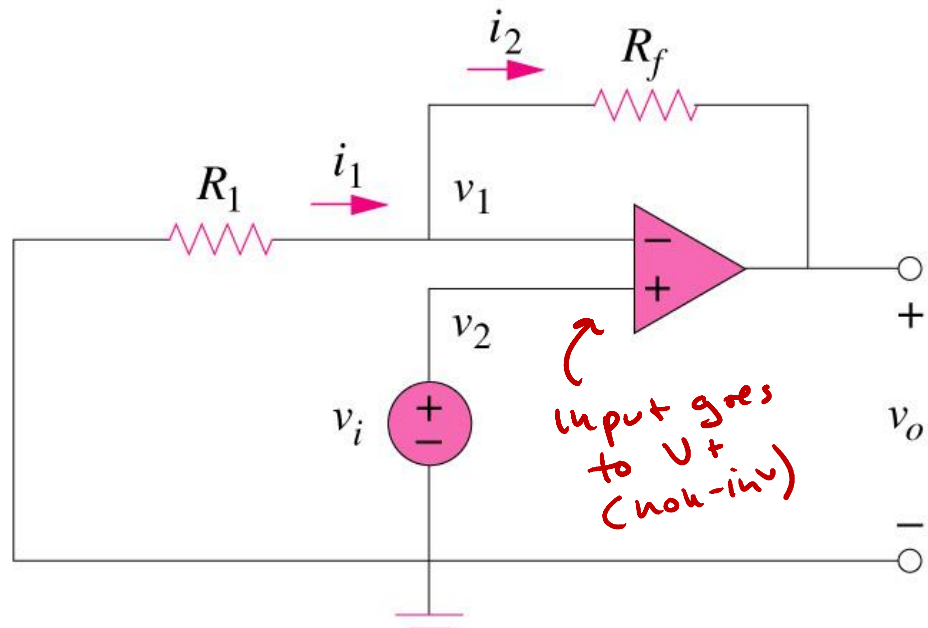
KCL at V_1

$$\frac{0 - V_1}{R_1} + \frac{V_o - V_1}{R_f} = 0$$

$$V_o = R_f\left(\frac{V_i}{R_1} + \frac{V_i}{R_f}\right)$$

$$V_o = V_i\left(\frac{R_f}{R_1} + 1\right)$$

Gain > 1



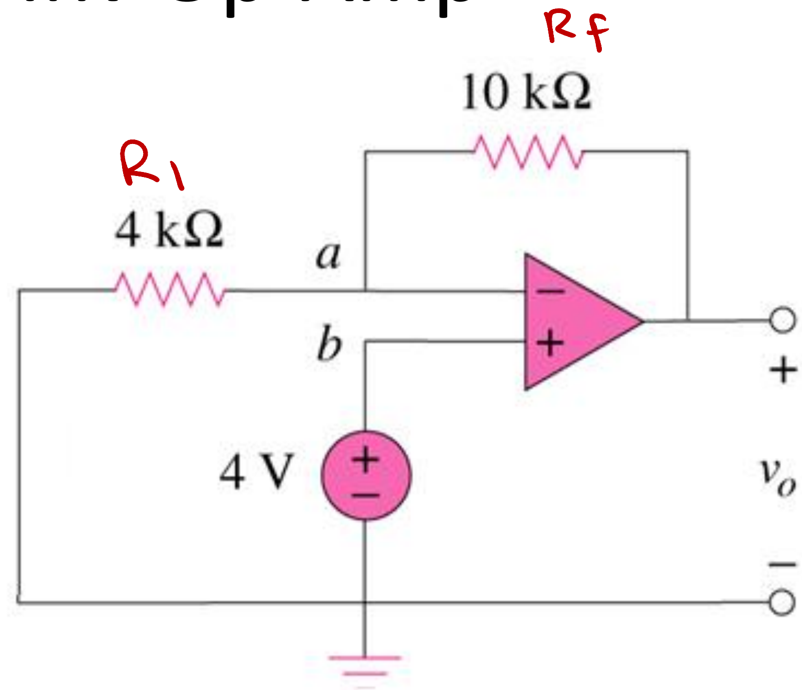
$$v_o = \left(1 + \frac{R_f}{R_1}\right)v_i$$

Find Vo for the Non-Inv Op-Amp

$$V_o = V_i\left(\frac{R_f}{R_1} + 1\right)$$

$$= 4\left(\frac{10}{4} + 1\right)$$

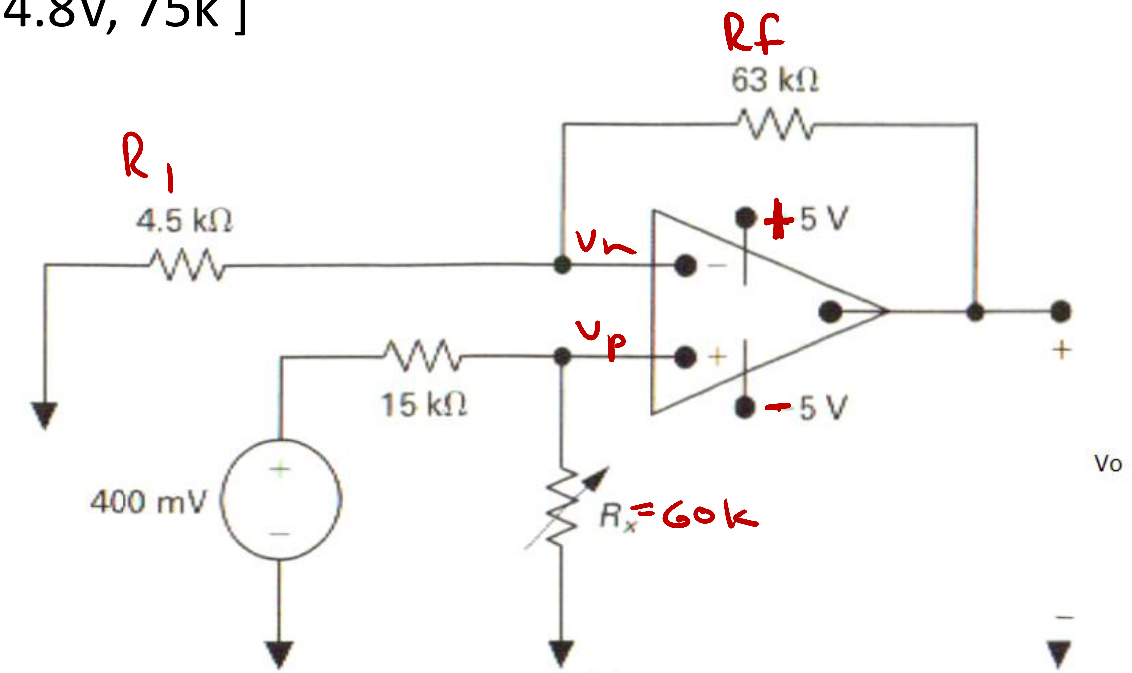
$$= 14\ V$$



3) Problem 3) Non-Inverting

Find the output Voltage when Rx is set to 60k

What Rx will cause saturation? [4.8V, 75k]



a) Non-inverting

Voltage divider on input

$$V_p = .4\left(\frac{60}{15+60}\right) = .32V$$

now use formula

$$v_o = v_p\left(\frac{RF}{R_1} + 1\right)$$

$$= .32\left(\frac{63}{4.5} + 1\right) = \underline{\underline{4.8\ V}}$$

b) Total formula for Vo

$$V_o = .4\left(\frac{Rx}{15+Rx}\right)(15) = +5 \text{ at saturation}$$

Solve for Rx

$$Rx(6) = 5(15+Rx), \quad \underline{Rx = 75\ k}$$

Difference Amplifier

Combine Non- & Inv Formulas:

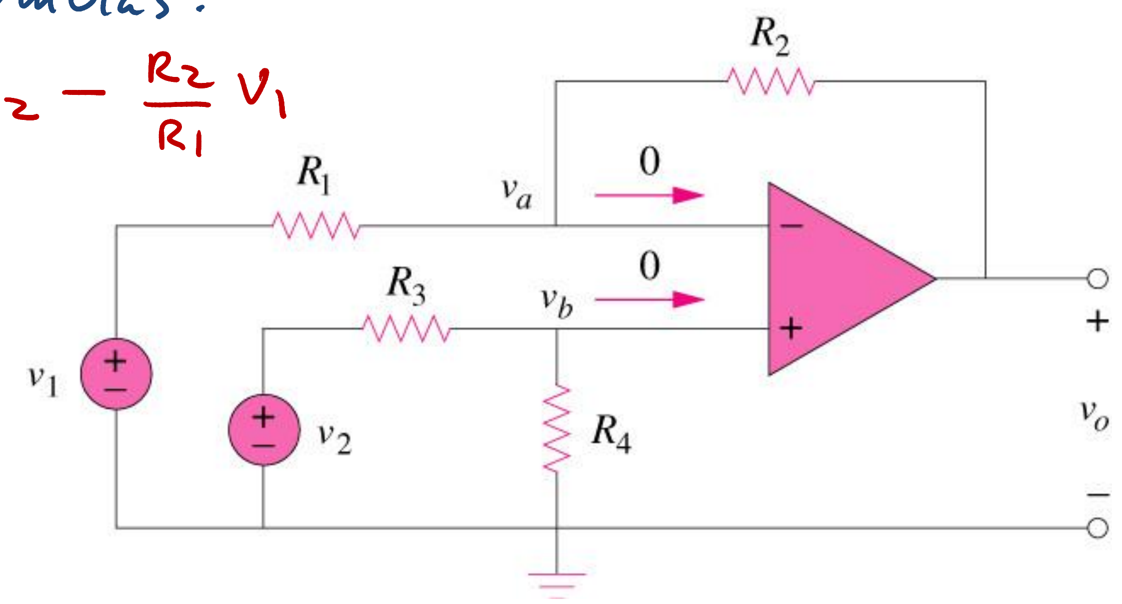
$$v_o = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_2 - \frac{R_2}{R_1}v_1$$

If we select

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}, \text{ then}$$

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

A balanced difference amp!



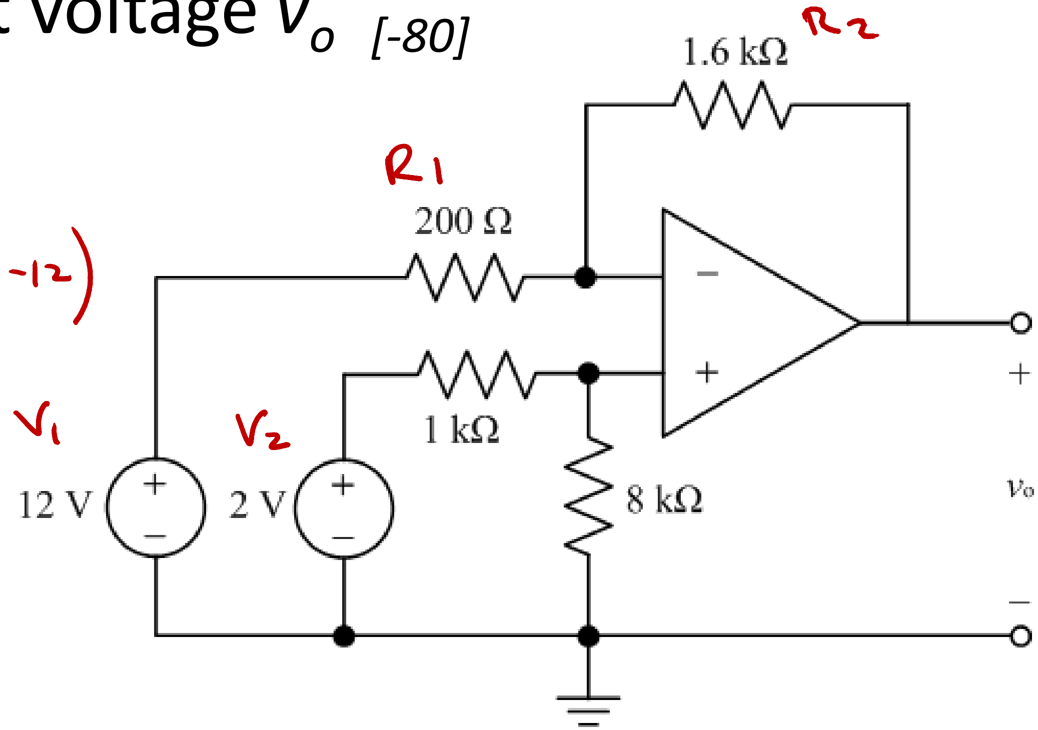
$$v_o = \frac{R_2}{R_1}(v_2 - v_1), \text{ if } \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Find the output voltage v_o *[-80]*

Note: $\frac{1600}{200} = \frac{8}{1}$

$\therefore \; V_o = \frac{R_2}{R_1}(V_2 - V_1) = 8(2-12)$

$= -80\,V$



4) Problem 4 Difference Amplifier

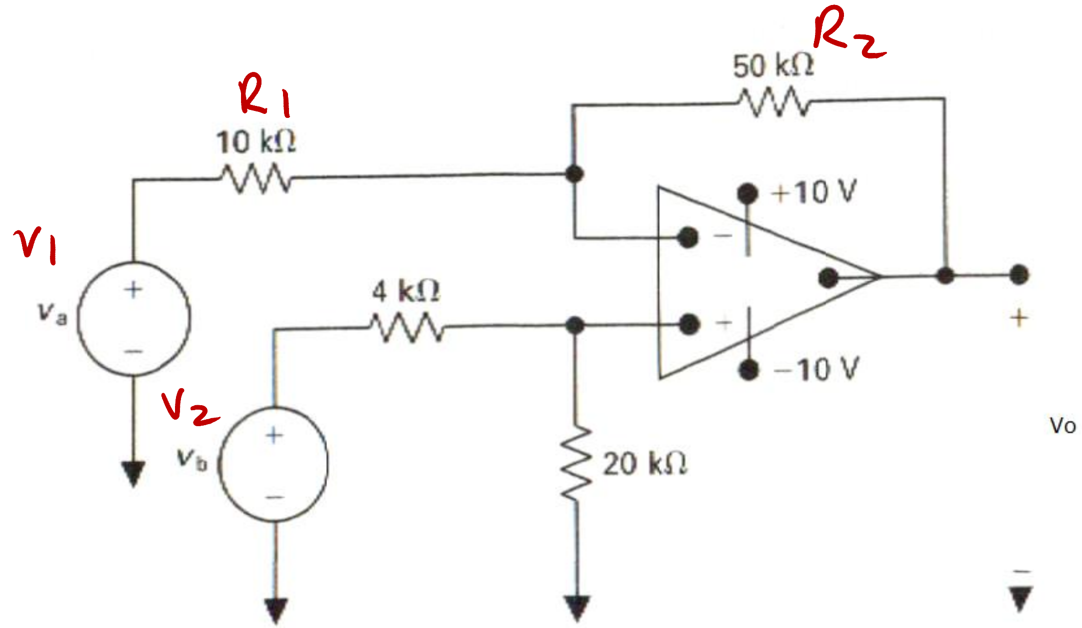
If Vb=4.0V, what values of Va will keep linear operation? [2<=va <=6]

Again, $\frac{50}{10} = \frac{20}{4}$

$\therefore V_o = \frac{R_2}{R_1}(V_2 - V_1)$

$V_o = 5(V_b - V_a)$

$= 5(4 - V_a)$



Solve for

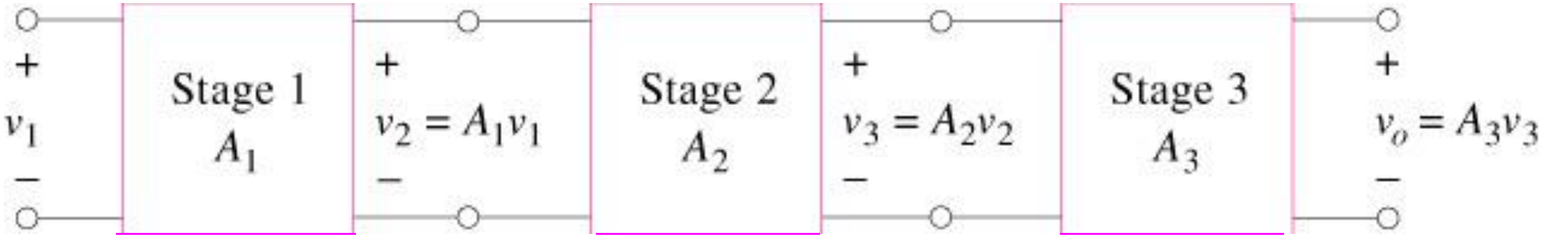
a) $10 = 5(4 - V_a) \rightarrow V_a = \frac{(20-10)}{5} = 2$

b) $-10 = 5(4 - V_a) \rightarrow V_a = \frac{(20+10)}{5} = 6$

$2 < V_a < 6$

Cascaded Op Amps

- A head-to-tail arrangement of two or more op amp circuits such that the output to one is the input of the next.

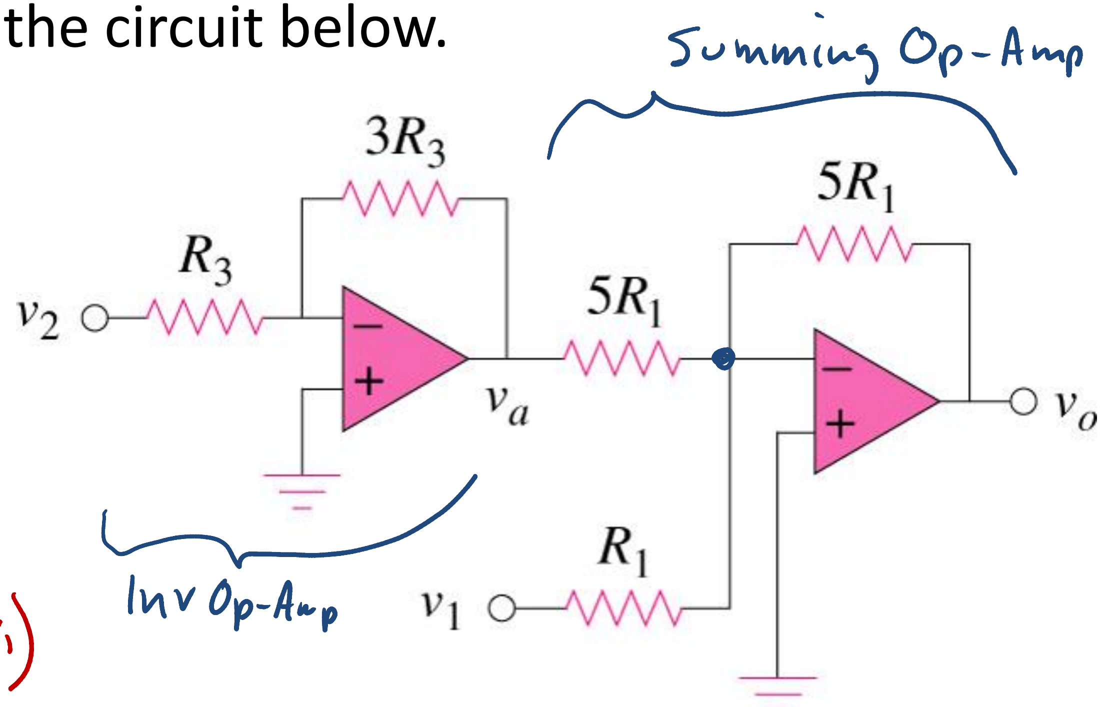


- Total Gain is the product of all stages

$$Gain = \frac{v_o}{v_i} = A_1 A_2 A_3$$

Another way to make a Difference Amp

Find the formula for Vo in the circuit below.



Inv Op-Amp

$$v_a = -\frac{3R_3}{R_3} v_2 = -3v_2$$

Summing Op-Amp

$$V_o = -\left(\frac{5R_1}{5R_1} v_a + \frac{5R_1}{R_1} v_1\right)$$

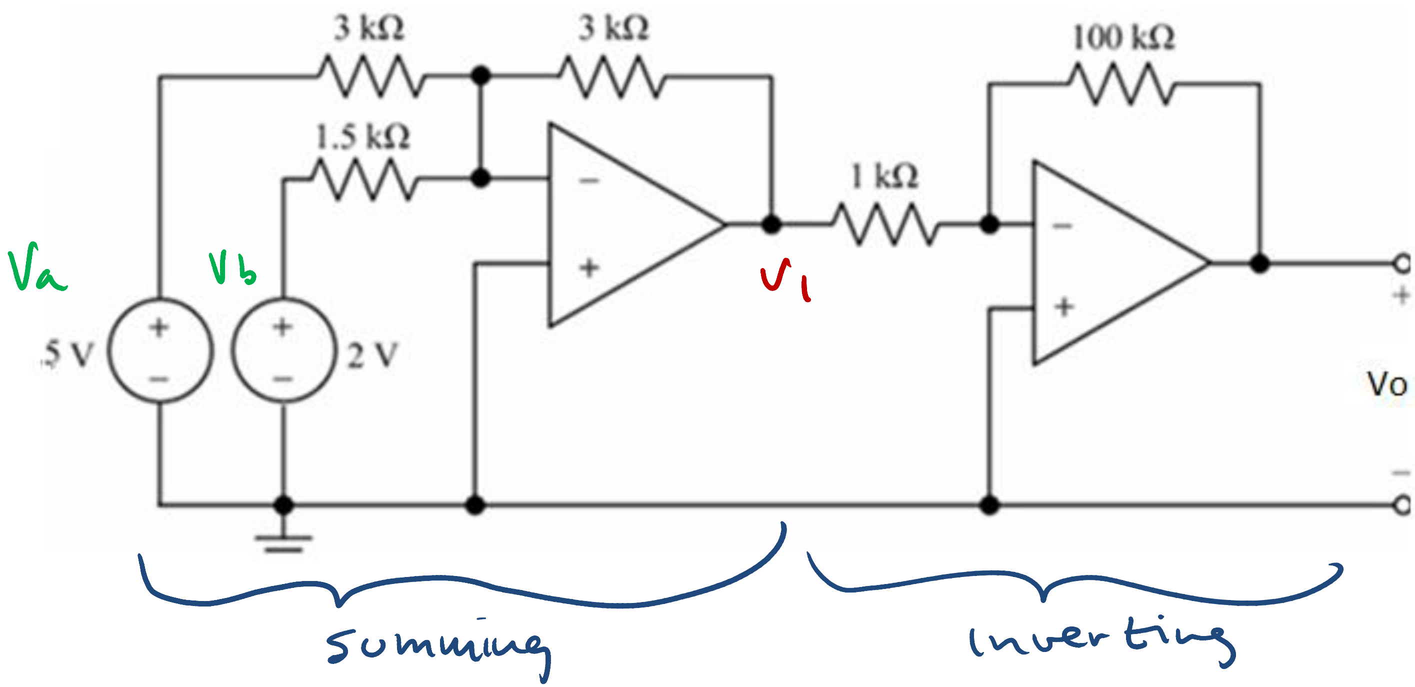
$$= -\left(-3v_2 + 5v_1\right)$$

$$V_o = 5v_1 - 3v_2$$

Ans:

$v_o = -5v_1+3v_2$

Find the output voltage v_o *[900 V]*



<u>summing</u>

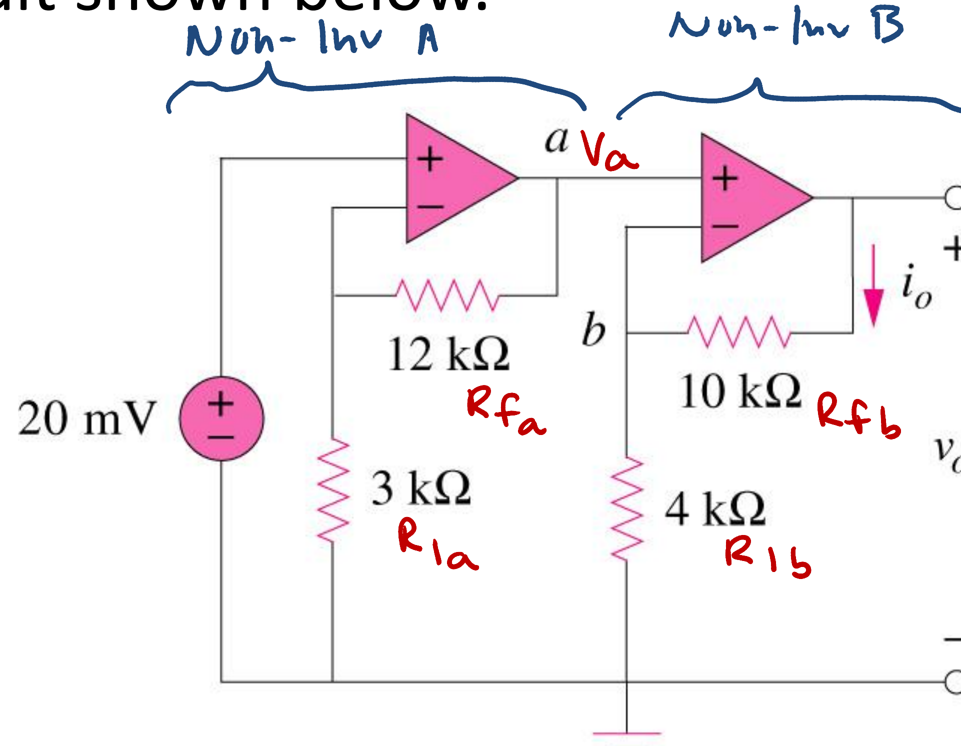
$$V_1 = -\left(\frac{3}{3}(5) + \frac{3}{1.5}(2)\right) = -9$$

<u>Inverting</u>

$$V_o = -\frac{100}{1} V_1 = \underline{\underline{900\,V}}$$ (assuming not saturated!)

Total Gain = 100 (Va + 2Vb)

Find v_o and i_o in the circuit shown below.



Non-Inv A

$$V_a = \left(1 + \frac{R_{fa}}{R_{1a}}\right) 20\,mV$$

$$= 5(.02) = .1\,V$$

Non-Inv B

$$V_o = \left(1 + \frac{R_{fb}}{R_{1b}}\right) V_a$$

$$= (3.5)(.1) = \underline{\underline{.35\,V}}$$

$$\text{Total Gain} = \left(1 + \frac{R_{fa}}{R_{1a}}\right)\left(1 + \frac{R_{fb}}{R_{1b}}\right) = (5)(3.5) = 17.5$$

Finding i_o

OL: $i_o = \frac{V_o}{14k} = \frac{.35}{14000} = \underline{\underline{25\,\mu A}}$

Ans: 350mV, 25μA

If v_1 = 1V and v_2 = 2V, find v_o in the op amp circuit shown below.

C

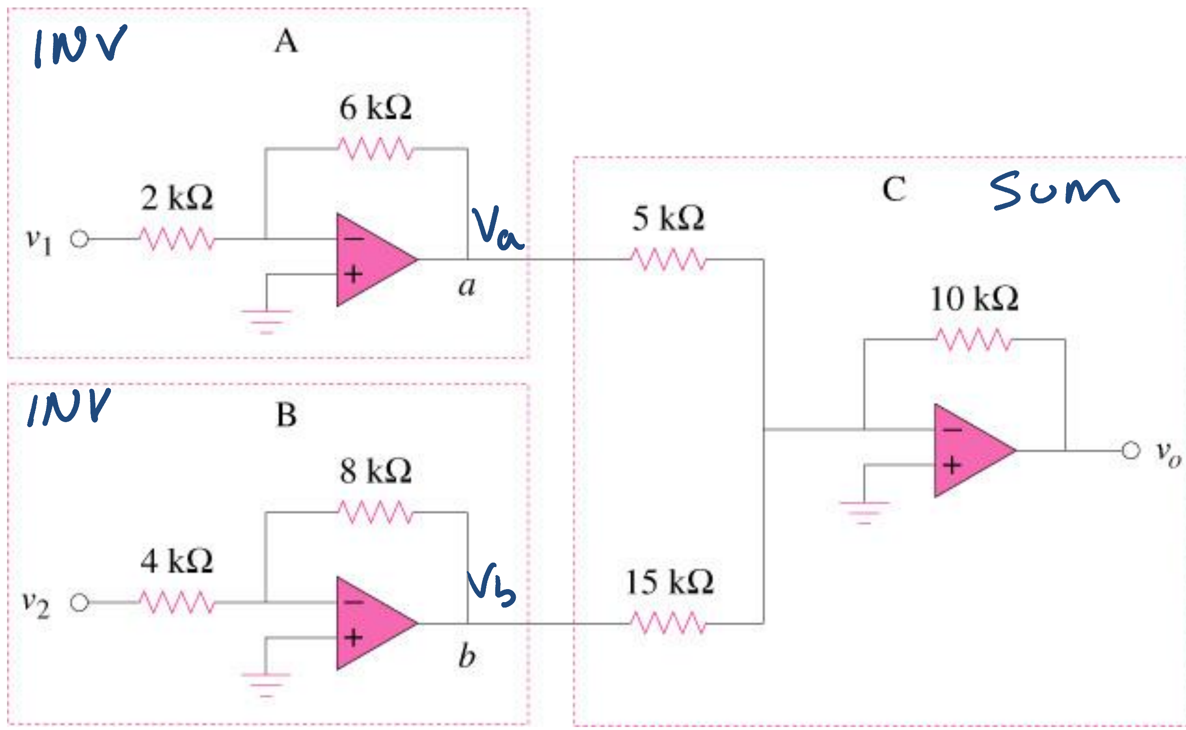
$$V_o = -\left(\frac{10}{5} V_a + \frac{10}{15} V_b\right)$$

A

$$V_a = -\frac{6}{2} V_1$$

B

$$V_b = -\frac{8}{2} V_2$$



$$\therefore V_o = 3V_1 + \frac{8}{3} V_2$$

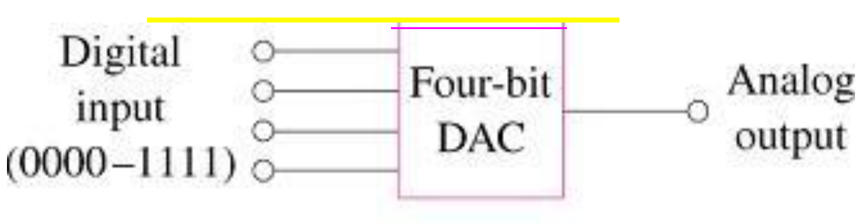
$$= 3 \cdot 1 + \frac{8}{3} \cdot 2 = 8.333\ V$$

Ans: 8.667 V

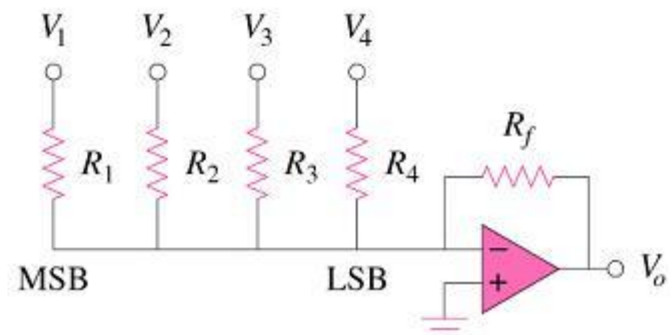
5.5 Application

- Digital-to Analog Converter (DAC) : a device which transforms digital signals into analog form.

Four-bit DAC: (a) block diagram (b) binary weighted ladder type



(a)



(b)

where

V_1 – MSB, V_4 – LSB

V_1 to V_4 are either 0 or 1 V

5.5 DAC Example

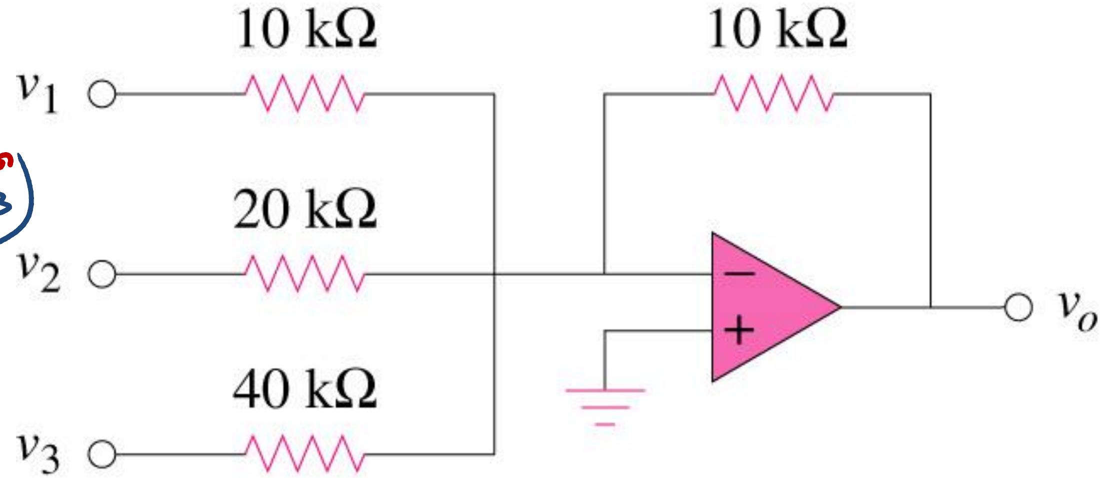
For the circuit shown below, calculate v_o if v_1= 0V,v_2=1V and v_3 = 1V.

Summing Op-Amp

$$V_o = -\left(\frac{10}{10}V_1 + \frac{10}{20}V_2 + \frac{10}{40}V_3\right)$$

$$= -(.5 + .25)$$

$$= -.75V$$

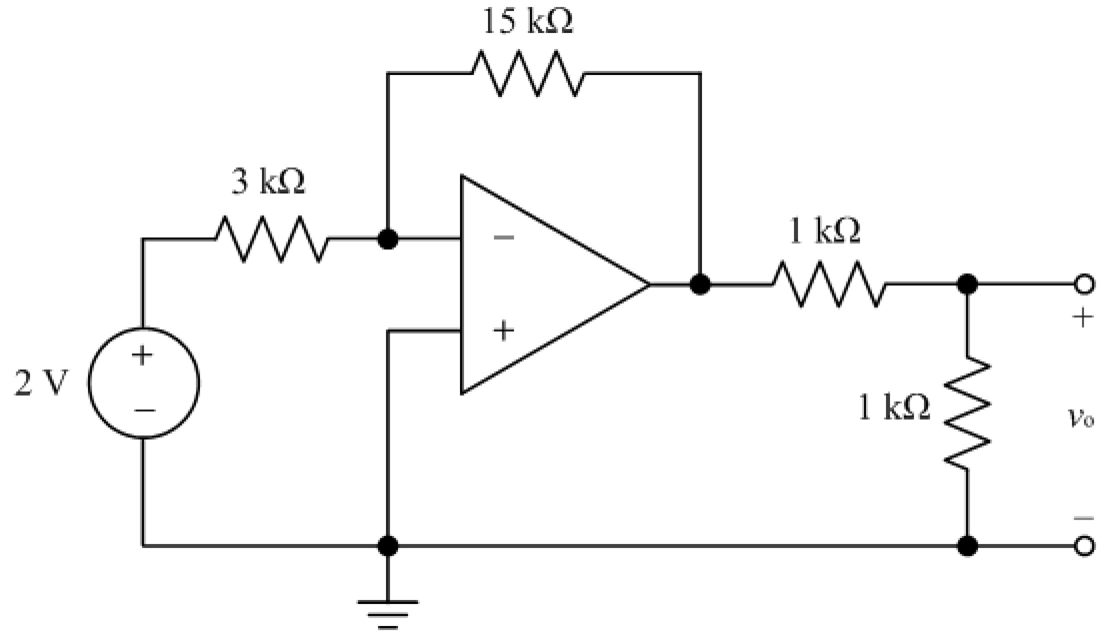


Ans:-0.75V

Handouts

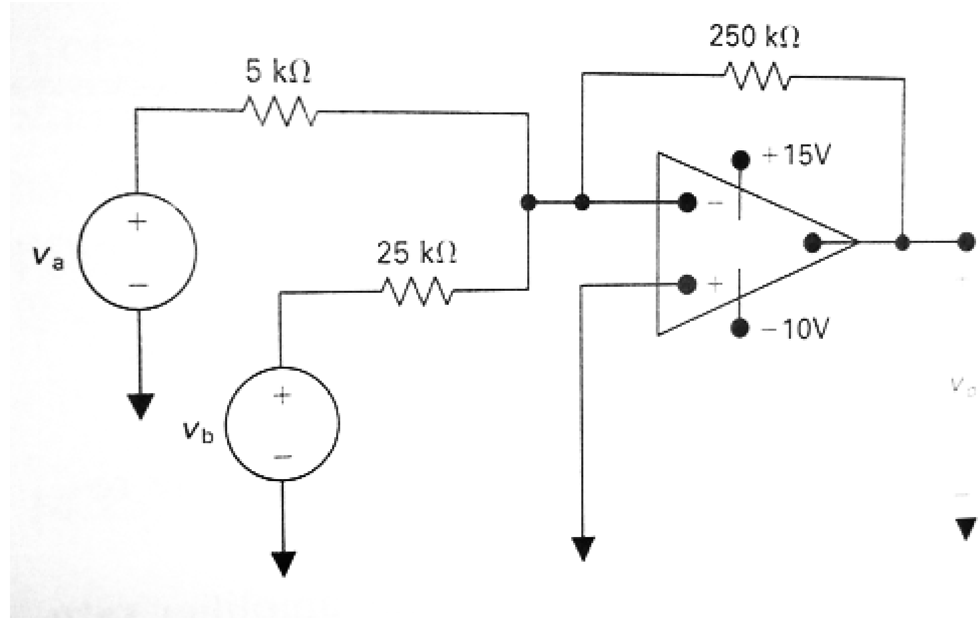
1) Problem 1 INVERTING OP AMP

find the voltage vo across the 1kΩ resistor. [-5V]



2) SUMMING OP-AMP

Find Vo in the circuit shown if Va=0.1V and Vb= 0.25V [-7.5]



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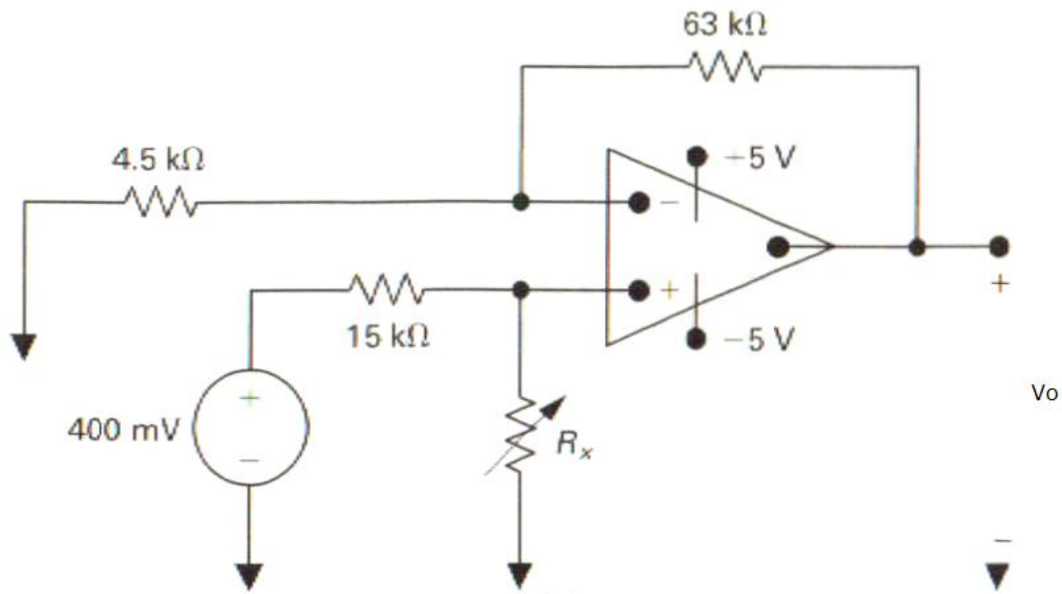

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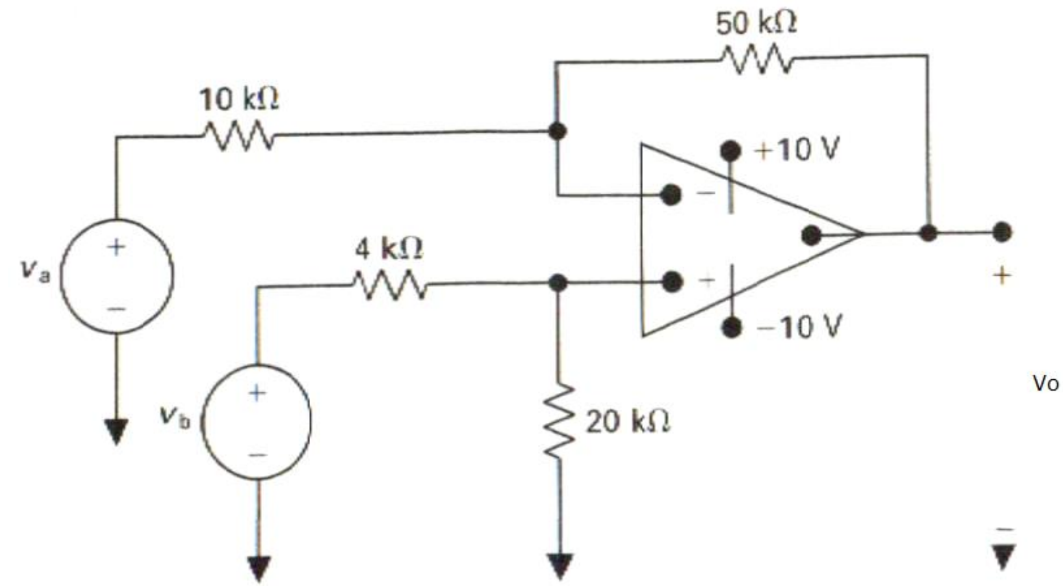
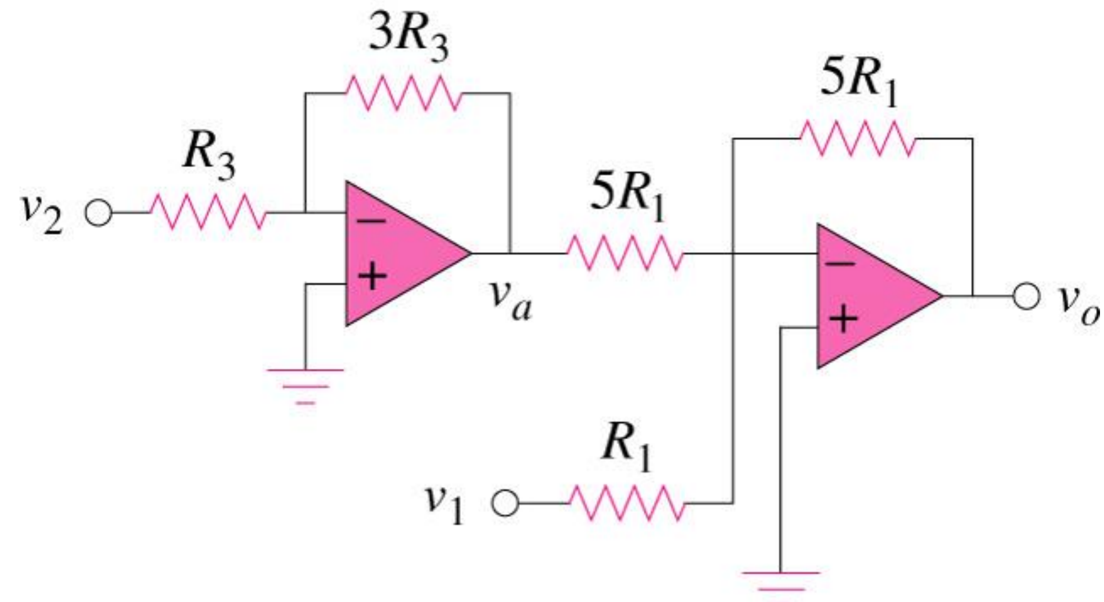


5. Another way to make a Difference Amp

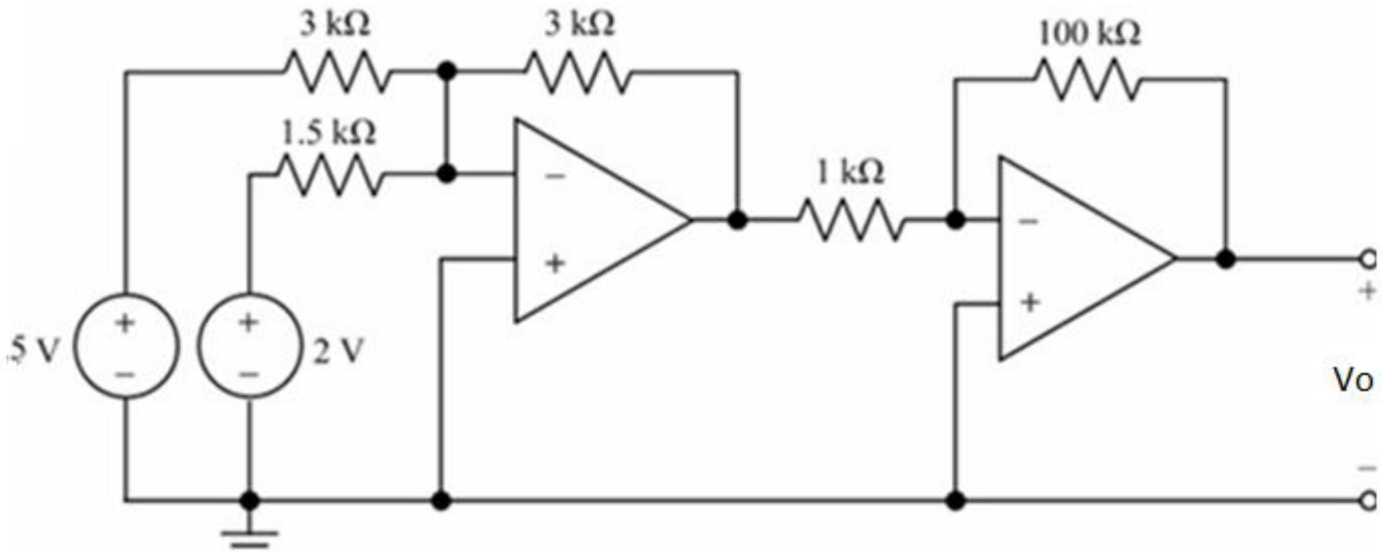
Find the formula for Vo in the circuit below.



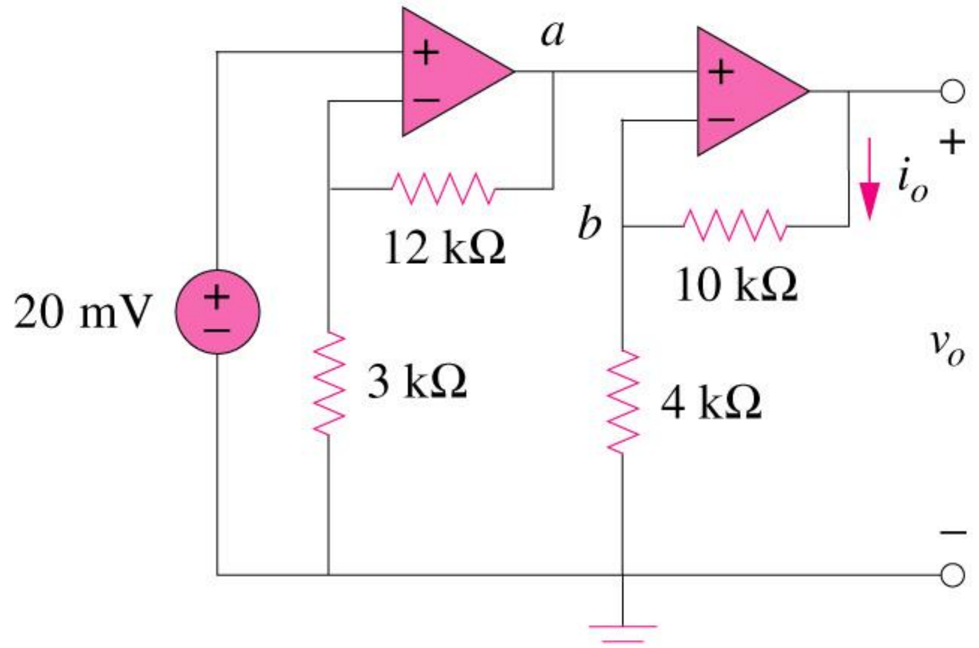
Ans:

$v_o = -5v_1+3v_2$

6. Find the output voltage v_o *[900 V]*



7. Find v_o and i_o in the circuit shown below.



Ans: 350mV, 25μA

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