Circuit Theory

Chapter 3

Methods of Analysis

Methods of Analysis - Chapter 3

3.1 Motivation
3.2 Nodal analysis.
3.3 Nodal analysis with voltage sources.
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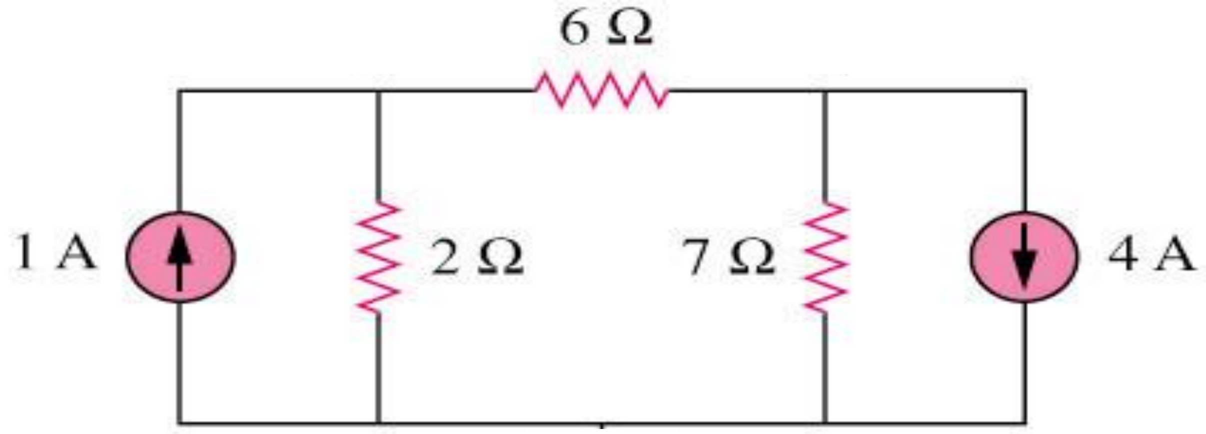
3.5 Mesh analysis with current sources.
3.6 Nodal and mesh analysis by inspection.
3.7 Nodal versus mesh analysis.

3.1 Motivation (1)

If you are given the following circuit, how can we determine (1) the voltage across each resistor, (2) current through each resistor. (3) power generated by each current source, etc.



What are the things which we need to know in order to determine the answers?

3.1 Motivation (2)

Things we need to know in solving any resistive circuit with current and voltage sources only:

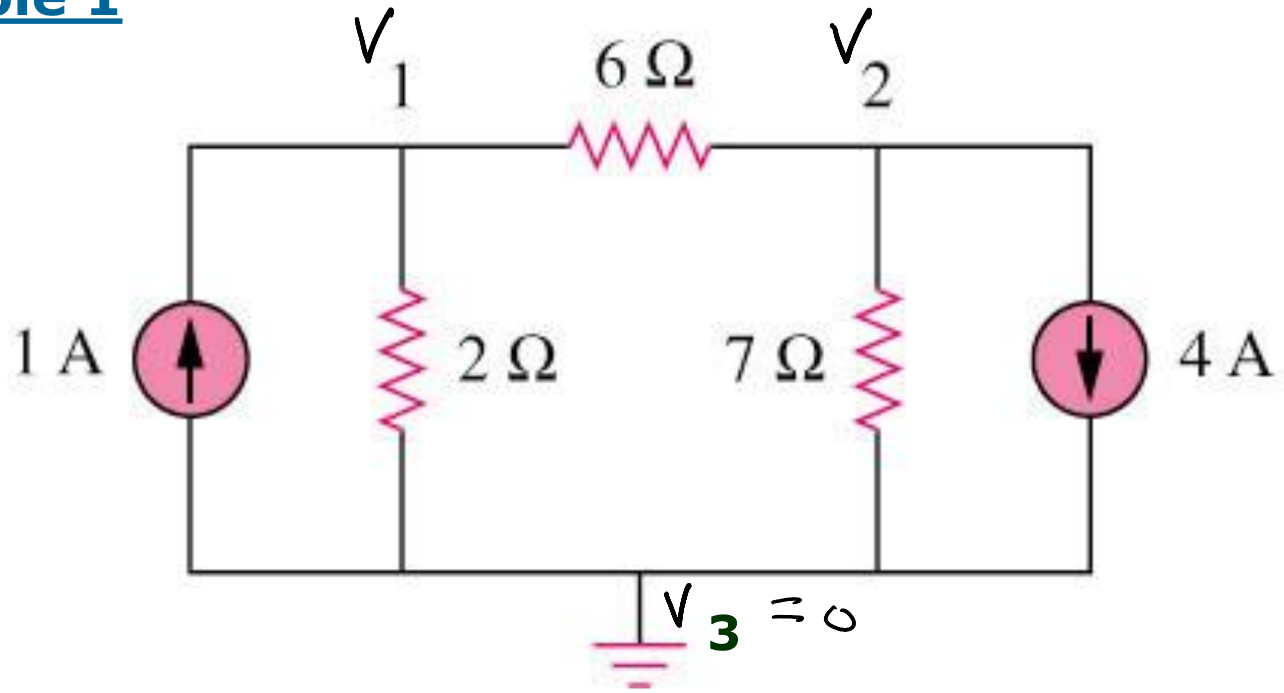
- Kirchhoff’s Current Laws (KCL)
- Kirchhoff’s Voltage Laws (KVL)
- Ohm’s Law

How should we apply these laws to determine the answers?

3.2 Nodal Analysis (1)

It provides a general procedure for analyzing circuits using node voltages as the circuit variables.

Example 1



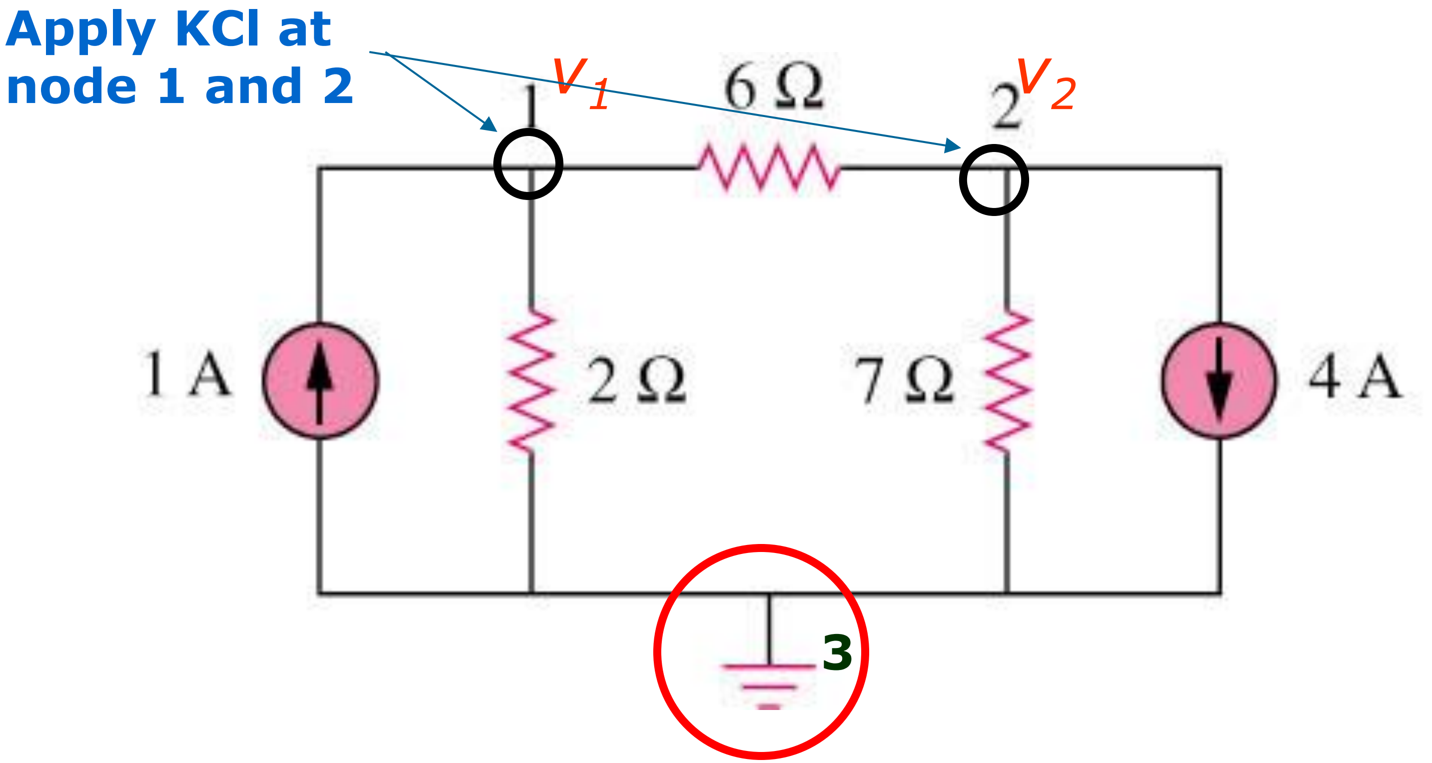
3.2 Nodal Analysis (2)

Steps to determine the node voltages:

1. Select a node as the reference node.
2. Assign voltages v1,v2,…,vn-1 to the remaining n-1 nodes. The voltages are referenced with respect to the reference node.
3. Apply KCL to each of the n-1 non-reference nodes. Use Ohm's law to express the branch currents in terms of node voltages.
4. Solve the resulting simultaneous equations to obtain the unknown node voltages.

3.2 Nodal Analysis (3)

<u>Example 2</u> – circuit independent current source only



*Refer to in-class illustration, textbook, answer v_1 = -2V, v_2 = -14V

Node Voltage Method

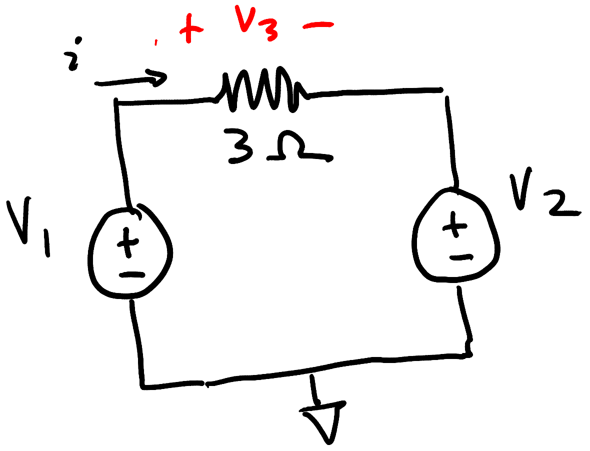
- Basic idea—what is current i ?

KVL

$$-V_1 + V_3 + V_2 = 0$$

$$V_3 = V_1 - V_2$$

$$i = \frac{V_3}{3} = \frac{V_1 - V_2}{3}$$



Node Voltage Method

- v1,v2 label unknown node voltages

KVL

$$-v_1 + v_3 + v_2 = 0$$

$$v_3 = v_1 - v_2$$

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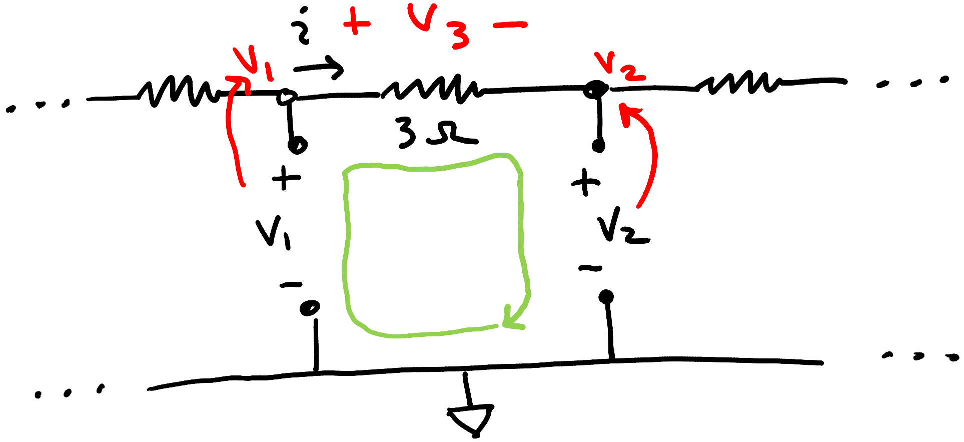
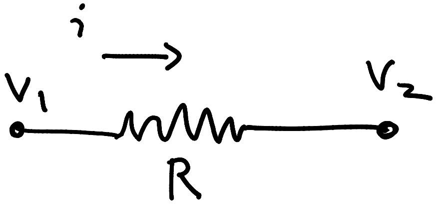


Same as last slide!

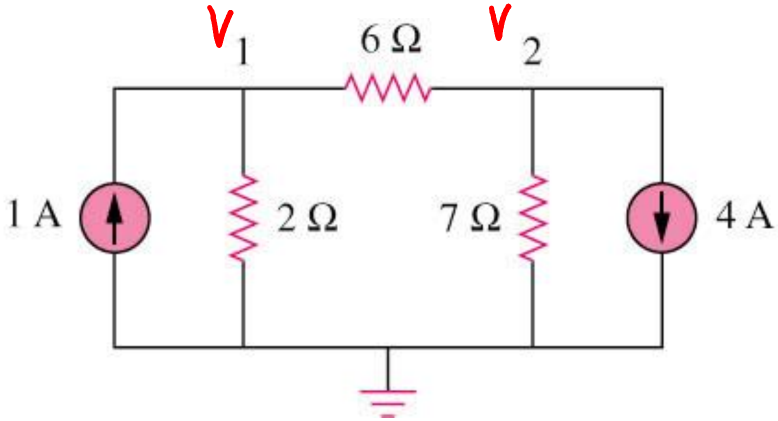
General Rule:

$$\boxed{i = \frac{v_1 - v_2}{R}}$$



Check: $v_1 = 10$, $v_2 = 3$, $R = 2$ → $i = \frac{10 - 3}{2} = 3.5\,A$

Write KCL at node 1 and 2 define currents as leaving nodes



Node 1 (V_1) KCL:

1) $-1 + \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{6} = 0$

Node 2 (V_2) KCL:

2) $4 + \frac{V_2 - 0}{7} + \frac{V_2 - V_1}{6} = 0$

$(1) * 6 \Rightarrow 3V_1 + V_1 - V_2 = 6 \Longrightarrow 4V_1 - V_2 = 6$

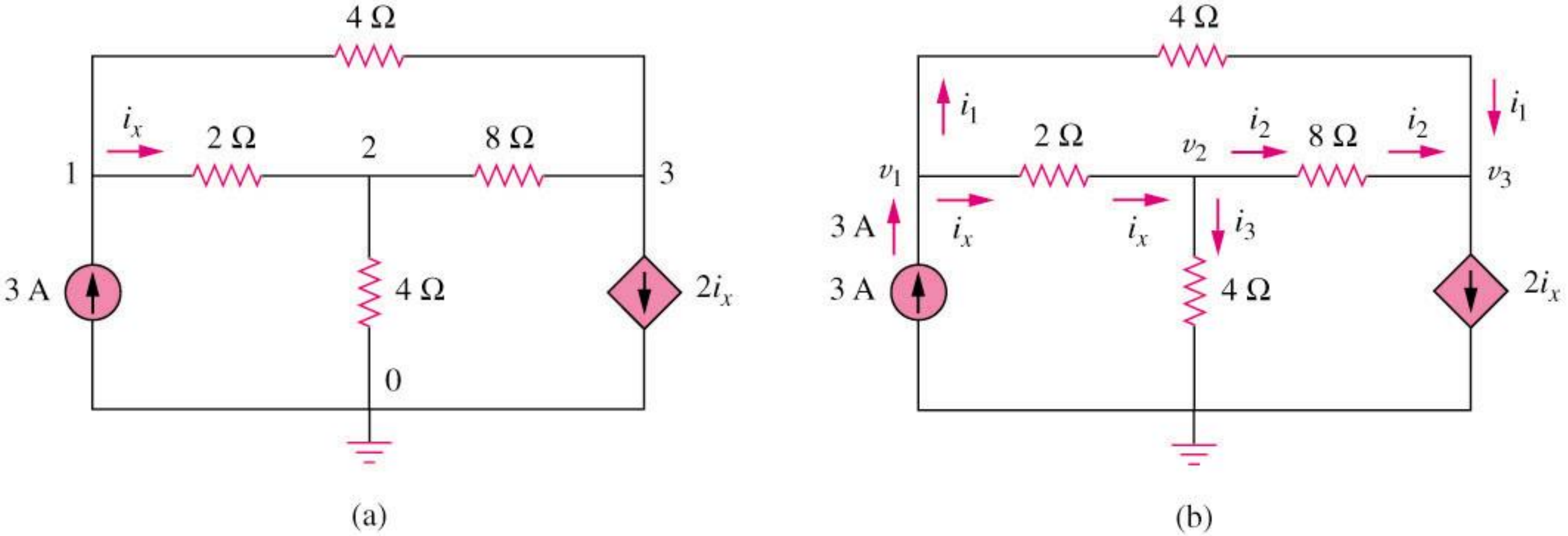
$(2) * 42 \Rightarrow -7V_1 + 7V_2 + 6V_2 = -168 \Longrightarrow -7V_1 + 13V_2 = -168$

$\left.\begin{matrix} 4V_1 - V_2 = 6 \\ -7V_1 + 13V_2 = -168 \end{matrix}\right\} \rightarrow$ Freemat

$v_1 = -2V, v_2 = -14V$

3.2 Nodal Analysis (4)

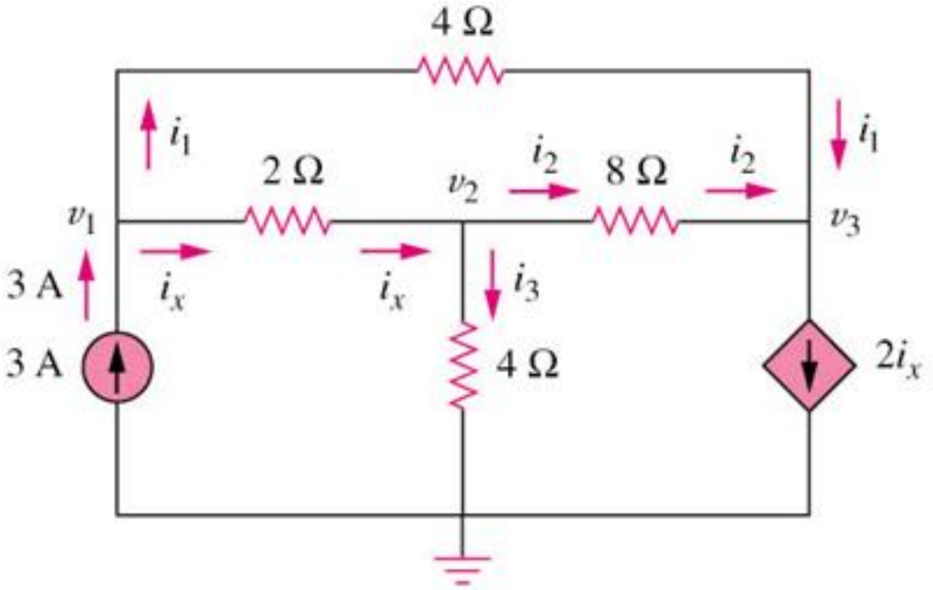
Example 3 – circuit with dependant current source



*Refer to in-class illustration, textbook, answer v_1= 4.8V, v_2 = 2.4V, v_3 = -2.4V

Node 1

$$-3 + \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = 0$$



Node 2

$$\frac{v_2}{4} + \frac{v_2 - v_1}{2} + \frac{v_2 - v_3}{8} = 0$$

Node 3

$$2i_x + \frac{v_3 - v_2}{8} + \frac{v_3 - v_1}{4} = 0$$

Constraint eq:

$$i_x = \frac{v_1 - v_2}{2}$$

$$\left.\begin{aligned} &(1) * 4 \Rightarrow 2v_1 + v_1 - 2v_2 - v_3 = 12 \\ &(2) * 8 \Rightarrow 2v_2 + 4v_2 - 4v_1 + v_2 - v_3 = 0 \\ &(3) * 8 \Rightarrow 16i_x + v_3 - v_2 + 2v_3 - 2v_1 = 0 \end{aligned}\right\} \rightarrow \left\{\begin{aligned} 3v_1 - 2v_2 - v_3 &= 0 \\ -4v_1 + 7v_2 - v_3 &= 0 \\ 6v_1 - 9v_2 + 3v_3 &= 0 \end{aligned}\right.$$

$v_1 = 4.8\text{V}$, $v_2 = 2.4\text{V}$, $v_3 = -2.4\text{V}$

Freemat

$$A = \begin{bmatrix} 3 & -2 & 1 \\ -4 & 7 & -1 \\ 6 & -9 & 3 \end{bmatrix}$$

$$b = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

$$V = A\backslash b \Rightarrow \begin{bmatrix} 4.8 \\ 2.4 \\ -2.4 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Node Voltage Method with Voltage Source attached to Ground

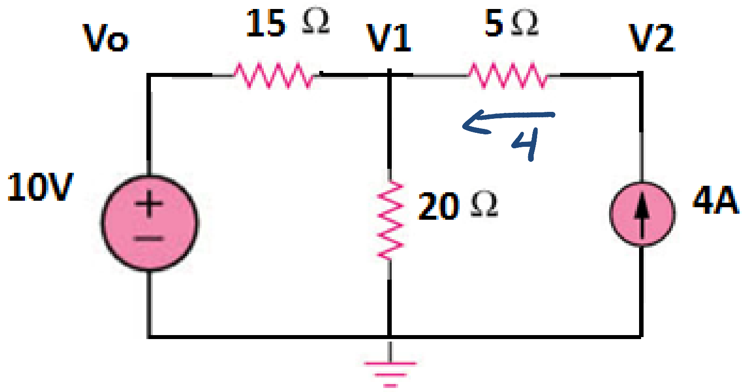
When a voltage source is connected from a node to ground, set the voltage at that node to the value of voltage source

KCL at V_o?

not needed!

$V_o \equiv 10$

(given)



ALSO: You can solve the circuit with fewer equations by focusing only on "essential nodes" – nodes with 3 or more branches

KCL at V_1

$$\frac{V_1 - \cancelto{10}{V_0}}{15} + \frac{V_1}{20} + \cancelto{-4}{\frac{V_1 - V_2}{5}} = 0 \qquad (1)$$

KCL at V_2 (not needed either)

$$-4 + \frac{V_2 - V_1}{5} = 0 \,, \quad \frac{V_2 - V_1}{5} = 4$$

(we could have seen that on cct)

Solving

$$\frac{V_1 - 10}{15} + \frac{V_1}{20} = 4 \overset{(\times 60)}{\Rightarrow} 4V_1 - 40 + 3V_1 = 240$$

$$7V_1 = 280 \,, \quad \underline{\underline{V_1 = 40}} \,, \quad \underline{\underline{V_2 = 60}}$$

Remember Matlab/Freemat

- To solve systems of equations such as

9v1 – v2 = 36

-3v1 + 11v2 = 180

use the following

```
A = [ 9  -1 ; -3 11];
b = [36; 180]
A\b
ans =
  6
 18
```

Your turn...

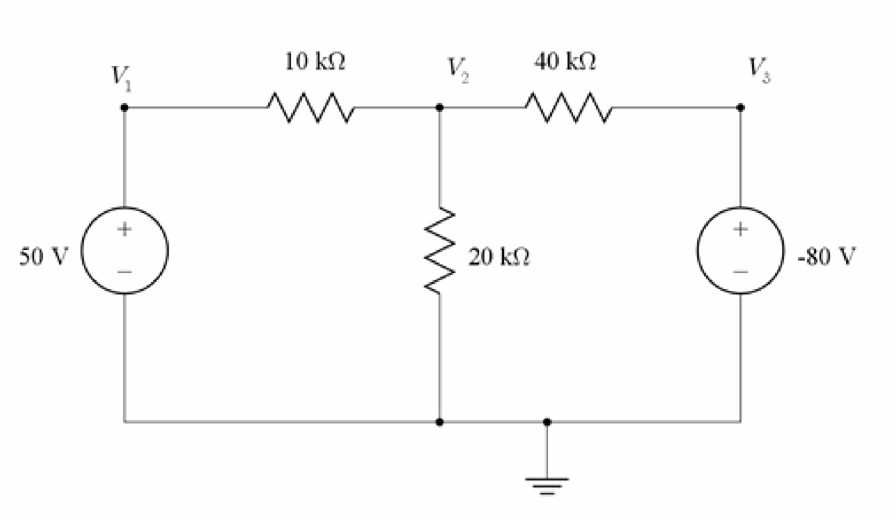
1) Cleo:Problem 1 Write 1 NV eq at V2 and solve

KCL at V_2:

$$\frac{V_2 - V_1}{10} + \frac{V_2}{20} + \frac{V_2 - V_3}{40} = 0$$

$$\frac{V_2 - 50}{10} + \frac{V_2}{20} + \frac{V_2 + 80}{40} = 0$$

$$\underline{\underline{V_2 = 17.1 \text{ V}}}$$

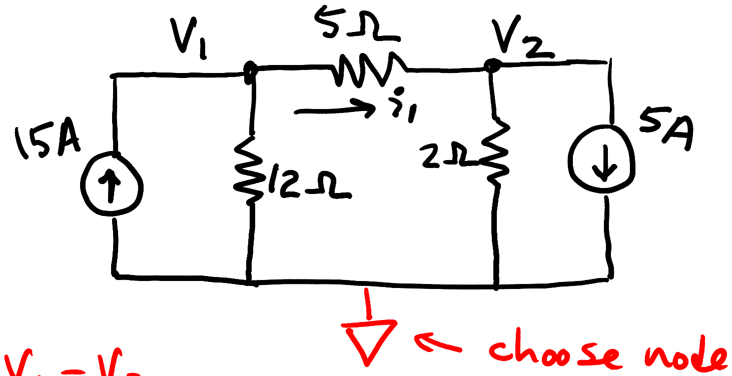


2) Use NV to find v1, v2 and i1 (60,10,10)

b) how much power delivered by 15A src? (-900W, del)

c) how about 5A source? (50W absorb)

Current leaving node is +



$i_1 = \frac{V_1 - V_2}{5}$

Node 1

$$-15 + \frac{V_1 - V_2}{5} + \frac{V_1}{12} = 0$$

Node 2

$$\frac{V_2}{2} + \frac{V_2 - V_1}{5} + 5 = 0$$

$$(1) \times 60 \Rightarrow 12V_1 - 12V_2 + 5V_1 = 900$$

$$(2) \times 10 \Rightarrow 5V_2 + 2V_2 - 2V_1 = -50$$

$$\rightarrow \begin{cases} 17V_1 - 12V_2 = 900 \\ -2V_1 + 7V_2 = -50 \end{cases}$$

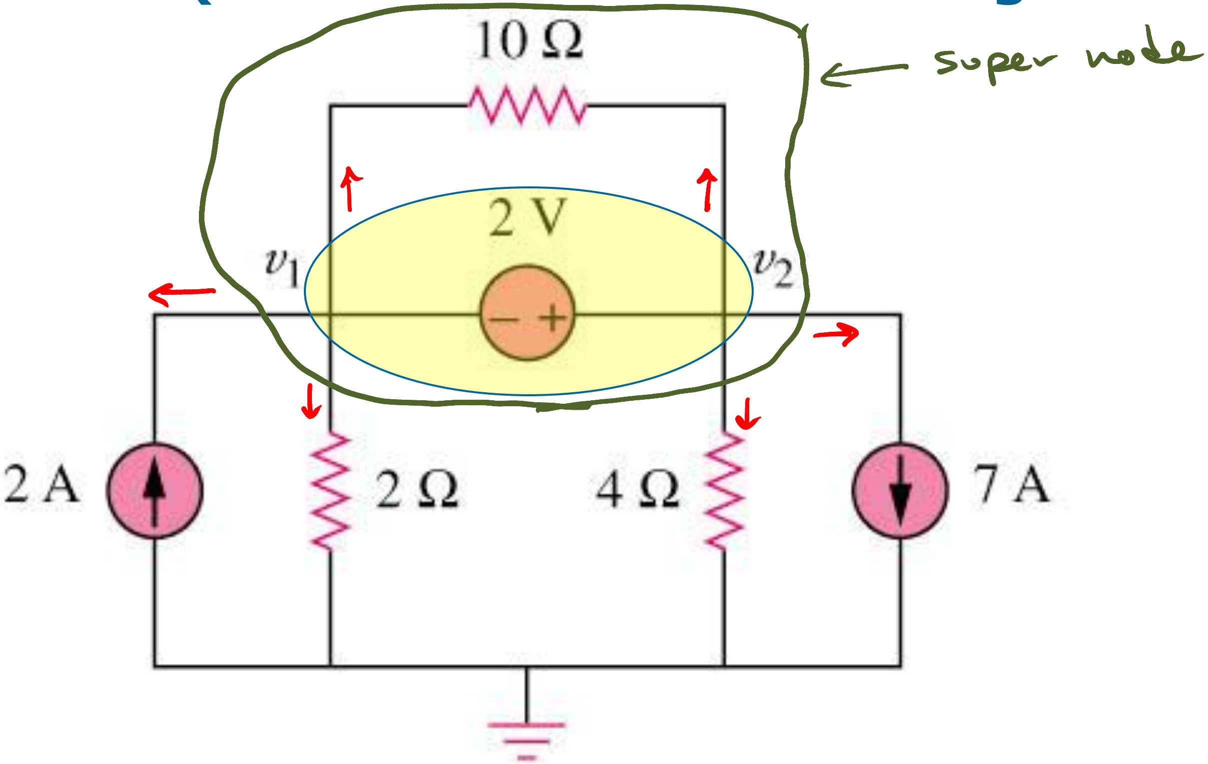
$$\underline{V_1 = 60V,\ V_2 = 10V,\ i_1 = 10A}$$

$P_{15A} = -60(15) = \underline{-900\,W, \text{ del}}$

$P_{5A} = 10(5) = \underline{50\,W, \text{ absorb}}$

3.3 Nodal Analysis with Floating Voltage Source (1)

Example 4 –circuit with voltage source between 2 reference nodes (2 nodes not connected to ground)



How to handle the 2V voltage source?

3.3 Nodal Analysis with Floating Voltage Source (2)

A super-node is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.

*Note: We analyze a circuit with super-nodes using the same three steps mentioned above except that the super-nodes are treated differently.

3.3 Nodal Analysis with Floating Voltage Source (3)

Basic steps:

1. Take off all voltage sources in super-nodes and apply KCL to super-nodes.

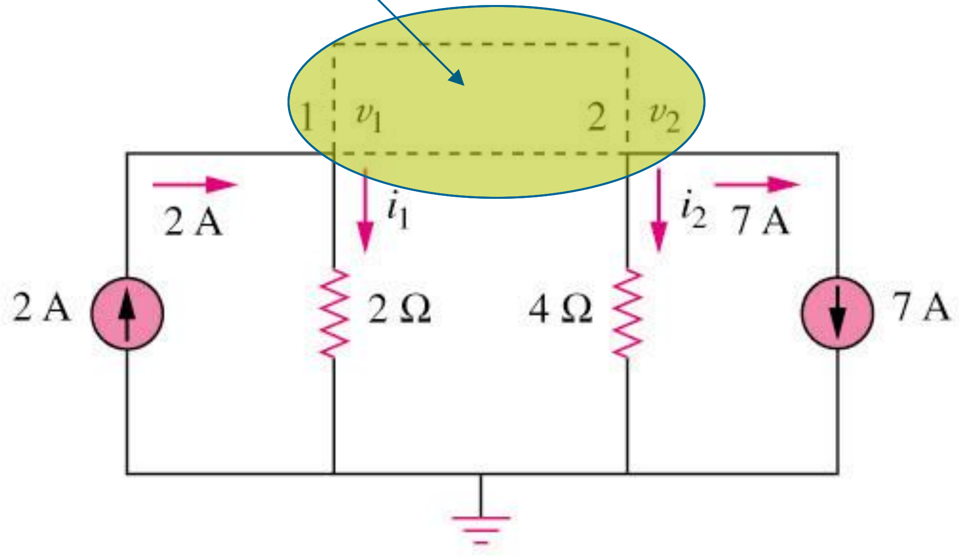
2. Put voltage sources back to the nodes and apply KVL to relative loops.

↑ adds a constraint to V_1 + V_2

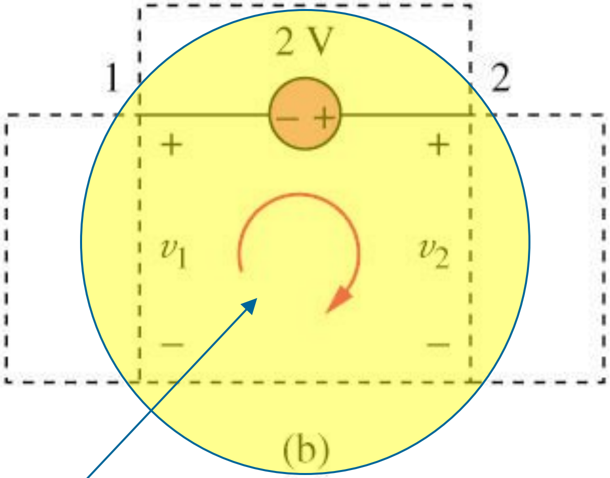
3.3 Nodal Analysis with Floating Voltage Source (4)

Example 5 – circuit with independent voltage source

Super-node => $-2+i_1+i_2+7 = 0$, oR $\frac{V_1}{2}+\frac{V_2}{2}-2+7=0$ (1)



(a)



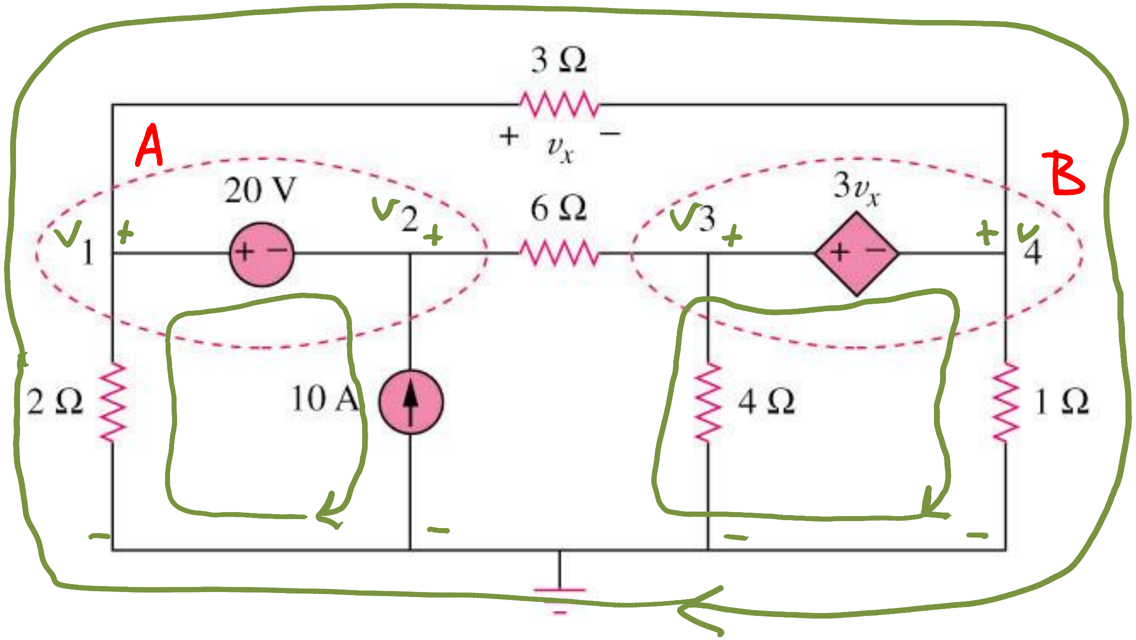
Apply KVL => $-v_1-2+v_2 = 0$

Constraint Eq: $V_2 = V_1 + 2$ (2)

then solve (1) & (2) ⇐

3.3 Nodal Analysis with Floating Voltage Source (5)

Example 6 – circuit with two independent voltage sources



Super Node A (KCL)

$$\frac{V_1}{2} - 10 + \frac{V_1 - V_4}{3} + \frac{V_2 - V_3}{6} = 0$$

Super Node B (KCL)

$$\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} + \frac{V_4 - V_1}{3} = 0$$

Constraint A (KVL)

$-V_1 + 20 + V_2 = 0$, $V_1 = V_2 + 20$

Constraint B (KVL)

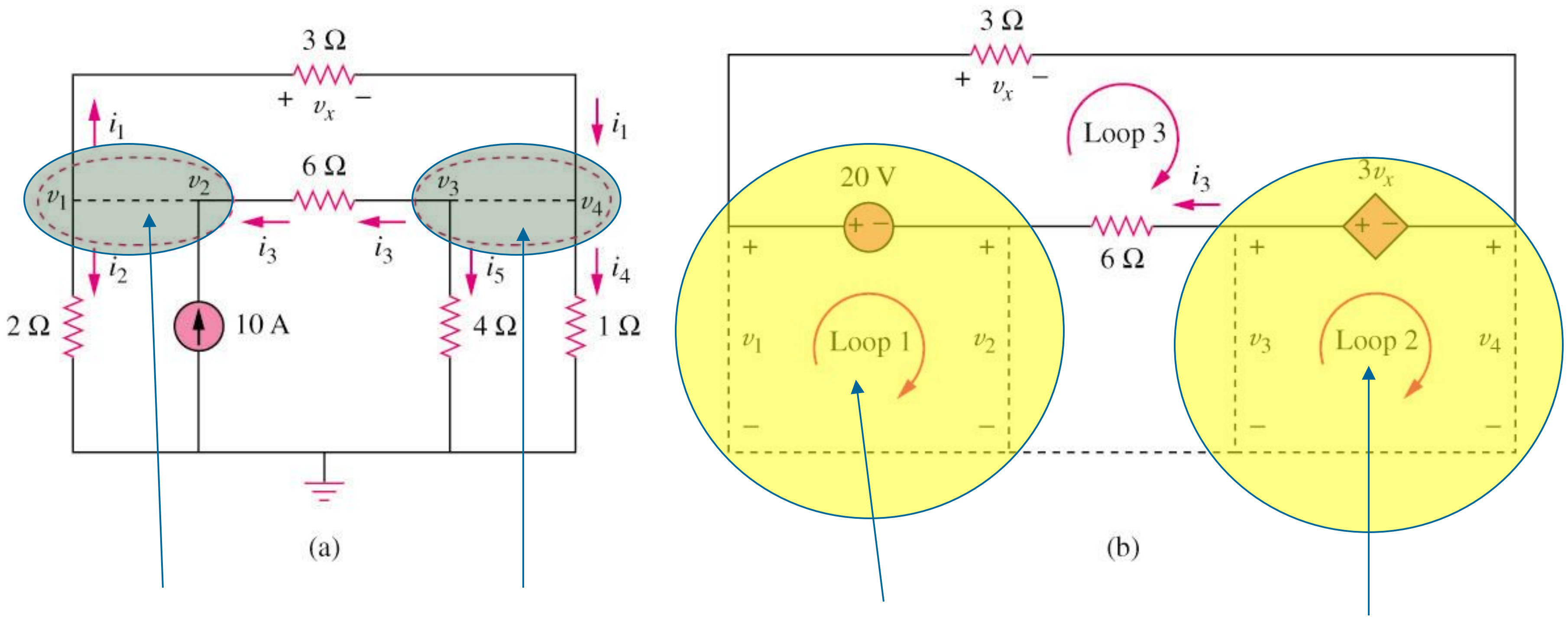
$-V_3 + 3V_x + V_4 = 0$, $V_3 = V_4 + 3V_x$

Constraint Vx (KVL)

$-V_1 + V_x + V_4 = 0$, $V_x = V_1 - V_4$

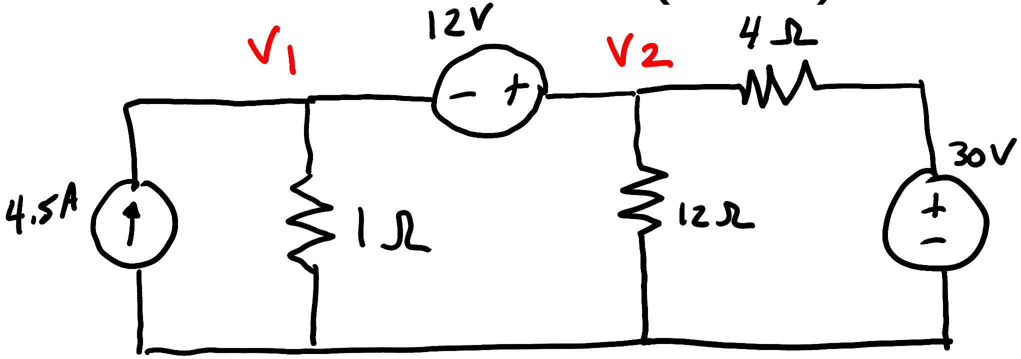
3.3 Nodal Analysis with Floating Voltage Source (6)

Example 7 – circuit with two independent voltage sources

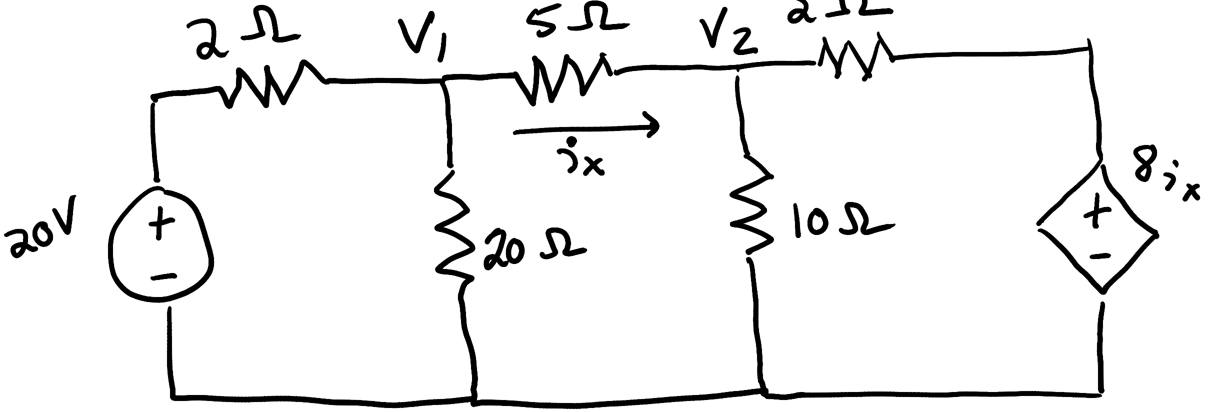


$-i_1-i_2+i_3=0$ $-i_3-i_5-i_4+i_1=0$ $v_1-20-v_2=0$ $v_3-3v_x-v_4=0$

3) Write a Supernode NV to find V1, V2(6,18)

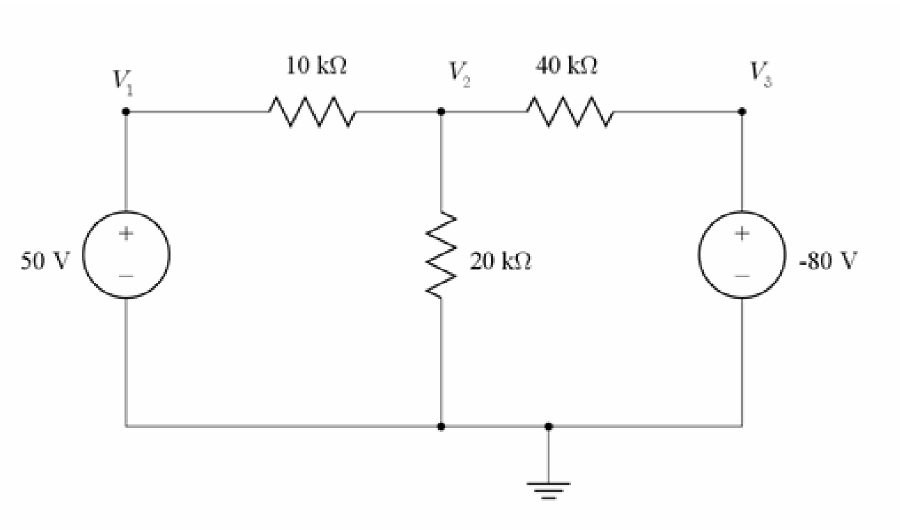


4) Find v1, v2, ix (16,10,6/5)



HANDOUTS

1) Cleo:Problem 1 Write 1 NV eq at V2 and solve



2) Use NV to find v1, v2 and i1 (60,10,10)

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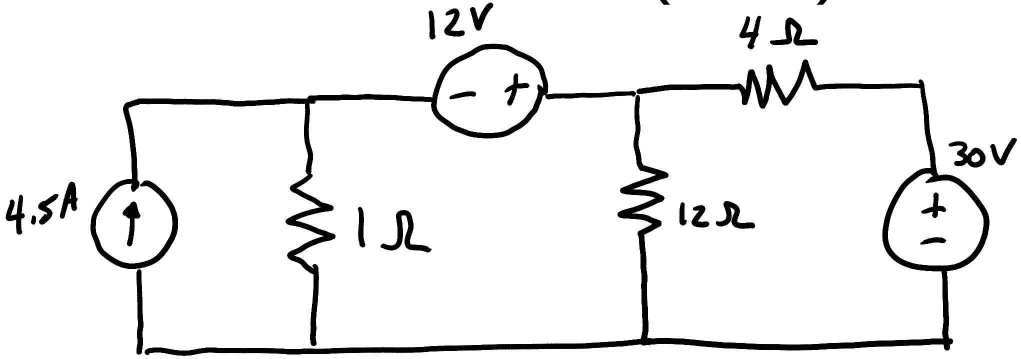
3) Write a Supernode NV to find V1, V2(6,18)



4) Find v1, v2, ix (16,10,6/5)