Circuit Theory

Chapter 3
Methods of Analysis

Methods of Analysis - Chapter 3

3.4 Mesh analysis.
3.5 Mesh analysis with current sources.
3.6 Nodal and mesh analysis by inspection.
3.7 Nodal versus mesh analysis.

3.4 Mesh Analysis (1)

1. Mesh analysis provides another general procedure for analyzing circuits using mesh currents as the circuit variables.

2. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

3. A mesh is a loop which does not contain any other loops within it.

3.4 Mesh Analysis (2)

Steps to determine the mesh currents:

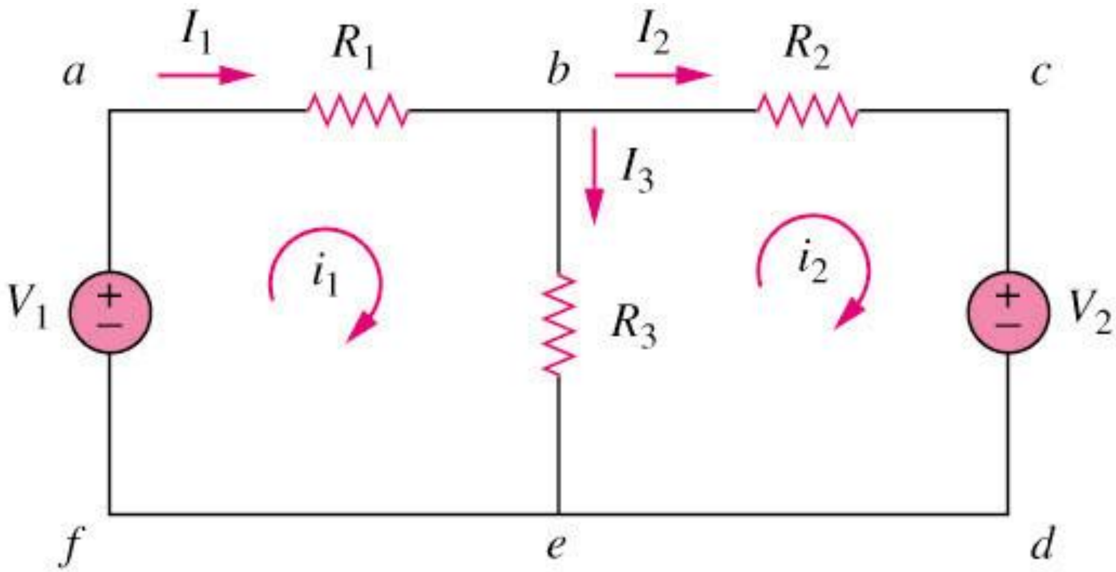
1. Assign mesh currents i_1, i_2, …, in to the n meshes.

2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

3. Solve the resulting n simultaneous equations to get the mesh currents.

3.4 Mesh Analysis (3)

Example 8 – circuit with independent voltage sources



Note:

i_1 and i_2 are mesh current (imaginative, not measurable directly)

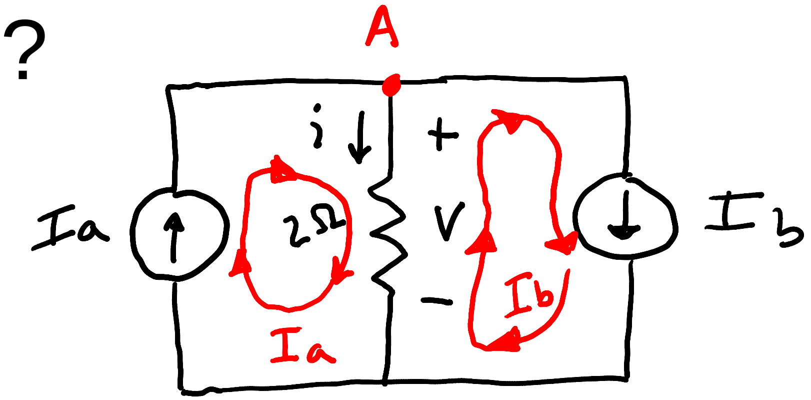
I_1, I_2 and I_3 are branch current (real, measurable directly)

$I_1 = i_1$; $I_2 = i_2$; $I_3 = i_1 - i_2$

*Refer to in-class illustration, textbook

Mesh Current Method

- What is branch current i?



KCL at Node A

$I_a - I_b - i = 0$

$i = I_a - I_b$

- voltage V?

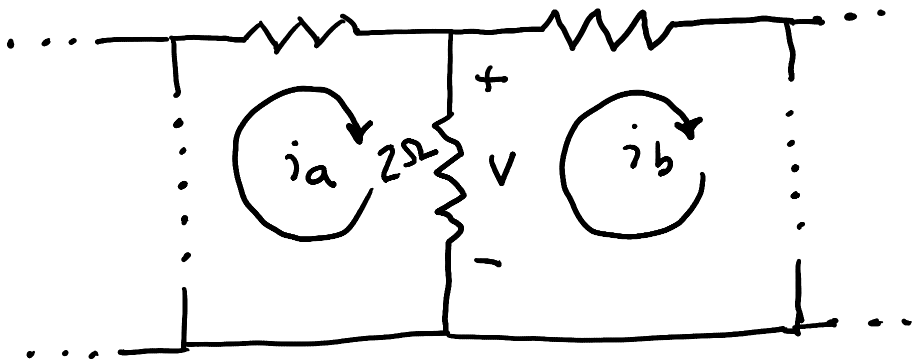
$V = 2i = 2(I_a - I_b)$

Mesh Current Method

- ia,ib label unknown mesh currents

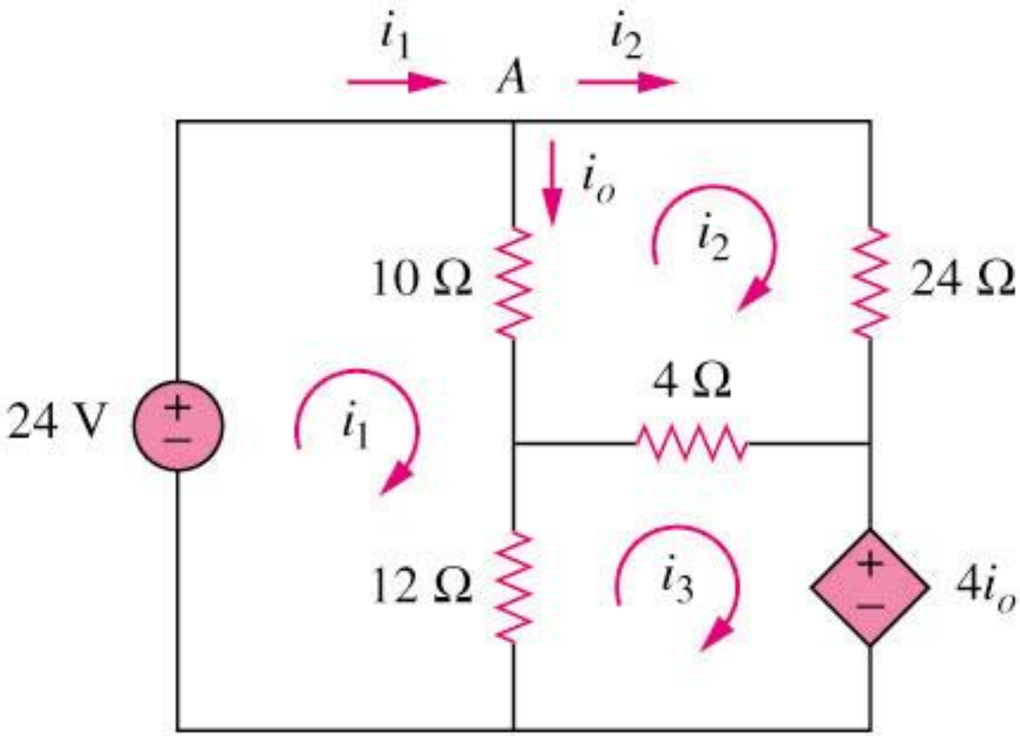
$v = 2(i_a - i_b)$

same as last slide!



3.4 Mesh Analysis (4)

<u>Example 9</u> – circuit with dependent voltage source



*Refer to in-class illustration, textbook, answer I_o = 1.5A

Mesh 1

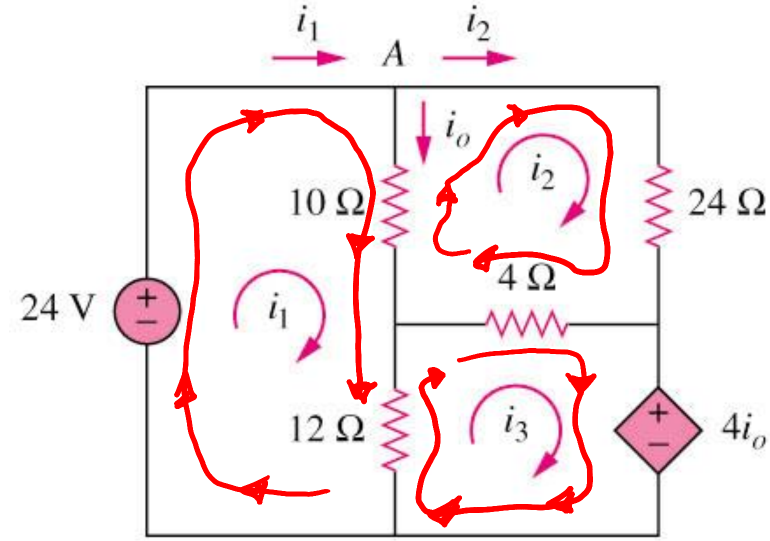
$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

Mesh 2

$$10(i_2 - i_1) + 24(i_2) + 4(i_2 - i_3) = 0$$

Mesh 3

$$4(i_3 - i_2) + 4i_o + 12(i_3 - i_1) = 0$$



Constraint Eq

$$i_o = i_1 - i_2$$

Simplifying

1) $22i_1 - 10i_2 - 12i_3 = 24$

2) $-10i_1 + 38i_2 - 4i_3 = 0$

3) $-12i_1 - 4i_2 + 16i_3 + 4i_o = 0$ $\quad (i_o \rightarrow (i_1 - i_2))$

$-8i_1 - 8i_2 + 16i_3 = 0$

Freemat $\rightarrow$

$i_1 = 2.25$ A

$i_2 = .75$ A

$i_3 = 1.5$ A

$i_o = i_1 - i_2 = 1.5$ A

$I_o = 1.5$A

1) Your Turn: Use mesh analysis to determine Ia and Ib

Ia = 639 mA; Ib = -48 mA

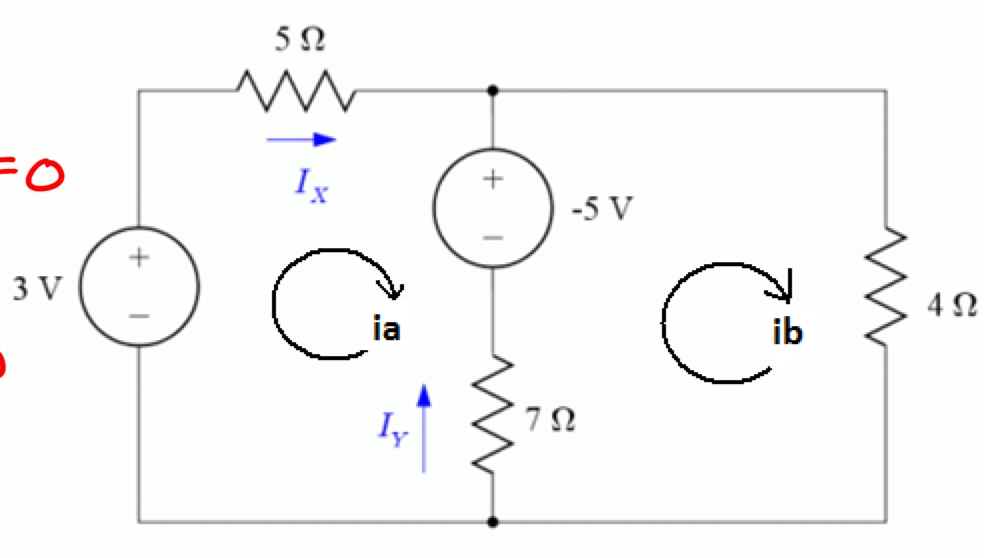
Then find BRANCH currents Ix, Iy (639,-687)

Mesh A

$-3 + 5i_a + (-5) + 7(i_a - i_b) = 0$

Mesh B

$7(i_b - i_a) - (-5) + 4i_b = 0$



$I_x = i_a$

$I_y = i_b - i_a$

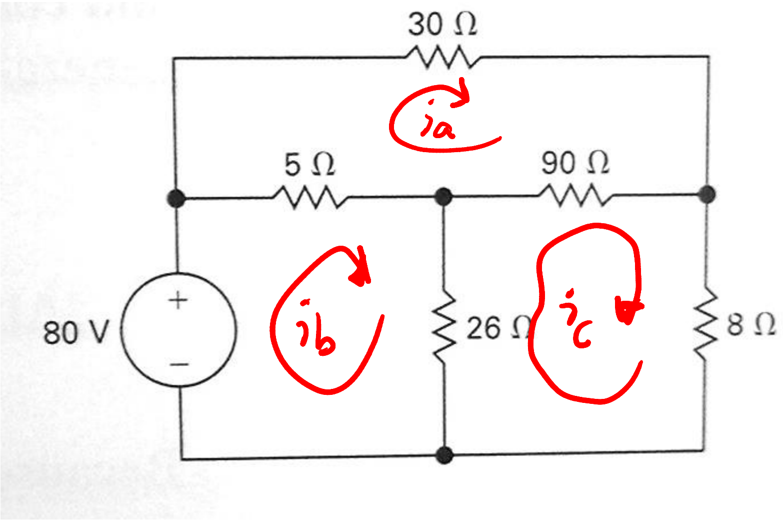
2) Find a) three mesh currents, [5,2.5,2]

b) Power of 80V source (-400W del)

$$-80 + 5(i_b - i_a) + 26(i_b - i_c) = 0$$

$$30 i_a + 90(i_a - i_c) + 5(i_a - i_b) = 0$$

$$26(i_c - i_b) + 90(i_c - i_a) + 8 i_c = 0$$

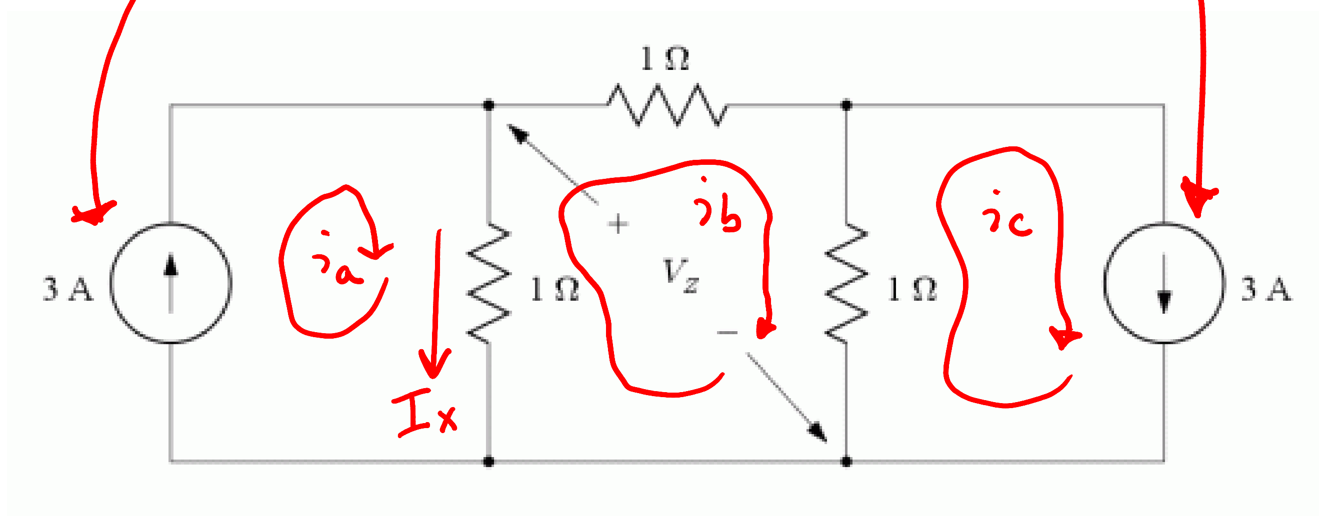


MC Special Cases

- Dependent sources
- Current Source in single mesh
- Current Source between two meshes (Supermesh)

Current Source in single mesh – eliminates unknown

Problem 2 Use MC to find Vz. [Vz = 1V]



$i_a \equiv 3A$

$i_c \equiv 3A$

only do Mesh B:

$$1(i_b - i_a) + 1\,i_b + 1(i_b - i_c) = 0$$

$$i_b - 3 + i_b + i_b - 3 = 0$$

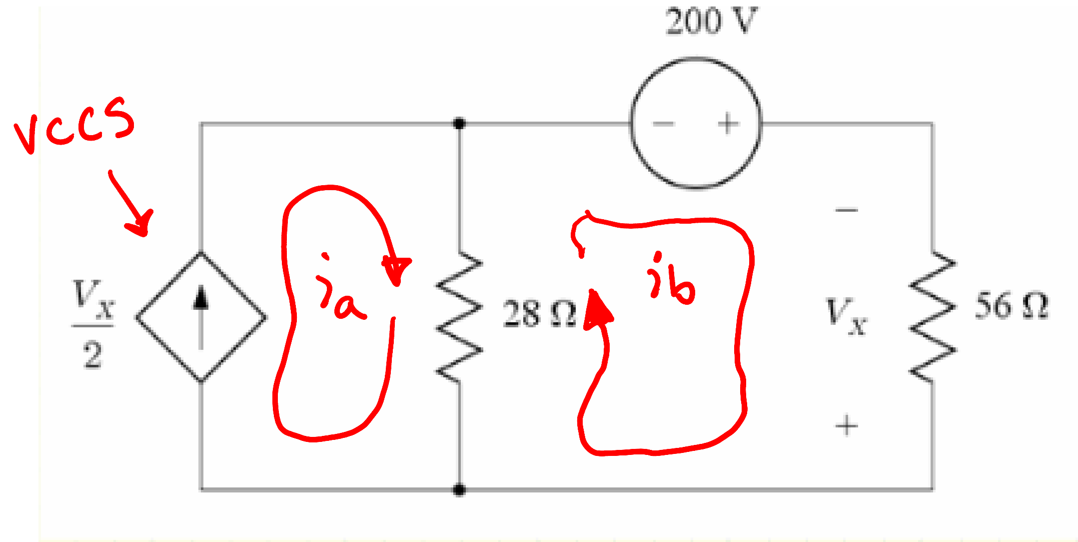
$$3 i_b = 6$$

$$i_b = 2$$

$$I_x = i_a - i_b = 3 - 2 = 1\ A$$

$$V_z = 1(1) = 1V$$

3) Your turn: Use MC to find Vx. [Vx = -12.9 V]



Mesh A

no KVL needed

$$i_a \equiv \frac{V_x}{2}$$

Mesh B

$$28(i_b - i_a) - 200 - V_x = 0$$

Constraint Eq

$$V_x = -56\, i_b, \text{ or } i_b = \frac{-V_x}{56}$$

$$28\left(\frac{-V_x}{56} - \frac{V_x}{2}\right) - 200 - V_x = 0$$

$$-\frac{V_x}{2} - 14V_x - 200 - V_x = 0$$

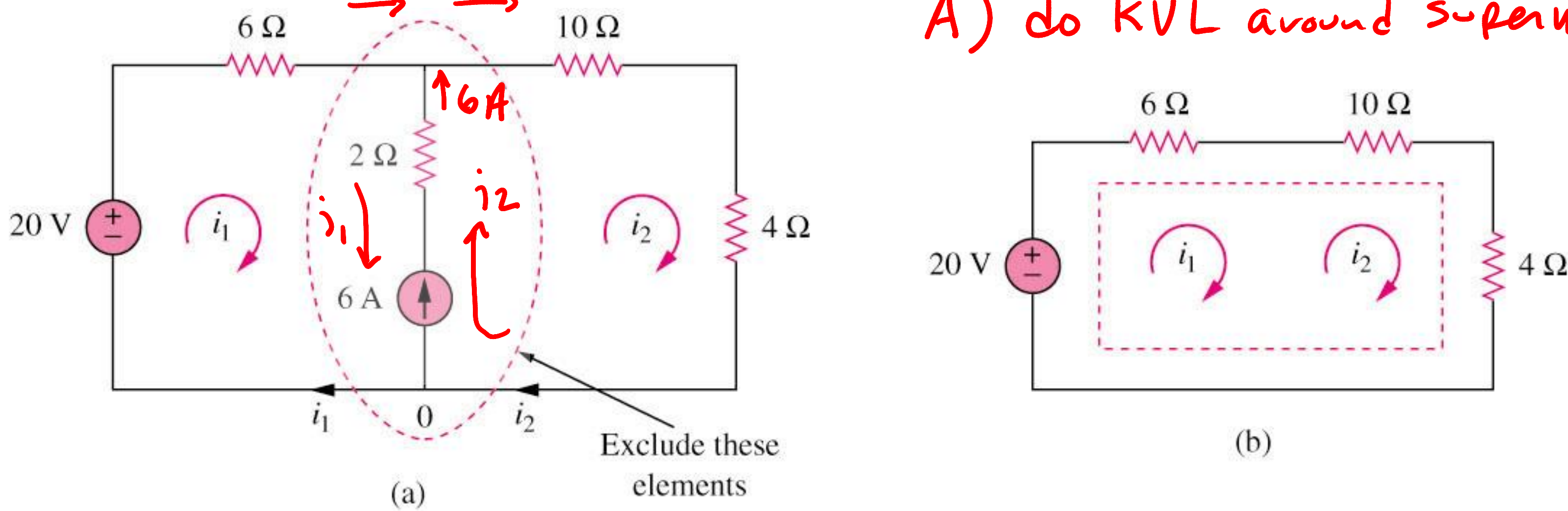
$$-15.5\, V_x = -200, \quad V_x = \frac{-200}{15.5} = \underline{-12.9\ V}$$

3.5 Current Source between 2 Meshes (1)

B) write constraint eq:
$6A = i_2 - i_1$ by inspection
or KVL: $6 + i_1 = i_2$

Circuit with current source between 2 meshes

A) do KVL around supermesh:



A **super-mesh** results when two meshes have a (dependent or independent) current source in common as shown in (a). We create a super-mesh by excluding the current source and any elements connected in series with it as shown in (b).

*Refer to in-class illustration, textbook

3.5 Current Source between 2 Meshes (2)

The properties of a super-mesh:

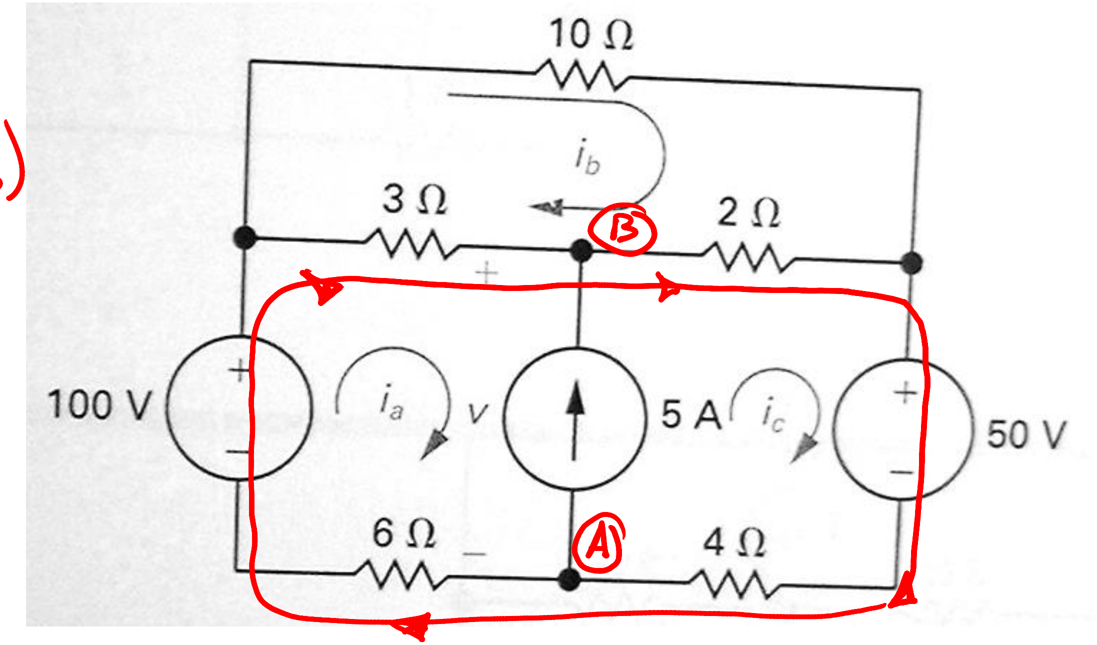
1. The current source in the super-mesh is not completely ignored; it provides the constraint equation necessary to solve for the mesh currents.

2. A super-mesh has no current of its own.

3. A super-mesh requires the application of both KVL and KCL.

Current Source In Two Meshes

Find ia, ib, ic [1.75,1.25,6.75]



Supermesh Eq:

$$-100 + 3(i_a - i_b) + 2(i_c - i_b) + 50 + 4i_c + 6i_a = 0$$

Constraint Eq:

$$5 = i_c - i_a \quad ✓$$

or, KCL at node A

$$i_c - 5 - i_a = 0 \quad ✓$$

or KCL at node B

$$i_a - i_b + 5 - (i_c - i_b) = 0 \quad ✓$$

all same eq

Mesh B

$$10 i_b + 2(i_b - i_c) + 3(i_b - i_a) = 0$$

simplifying

$$9i_a - 5i_b + 6i_c = 50$$

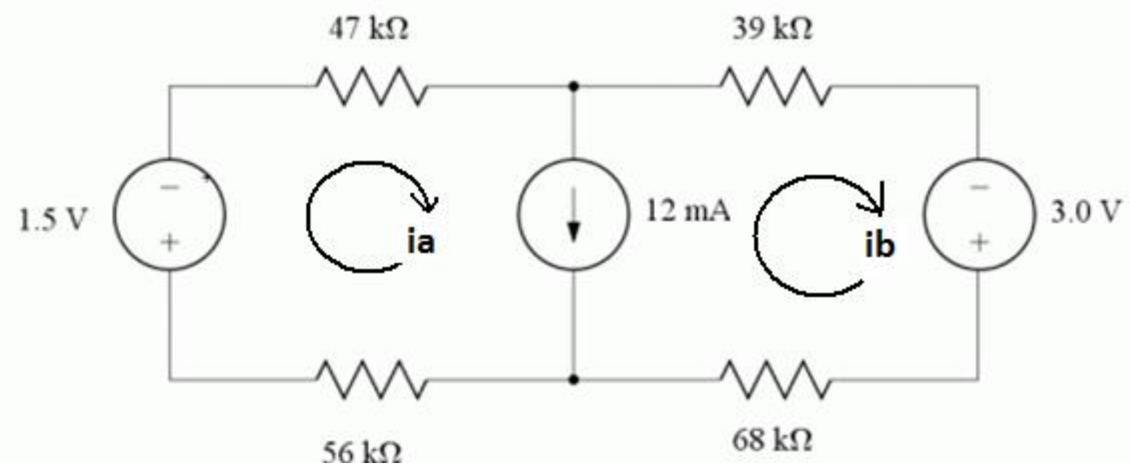
$$-i_a + 0i_b + i_c = 5$$

$$-3i_a + 15i_b - 2i_c = 0$$

+ use Freemat to solve

4) Your turn: Use SUPERMESH to find the ia, ib and power associated with each voltage source.

Ia = 6.12 mA. Ib = -5.88 mA,   P1.5 = 9.18 mW; P3.0 = 17.6 mW



Supermesh

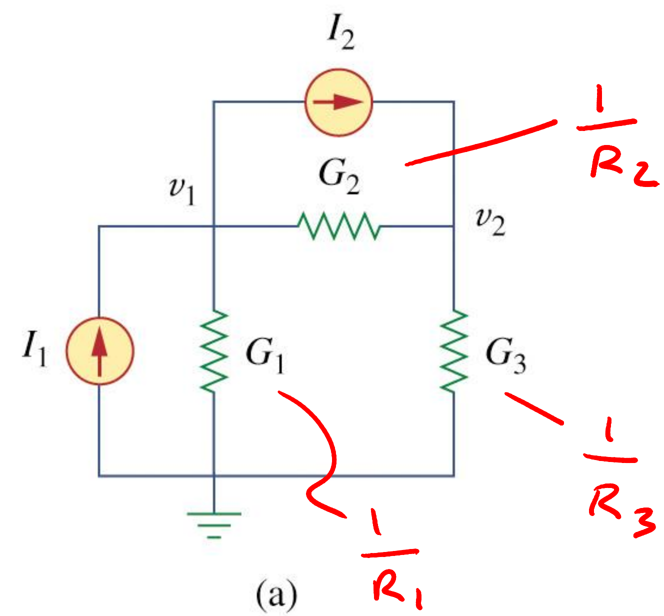
$1.5 + 47 i_a + 39 i_b - 3 + 68 i_b + 56 i_a = 0$

Constraint

$i_a - i_b = 12 \text{ mA}$

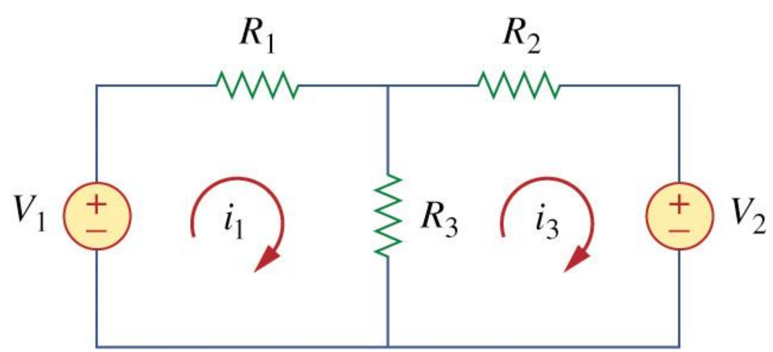
3.6 Nodal and Mesh Analysis by Inspection

The analysis equations can be obtained by direct inspection



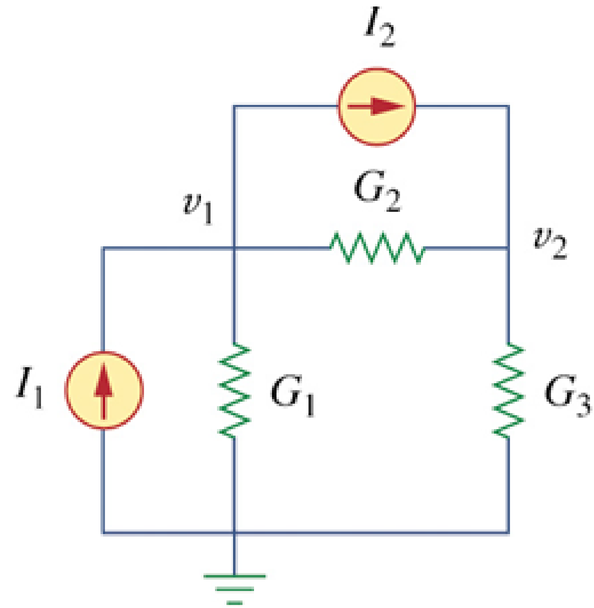
(a)

(a)For circuits with only resistors and independent current sources
(b)For planar circuits with only resistors and independent voltage sources



(b)

- the circuit has two nonreference nodes and the node equations were derived as:



$$I_1 = I_2 + G_1 v_1 + G_2 (v_1 - v_2) \quad (3.7)$$

$$I_2 + G_2 (v_1 - v_2) = G_3 v_2 \quad (3.8)$$

$\rightarrow MATRIX$

Σ G's attached to node 1

$$\begin{bmatrix} G_1 + G_2 & -G_2 \\ -G_2 & G_2 + G_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 - I_2 \\ I_2 \end{bmatrix}$$

Σ G's attached to node 2

G from $n_1 \leftrightarrow n_2$

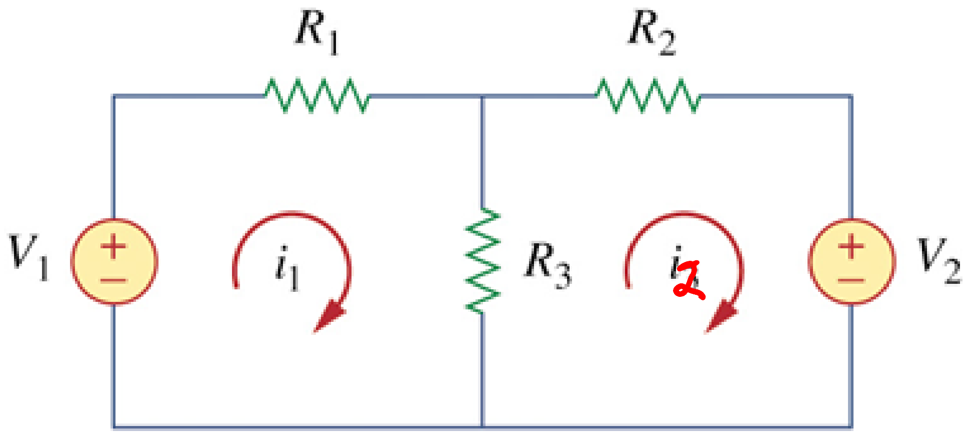
- In general, the node voltage equations in terms of the conductances is

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1N} \\ G_{21} & G_{22} & \cdots & G_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ G_{N1} & G_{N2} & \cdots & G_{NN} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

or simply

Gv = i

where *G* : the conductance matrix,
v : the output vector, *i* : the input vector



- The circuit has two meshes and the mesh equations were derived as

$$\begin{bmatrix} R_1 + R_3 & -R_3 \\ -R_3 & R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ -v_2 \end{bmatrix}$$

Σ R's in mesh 1

Σ R's in mesh 2

R in both meshes

- In general, if the circuit has N meshes, the mesh-current equations as the resistances term is

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1N} \\ R_{21} & R_{22} & \cdots & R_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ R_{N1} & R_{N2} & \cdots & R_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}$$

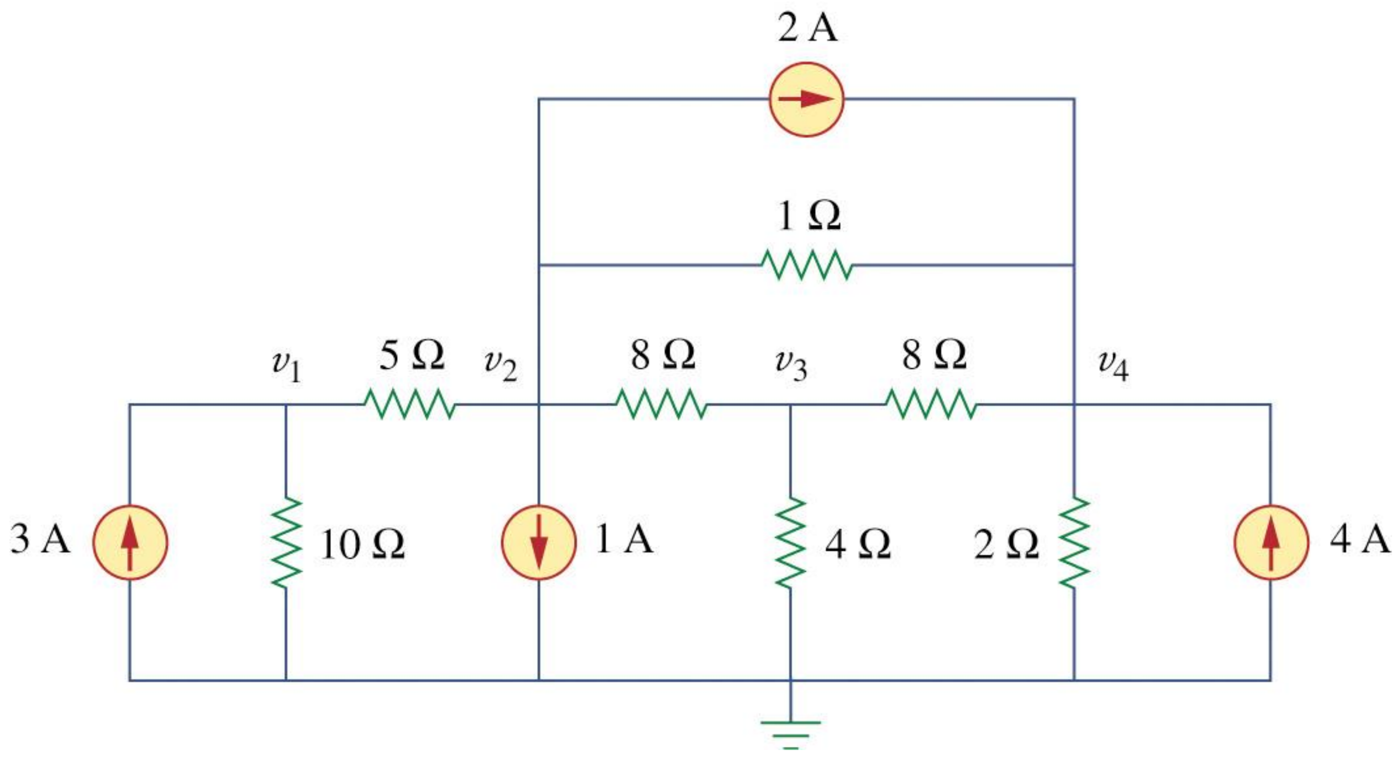
or simply

Rv = i

where *R* : the resistance matrix,

i : the output vector, *v* : the input vector

- Write the node voltage matrix equations



- The circuit has 4 nonreference nodes, so

$$G_{11} = \frac{1}{5} + \frac{1}{10} = 0.3, \quad G_{22} = \frac{1}{5} + \frac{1}{8} + \frac{1}{1} = 1.325$$

$$G_{33} = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = 0.5, \quad G_{44} = \frac{1}{8} + \frac{1}{2} + \frac{1}{1} = 1.625$$

- The off-diagonal terms are

$$G_{12} = -\frac{1}{5} = -0.2, \quad G_{13} = G_{14} = 0$$

$$G_{21} = -0.2, \; G_{23} = -\frac{1}{8} = -0.125, \quad G_{24} = -\frac{1}{1} = -1$$

$$G_{31} = 0, \; G_{32} = -0.125, \; G_{34} = -0.125$$

$$G_{41} = 0, \; G_{42} = -1, \; G_{43} = -0.125$$

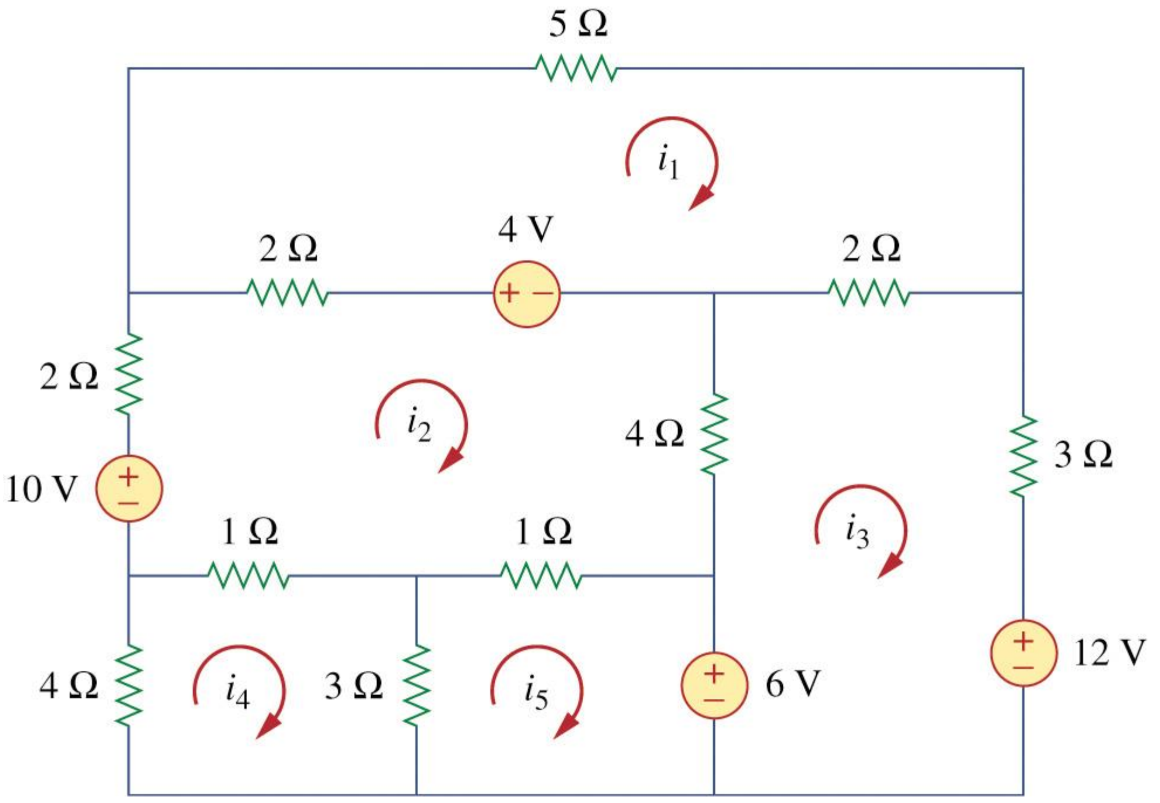
- The input current vector *i* in amperes

$$i_1 = 3, \quad i_2 = -1 - 2 = -3, \quad i_3 = 0, \quad i_4 = 2 + 4 = 6$$

- The node-voltage equations are

$$\begin{bmatrix} 0.3 & -0.2 & 0 & 0 \\ -0.2 & 1.325 & -0.125 & -1 \\ 0 & -0.125 & 0.5 & -0.125 \\ 0 & -1 & -0.125 & 1.625 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \\ 6 \end{bmatrix}$$

- Write the mesh current equations



- The input voltage vector *v* in volts

$$v_1 = 4,\ v_2 = 10 - 4 = 6,$$

$$v_3 = -12 + 6 = -6,\ v_4 = 0,\ v_5 = -6$$

- The mesh-current equations are

$$\begin{bmatrix} 9 & -2 & -2 & 0 & 0 \\ -2 & 10 & -4 & -1 & -1 \\ -2 & -4 & 9 & 0 & 0 \\ 0 & -1 & 0 & 8 & -3 \\ 0 & -1 & 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ -6 \\ 0 \\ -6 \end{bmatrix}$$

3.7 Nodal versus Mesh Analysis (1)

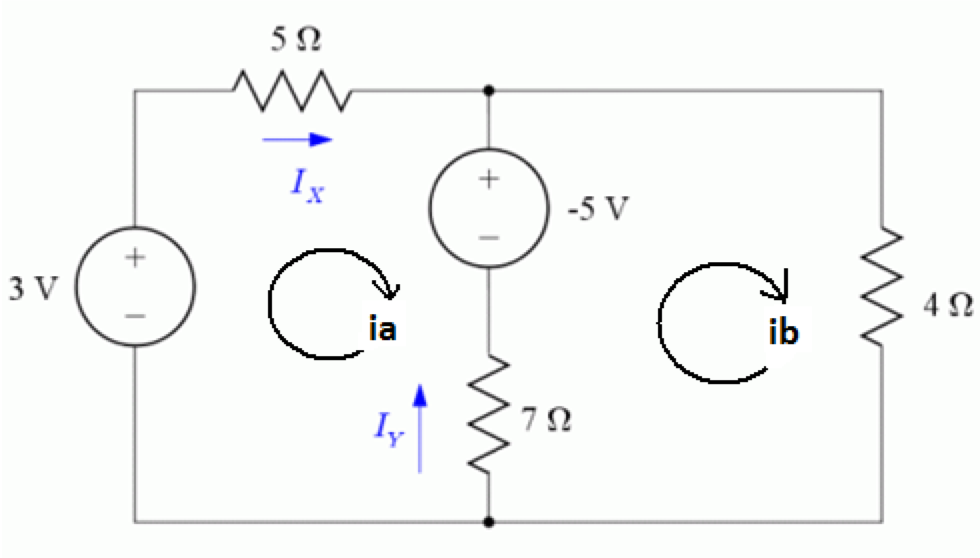
To select the method that results in the smaller number of equations. For example:

1. Choose nodal analysis for circuit with fewer nodes than meshes.
   *Choose mesh analysis for circuit with fewer meshes than nodes.
   *Networks that contain many series connected elements, voltage sources, or supermeshes are more suitable for mesh analysis.
   *Networks with parallel-connected elements, current sources, or supernodes are more suitable for nodal analysis.
2. If node voltages are required, it may be expedient to apply nodal analysis. If branch or mesh currents are required, it may be better to use mesh analysis.

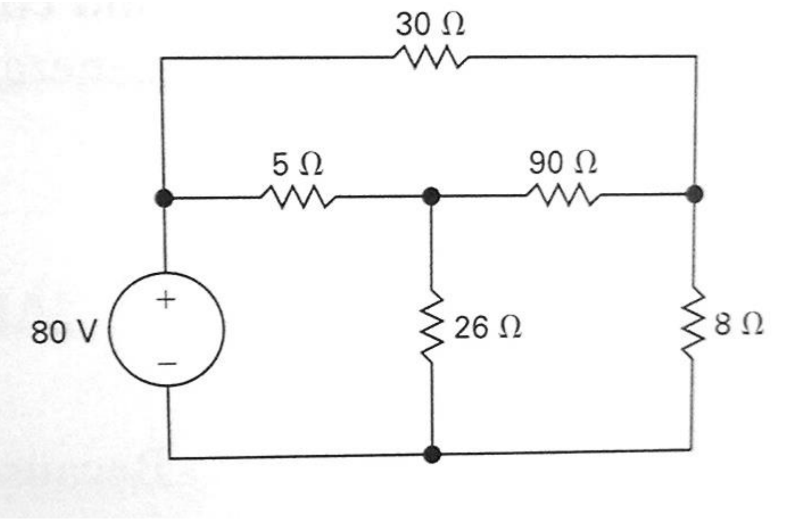
1) Use mesh analysis to determine Ia and Ib

Ia = 639 mA; Ib = -48 mA

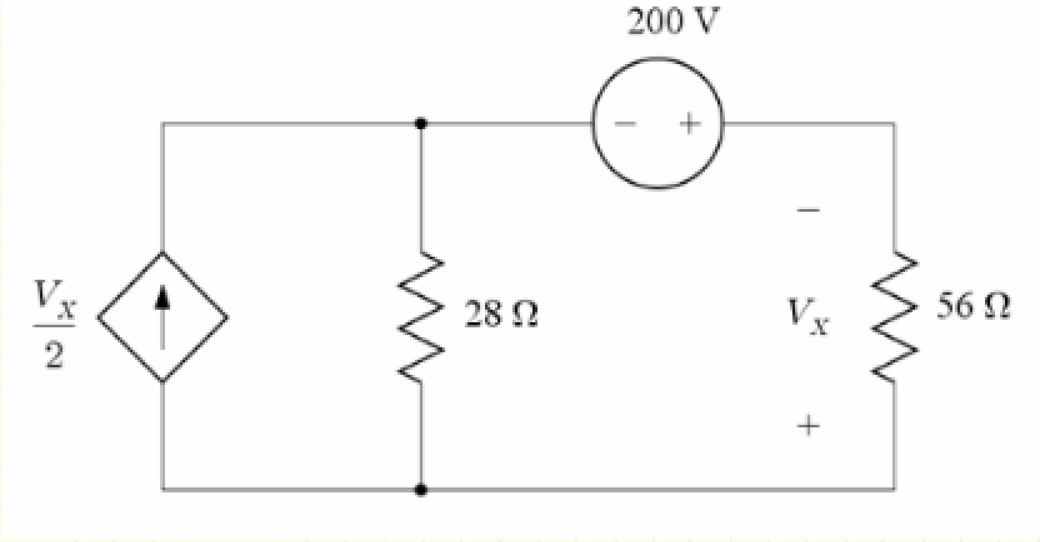
Then find BRANCH currents Ix, Iy (639,-687)



2) Find a) three mesh currents, [5,2.5,2]
b) Power of 80V source (-400W del)



3) Use MC to find Vx. [Vx = -12.9 V]



4) Use SUPERMESH to find the ia, ib and power associated with each voltage source.

Ia = 6.12 mA. Ib = -5.88 mA,   P1.5 = 9.18 mW; P3.0 = 17.6 mW