ENGR12

Chapter 2.4 – 2.7

Equivalent Resitance and

Wye-Delta Circuits

Basic Laws of Circuits

Equivalent Resistance:

We know the following for series resistors:

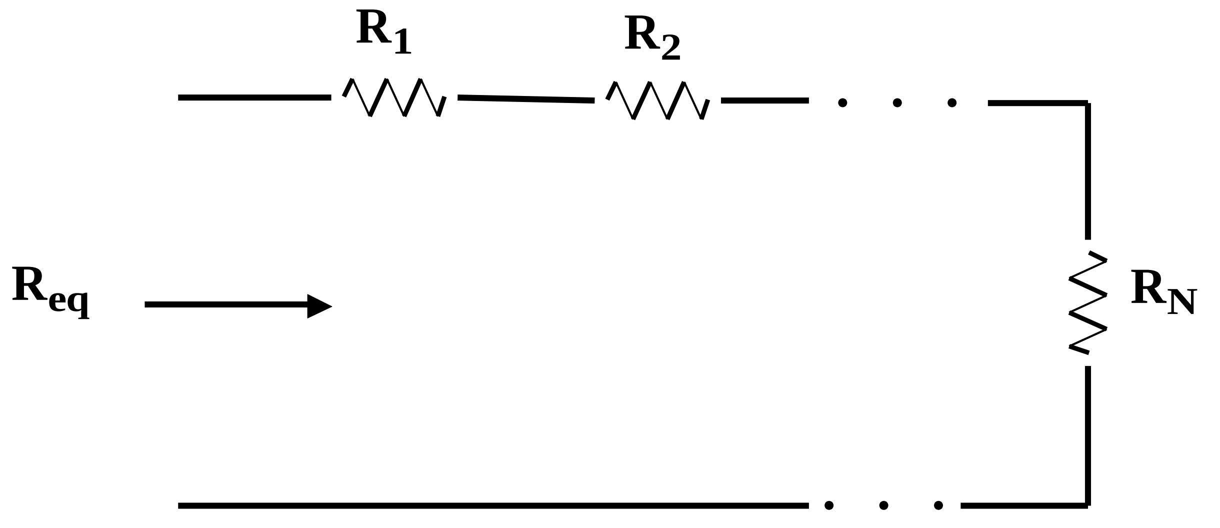


Figure 5.1: Resistors in series.

$$R_{eq} = R_1 + R_2 + \ldots + R_N$$

Basic Laws of Circuits

Equivalent Resistance:

We know the following for parallel resistors:

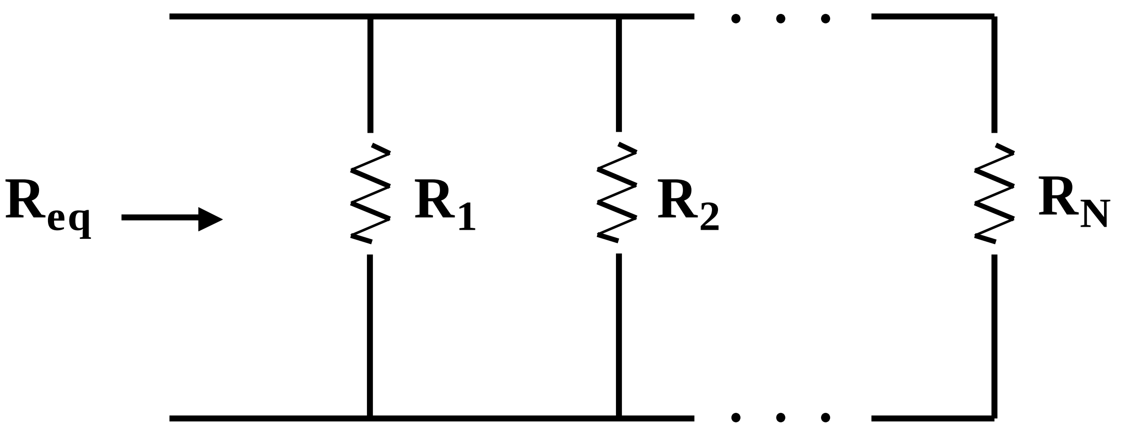


Figure 5.2: Resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ . \ . \ . + \frac{1}{R_N}$$

Basic Laws of Circuits

Equivalent Resistance:

For the special case of two resistors in parallel:

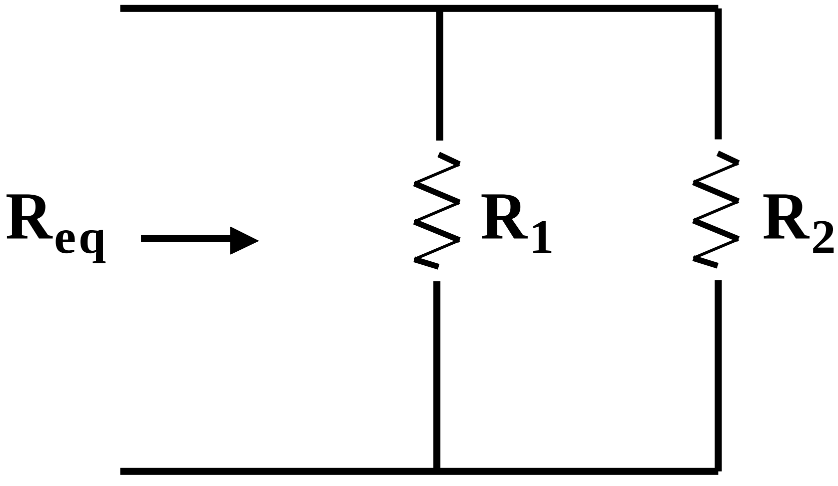


Figure 5.3: Two resistors in parallel.

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Basic Laws of Circuits

Equivalent Resistance: Resistors in combination.

By combination we mean we have a mix of series and Parallel. This is illustrated below.

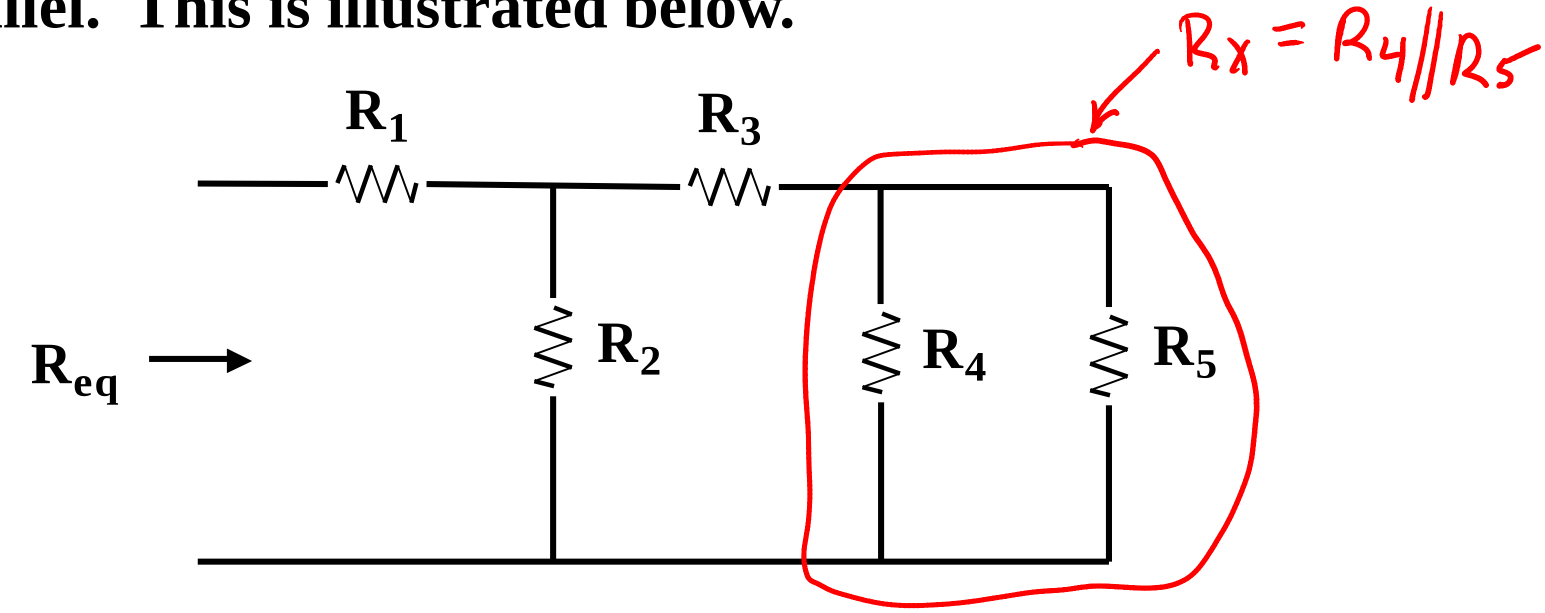
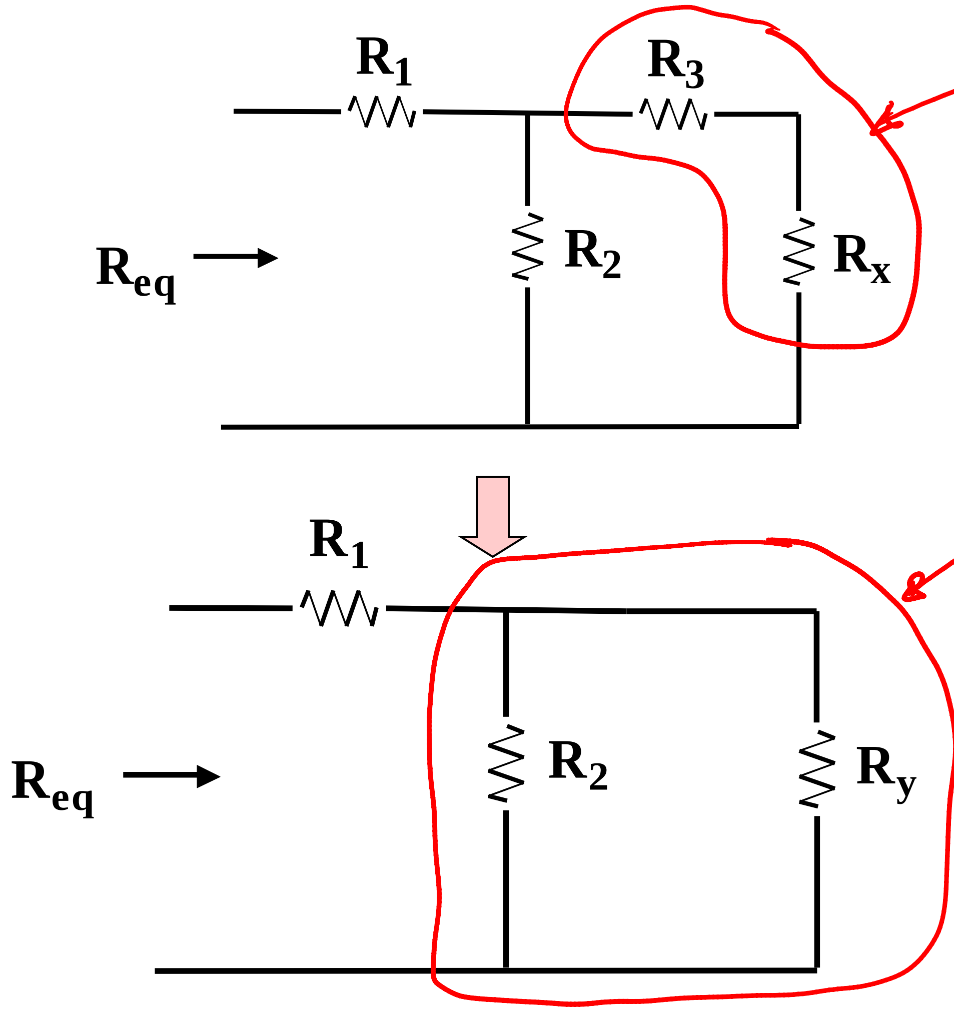


Figure 5.4: Resistors In Series – Parallel Combination

To find the equivalent resistance we usually start at the output of the circuit and work back to the input.

Basic Laws of Circuits

Equivalent Resistance: Resistors in combination.



$R_y = R_x + R_3$

$= R_4 \| R_5 + R_3$

$R_z = R_2 \| R_y$

Easier to compute R_y, R_z numerically than symbolically

Figure 5.5: Resistance reduction.

Basic Laws of Circuits

Equivalent Resistance: Resistors in combination.

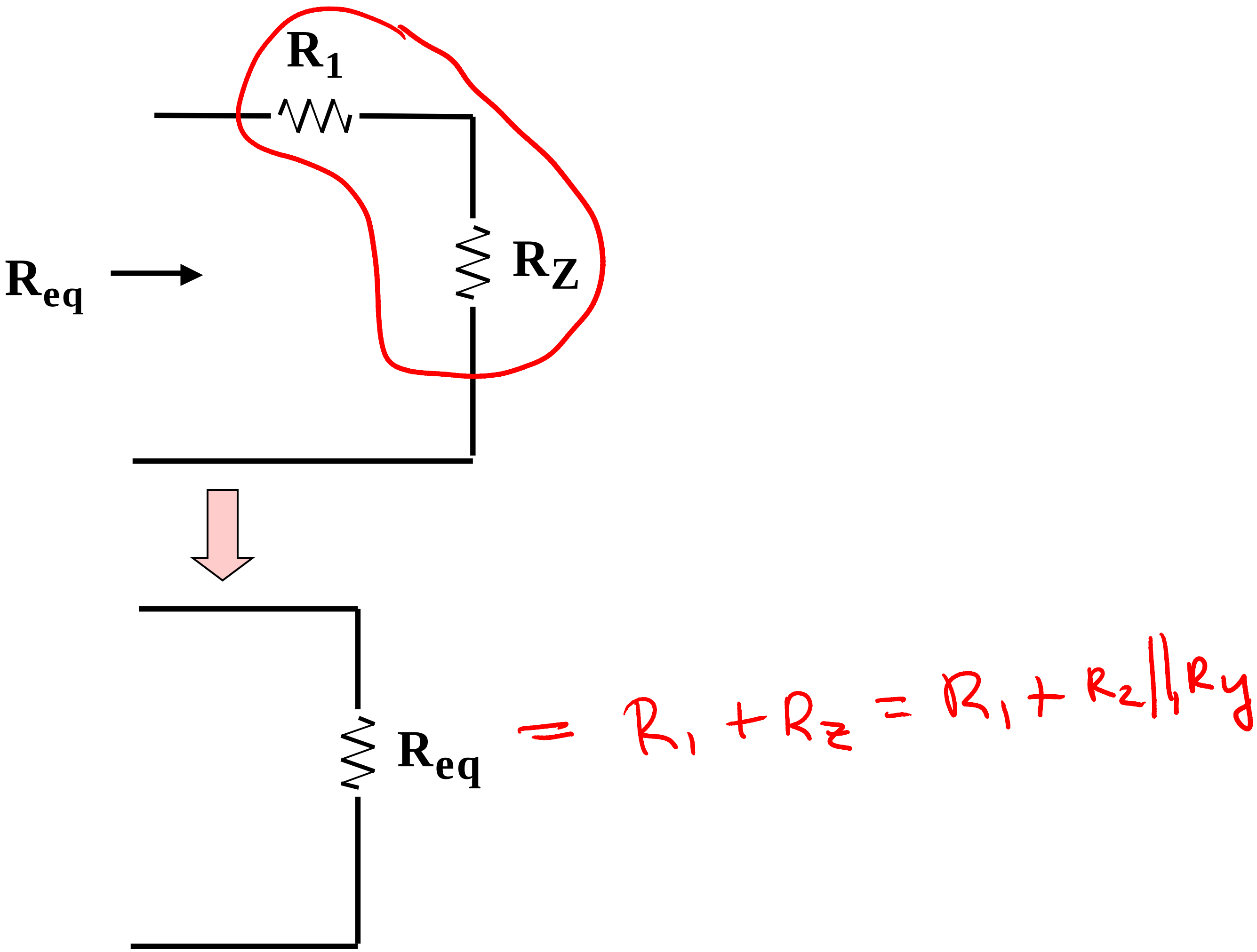


Figure 5.6: Resistance reduction, final steps.

Basic Laws of Circuits

Equivalent Resistance: Resistors in combination.

It is easier to work the previous problem using numbers than to work out a general expression. This is illustrated below.

Example 5.1: Given the circuit below. Find R_{eq}.

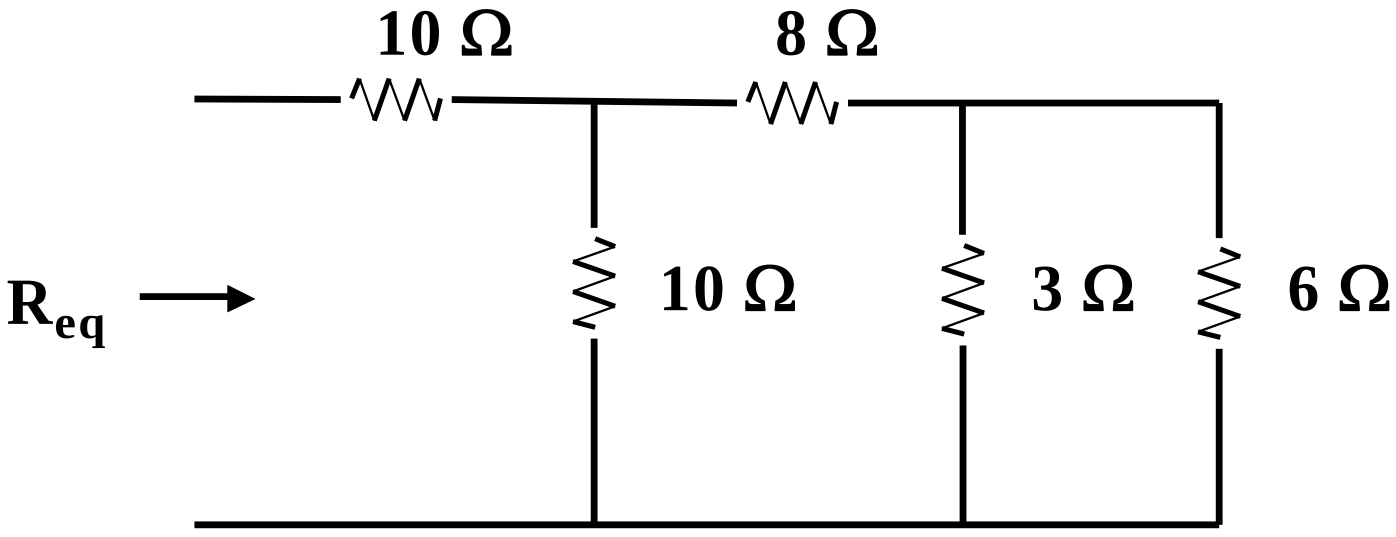
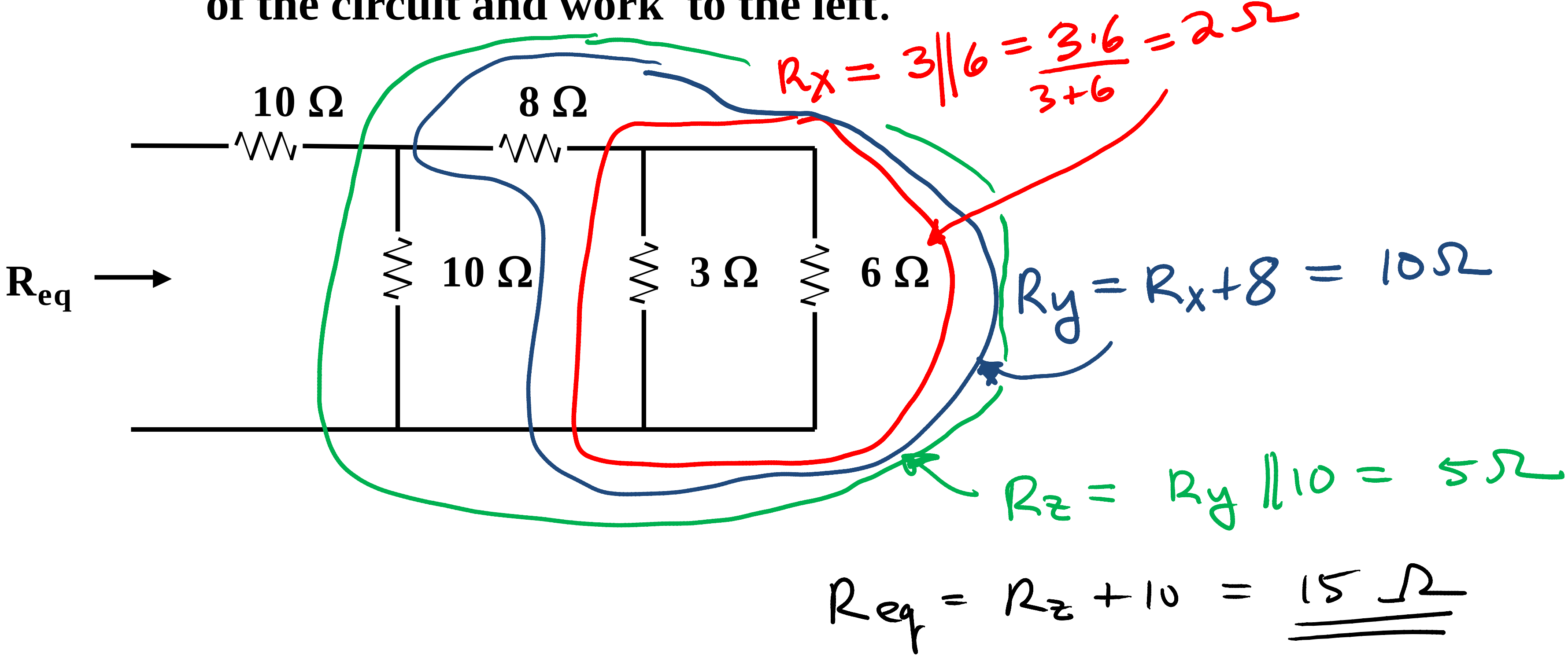


Figure 5.7: Circuit for Example 5.1.

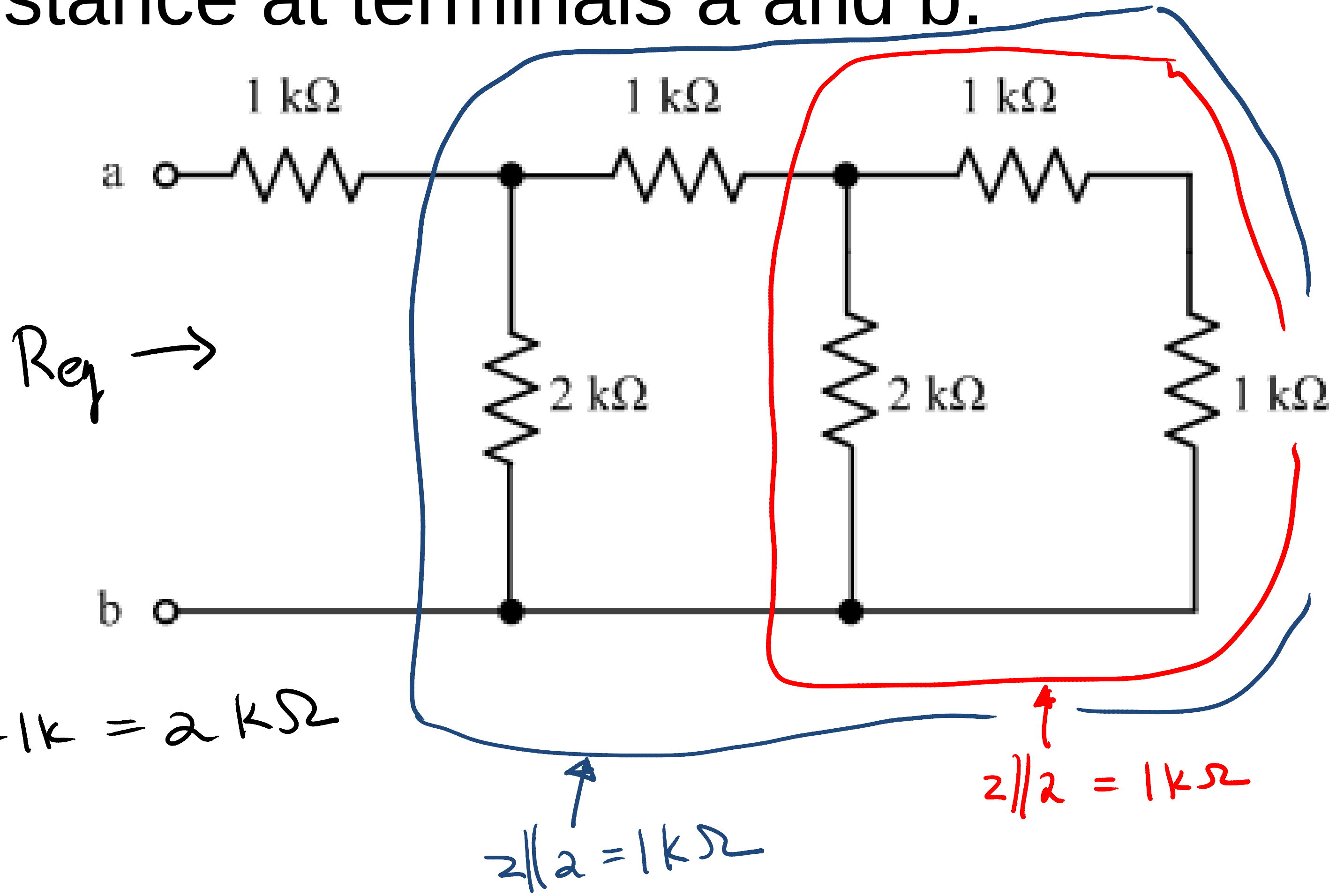
Basic Laws of Circuits

Equivalent Resistance: **Resistors in combination.**

Example 5.1: Continued. **We start at the right hand side of the circuit and work to the left.**



Cleo: Problem 1 Find the equivalent resistance at terminals a and b.



$R_{eq} = 1k + 1k = 2k\Omega$

(2k-ohm)

Basic Laws of Circuits

Equivalent Resistance: **Resistors in combination.**

Example 5.2: **Given the circuit shown below. Find $R_{eq.}$**

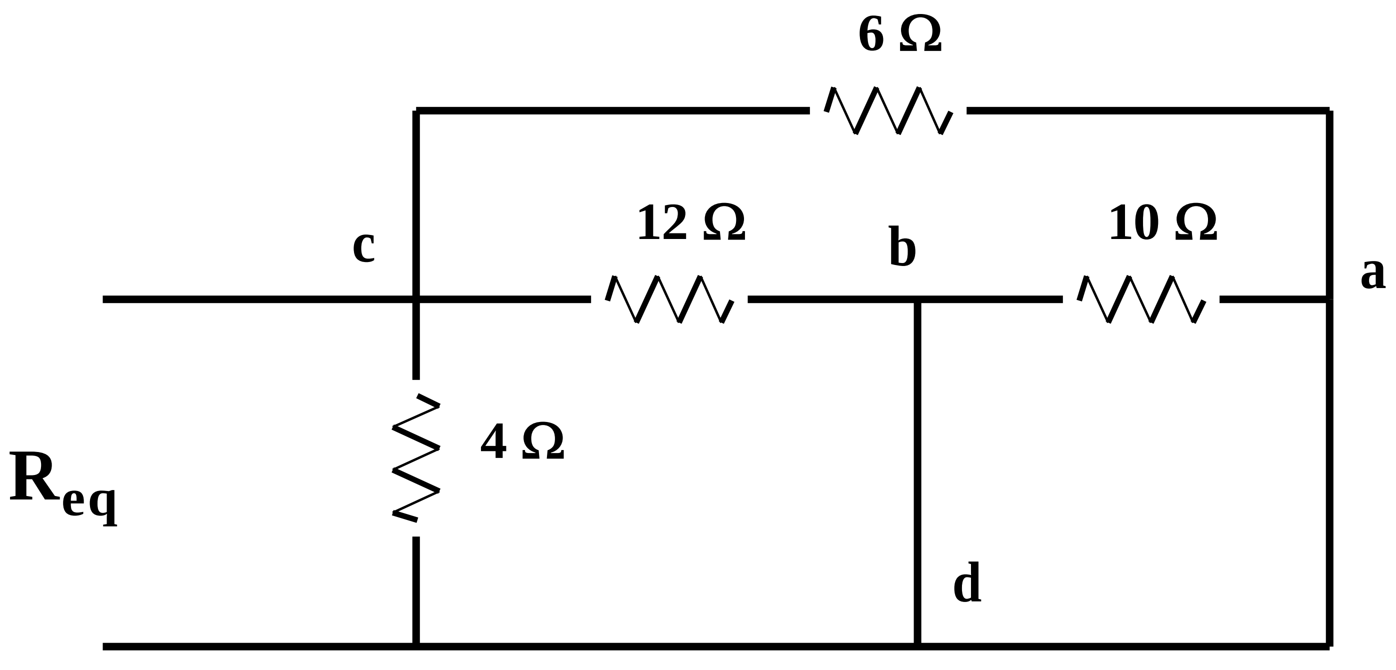
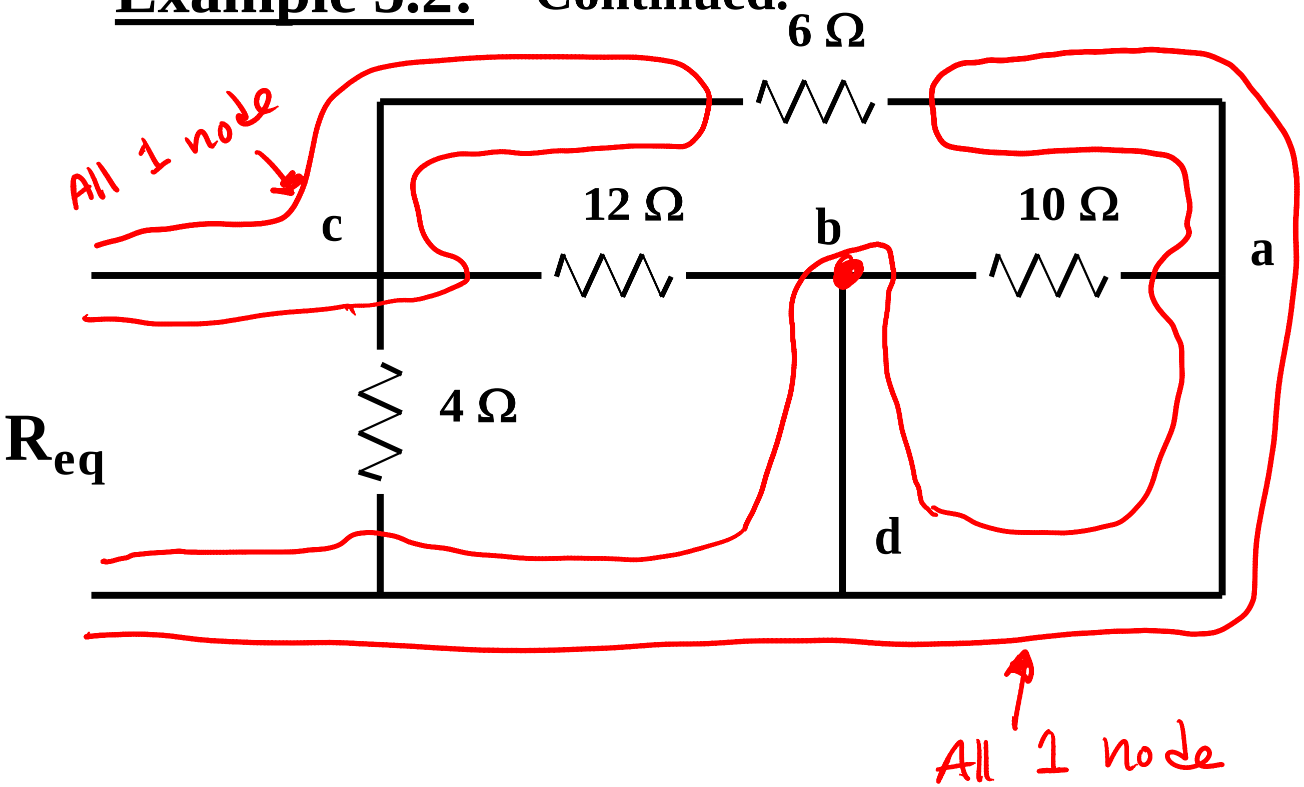


Figure 5.9: Diagram for Example 5.2.

Basic Laws of Circuits

Equivalent Resistance: **Resistors in combination.**

Example 5.2: **Continued.**



Only 2 nodes!

Redraw as Parallel Cct:

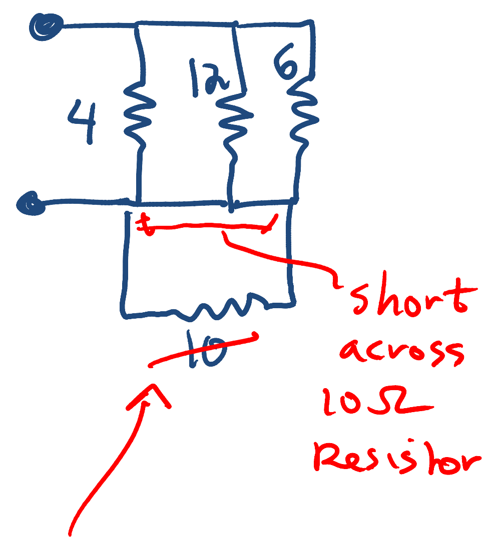


Fig 5.10: Reduction steps.

When a Resistor has a short across it

$$R_T = \frac{R \cdot 0}{R + 0} = 0\,\Omega$$

Basic Laws of Circuits

Equivalent Resistance: Resistors in combination.

Example 5.2: Continued.

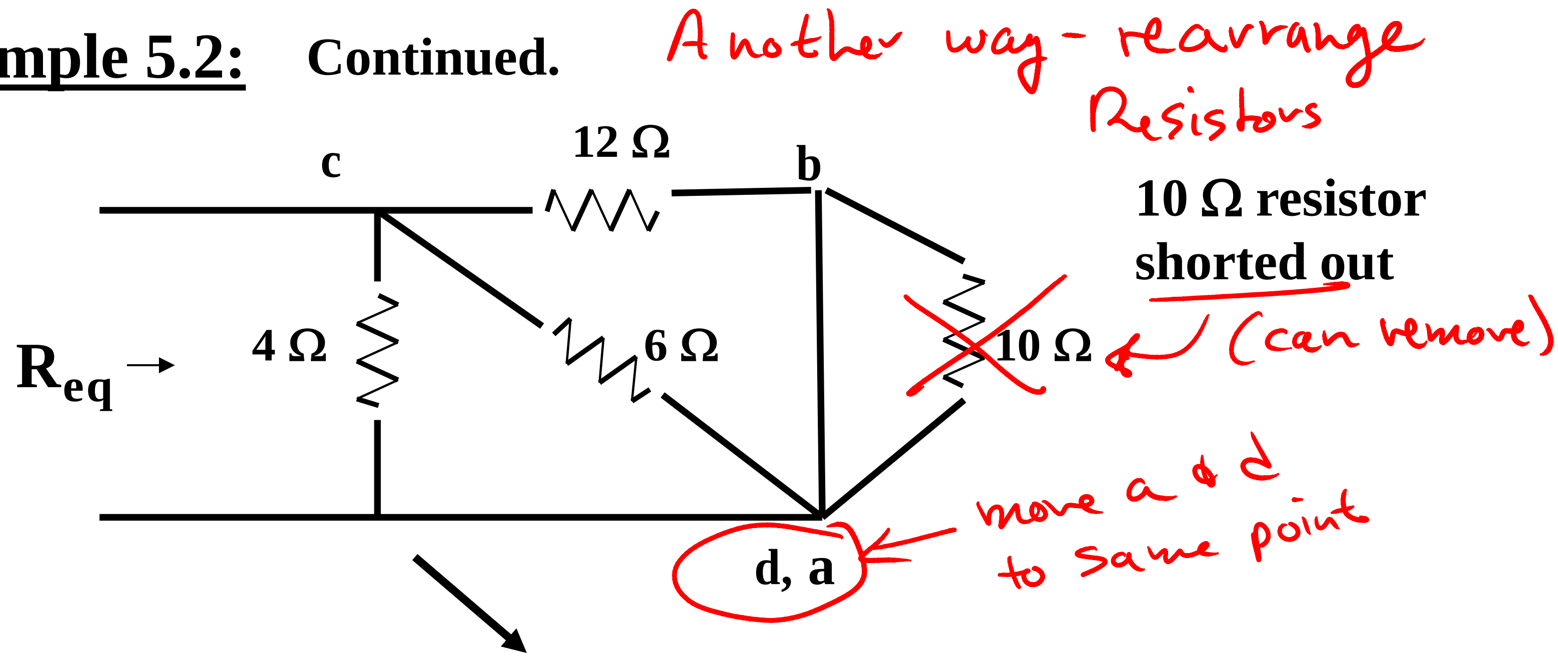
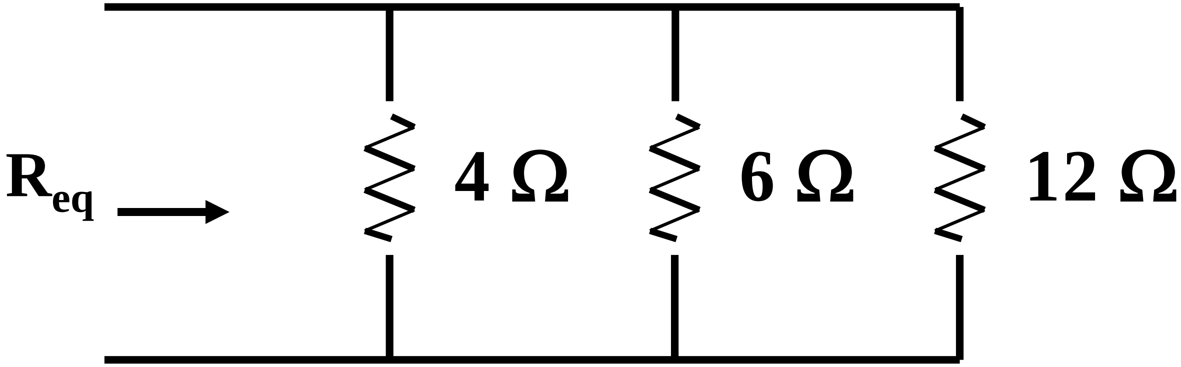


Fig 5.11: Reduction steps.

Basic Laws of Circuits

Equivalent Resistance: **Resistors in combination.**

Example 5.2: **Continued.** Simplified



$$R_{eq} = \frac{1}{\frac{1}{4} + \frac{1}{6} + \frac{1}{12}} = 2\,\Omega$$

Fig 5.12: Reduction steps.

Cleo: Problem 2 Reduce the circuit to a single resistor at terminals a and b.

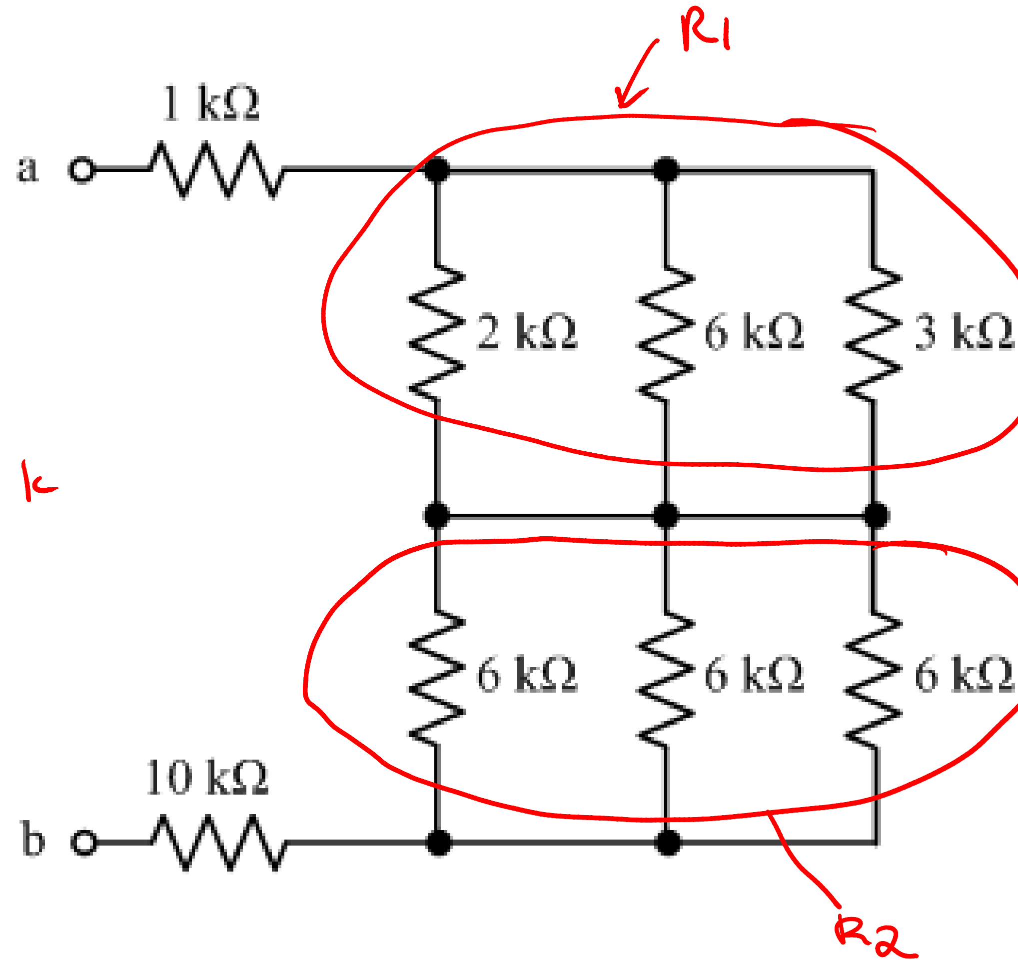
$$R_1 = 2 \| 6 \| 3 = \frac{1}{\frac{1}{2} + \frac{1}{6} + \frac{1}{3}}$$

$$= 1\ k\Omega$$

$$R_2 = 6 \| 6 \| 6 = 2\ k\Omega$$

$$R_{eq} = 1k\Omega + R_1 + R_2 + 10\ k$$

$$= 14\ k\Omega$$



(14 k-ohm)

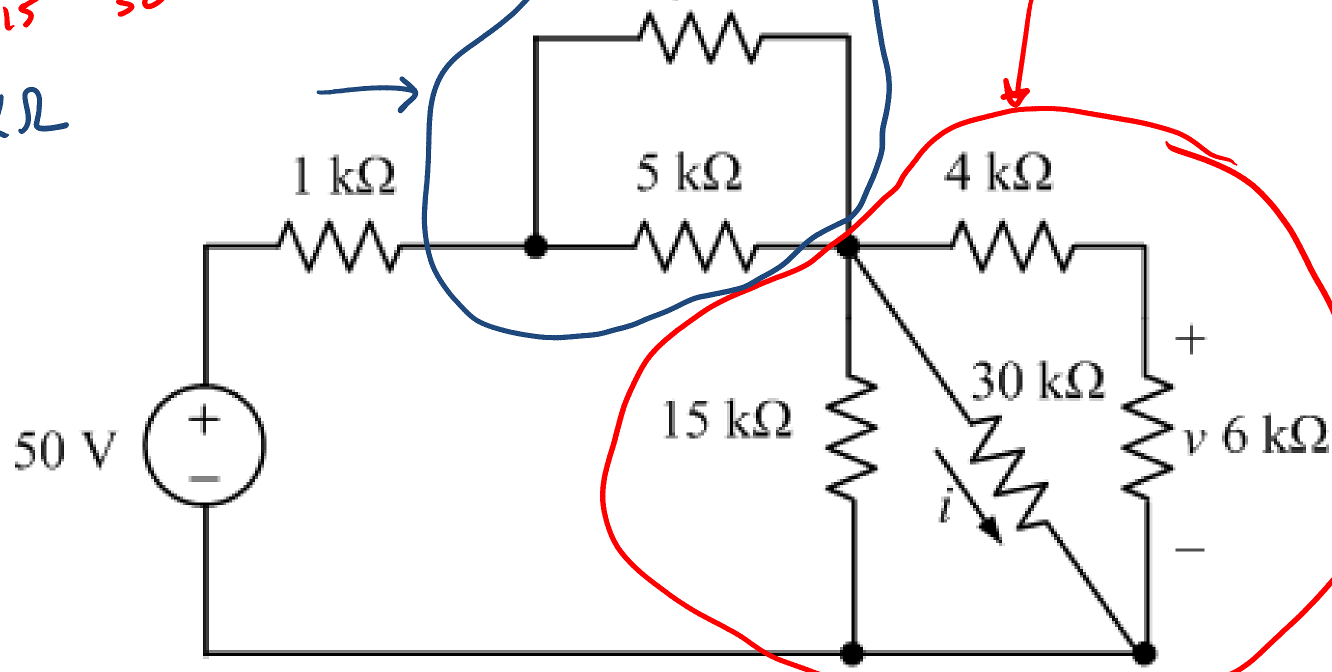
Find the equivalent resistance seen by the 50V source

$R_x = 15 \| 30 \| 10 = \dfrac{1}{\frac{1}{15} + \frac{1}{30} + \frac{1}{10}} = \dfrac{30}{6} = 5\ k\Omega$

$R_x = 15 \| 30 \| 10\ k\Omega$

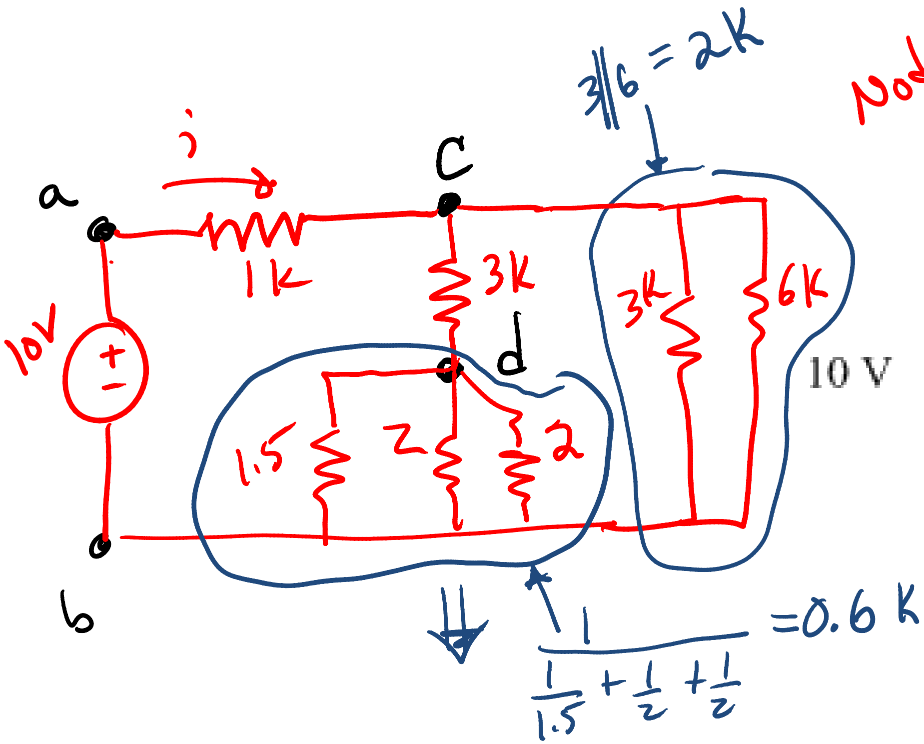
$R_y = 20 \| 5 = 4\ k\Omega$

$R_{eq} = 1 + R_x + R_y$
$= 10\ k\Omega$



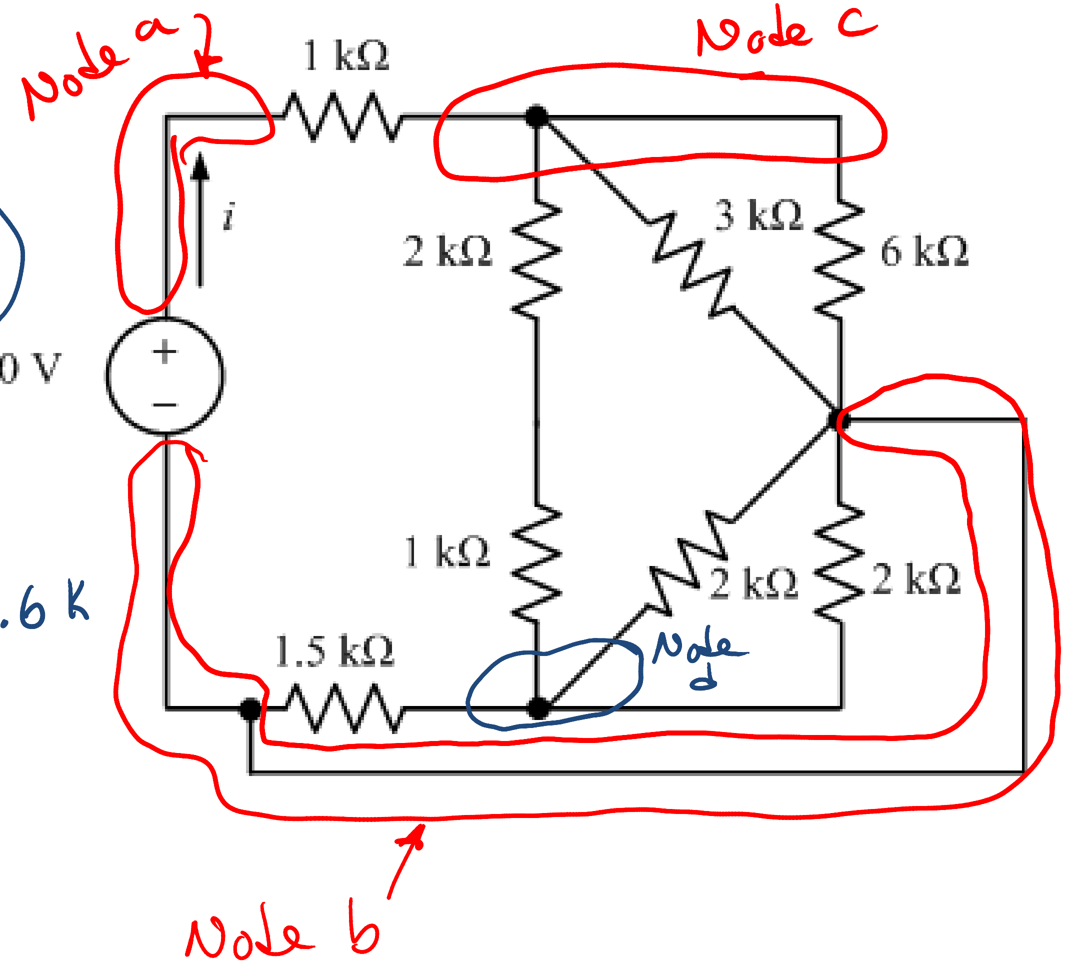
10k-ohm

Cleo: Problem 3 Find the current i in the circuit. (First find equivalent R seen by 10V, by carefully rearranging the circuit elements)



$R_{eq} = (3+.6)\,||\,2 + 1 = 2.286$

$i = \frac{10V}{2.286} = 4.38\ mA$



4.38 mA

Basic Electric Circuits

Wye to Delta Transformation:

You are given the following circuit. Determine R_{eq}.

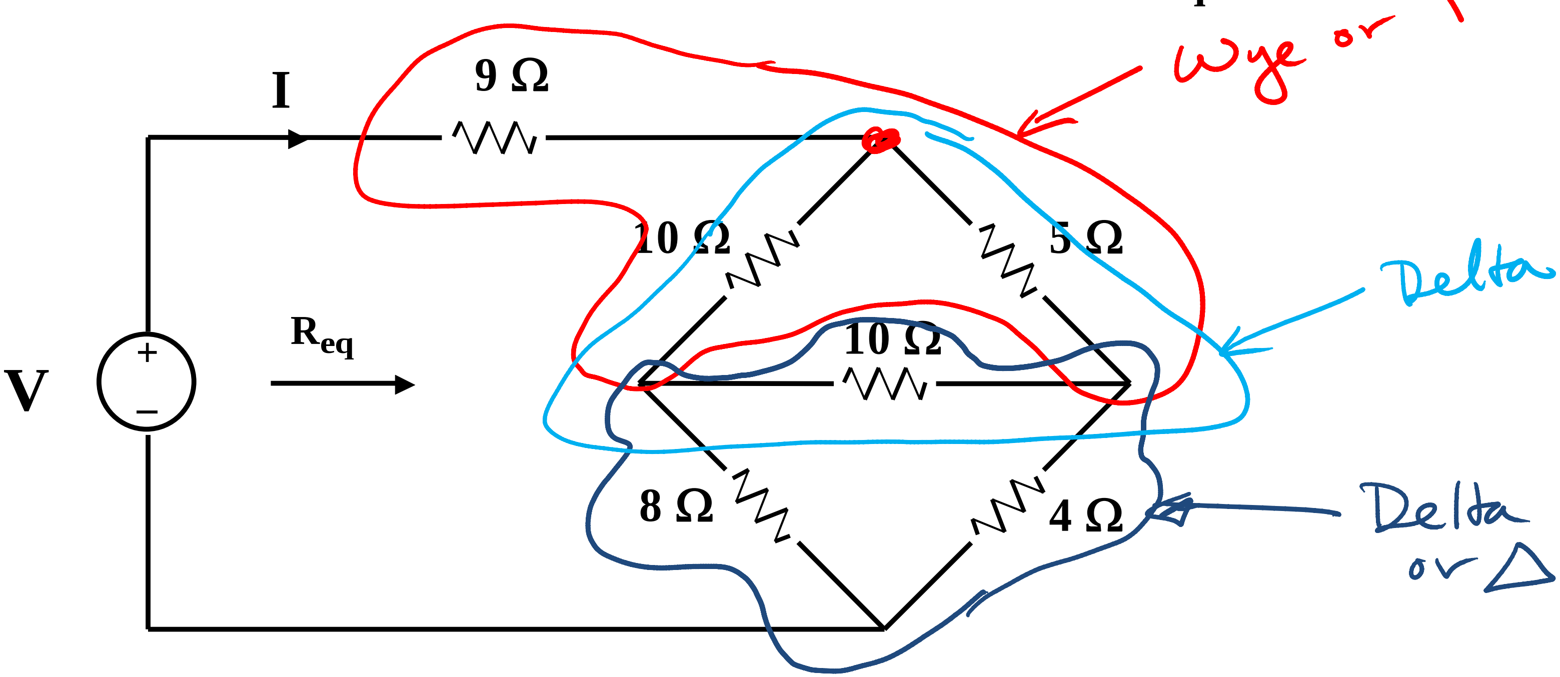
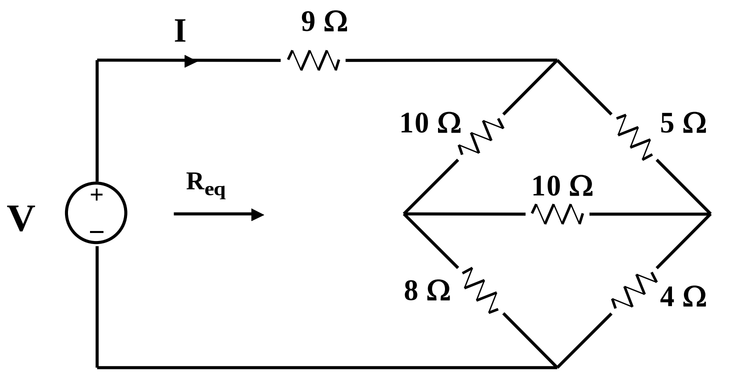


Figure 5.1: Diagram to start wye to delta.

Basic Electric Circuits

Wye to Delta Transformation:



We cannot use resistors in parallel. We cannot use resistors in series. If we knew V and I we could solve

$$R_{eq} = \frac{V}{I}$$

There is another way to solve the problem without solving for I (given, assume, V) and calculating R_{eq} for V/I.

Basic Electric Circuits

Wye to Delta Transformation:

Consider the following:

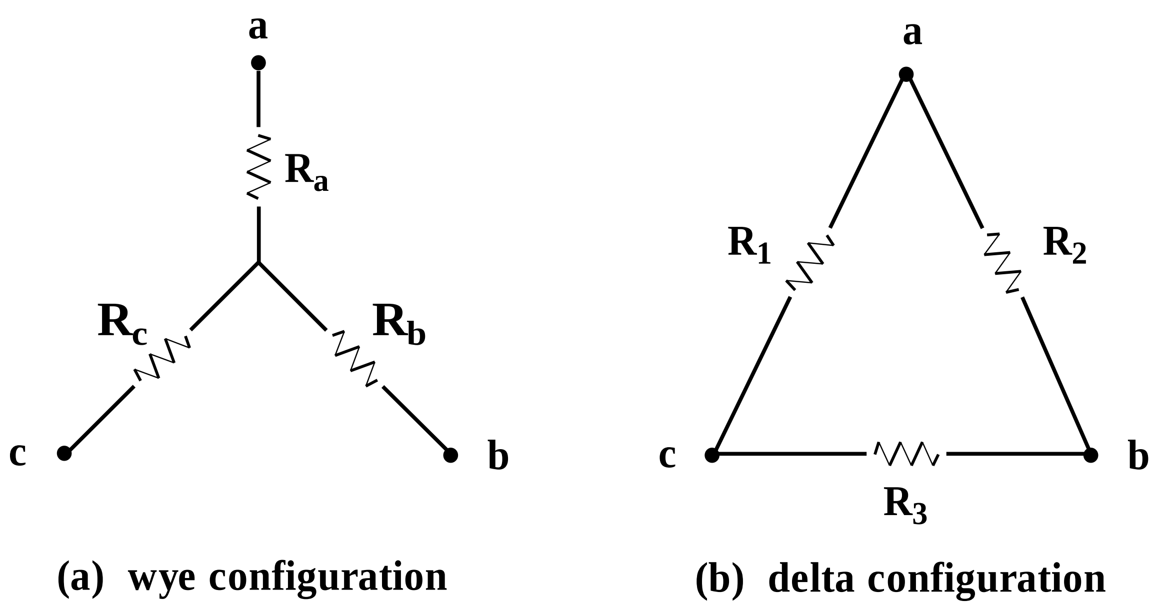


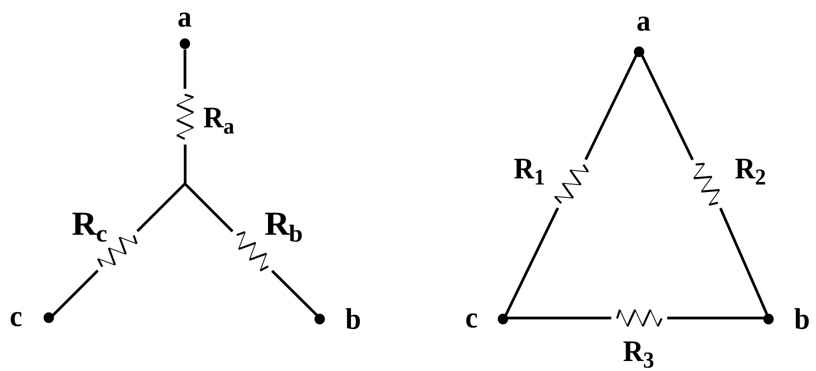
Figure 5.14: Wye to delta circuits.

We equate the resistance of R_{ab}, R_{ac} and R_{ca} of (a) to R_{ab} , R_{ac} and R_{ca} of (b) respectively.

Basic Electric Circuits

Wye to Delta Transformation:

Consider the following:



(a) wye configuration (b) delta configuration

Wye

Delta

$$R_{ab} = R_a + R_b = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3} \quad \text{Eq 5.1}$$

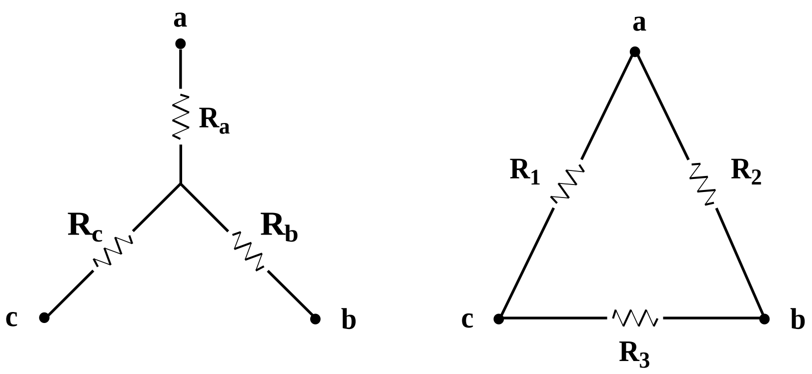
$$R_{ac} = R_a + R_c = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \quad \text{Eq 5.2}$$

$$R_{bc} = R_b + R_c = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} \quad \text{Eq 5.3}$$

Basic Electric Circuits

Wye to Delta Transformation:

Previous Equations reduce to:



(a) wye configuration (b) delta configuration

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3} \qquad R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b} \qquad \text{Eq 5.4}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3} \qquad R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c} \qquad \text{Eq 5.5}$$

$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3} \qquad R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a} \qquad \text{Eq 5.6}$$

Basic Electric Circuits

Wye to Delta Transformation:

Summary:

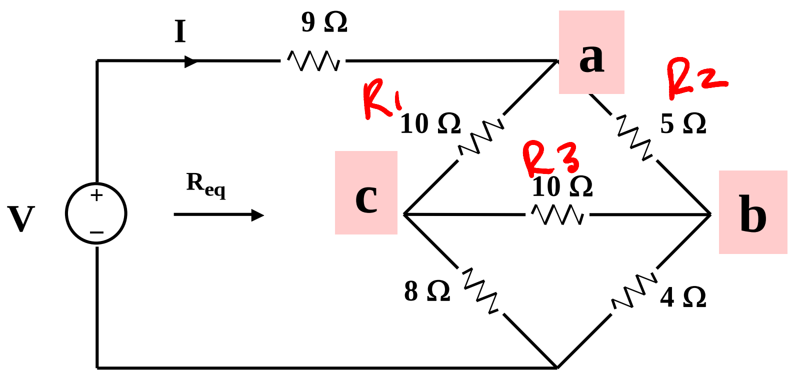
Go to wye	Go to delta	
$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$	$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$	Eq 5.4
$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$	$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$	Eq 5.5
$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$	$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$	Eq 5.6

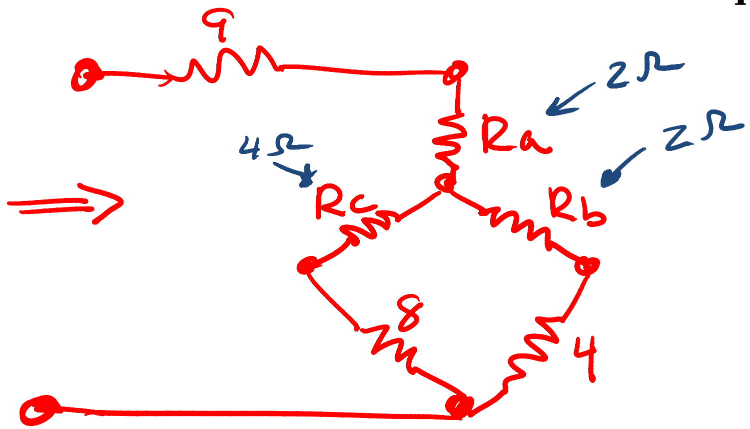
We note that the denominator for R_a, R_b, R_c is the same.
We note that the numerator for R_1, R_2, R_3 is the same.
We could say "Y" below: "D"

Basic Electric Circuits

Wye to Delta Transformation:

Example 5.3: Return to the circuit of Figure 5.13 and find R_{eq}.





Convert the delta around a – b – c to a wye.

$\Delta \rightarrow Y$

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{(10)(5)}{25} = 2\,\Omega$$

$$R_b = \frac{R_2 R_3}{25} = \frac{50}{25} = 2\,\Omega$$

$$R_c = \frac{R_1 R_3}{25} = \frac{100}{25} = 4\,\Omega$$

Basic Electric Circuits

Wye to Delta Transformation:

Example 5.3: continued

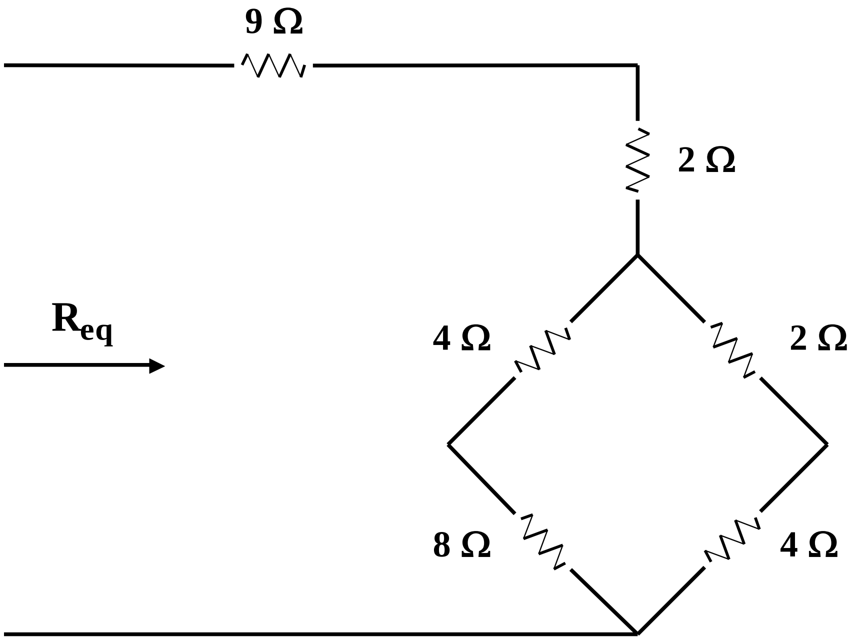


Figure 5.15: Example 5.3 diagram.

It is easy to see that R_{eq} = 15 Ω

$= 9 + 2 + (4+8) \| (4+2)$

$= 9 + 2 + \frac{(12)6}{18} = 15\Omega$

Basic Electric Circuits

Wye to Delta Transformation:

Example 5.4: Using wye to delta. The circuit of 5.13 may be redrawn as shown in 5.16.

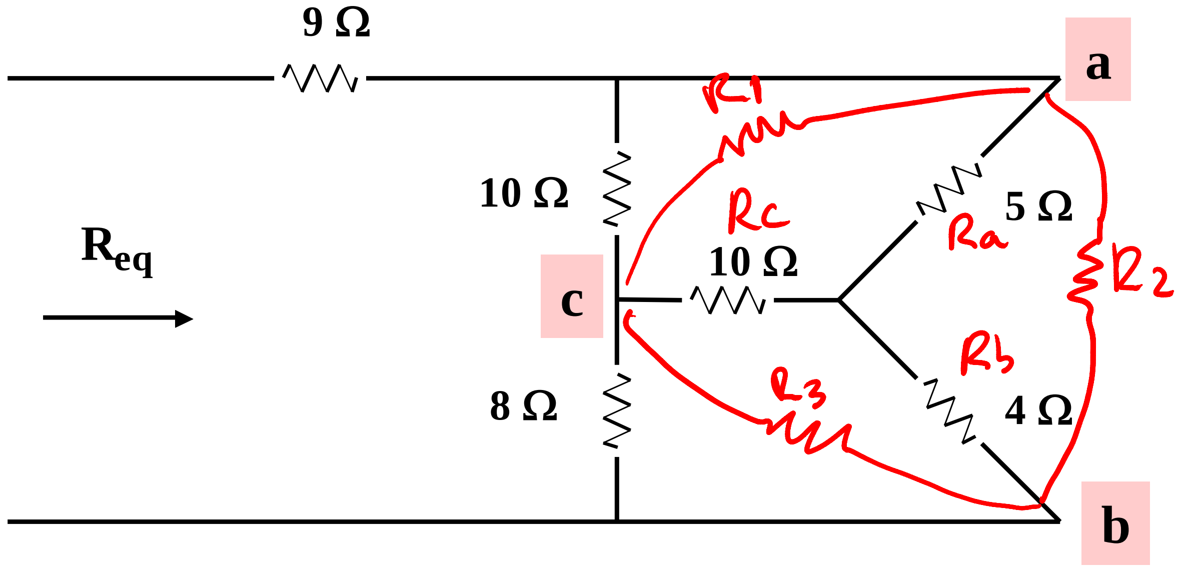


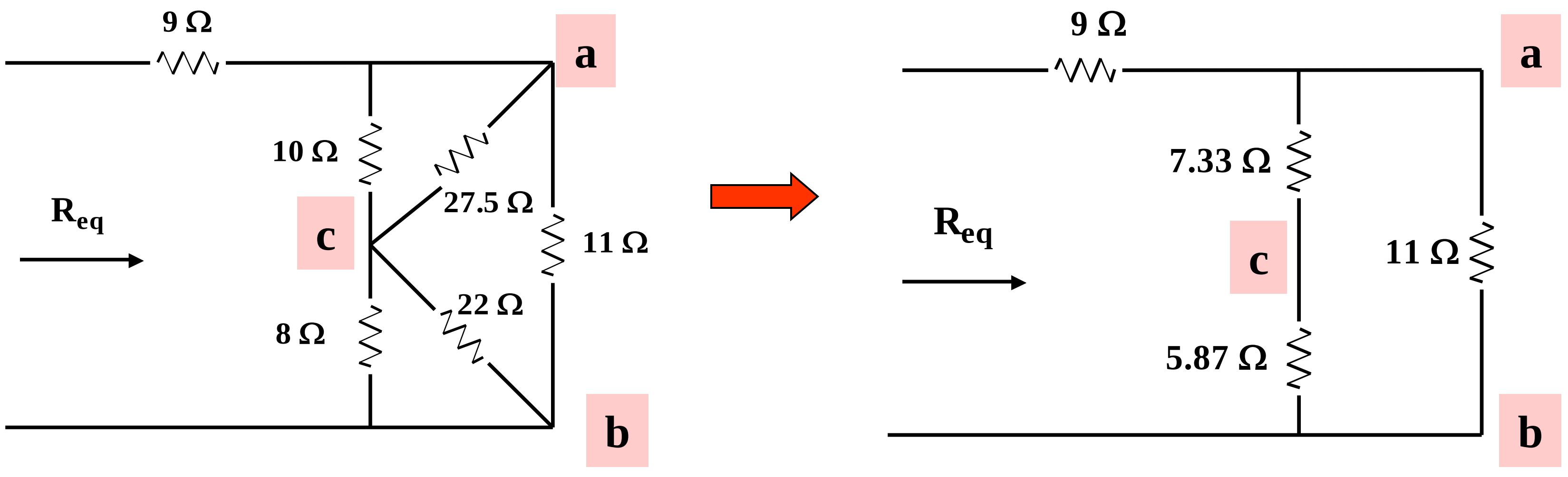
Figure 5.16: *"Stretching"* (rearranging) the circuit.

Convert the wye of a – b – c to a delta.

Basic Electric Circuits

Wye to Delta Transformation:

Example 5.4: continued



(a) (b)

Basic Electric Circuits

Wye to Delta Transformation:

Example 5.4: continued

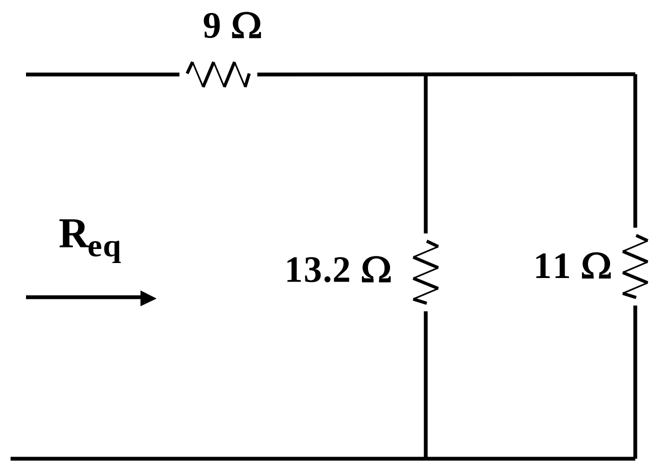


Figure 5.18: Reduction of Figure 5.17.

$$R_{eq} = 15\ \Omega$$

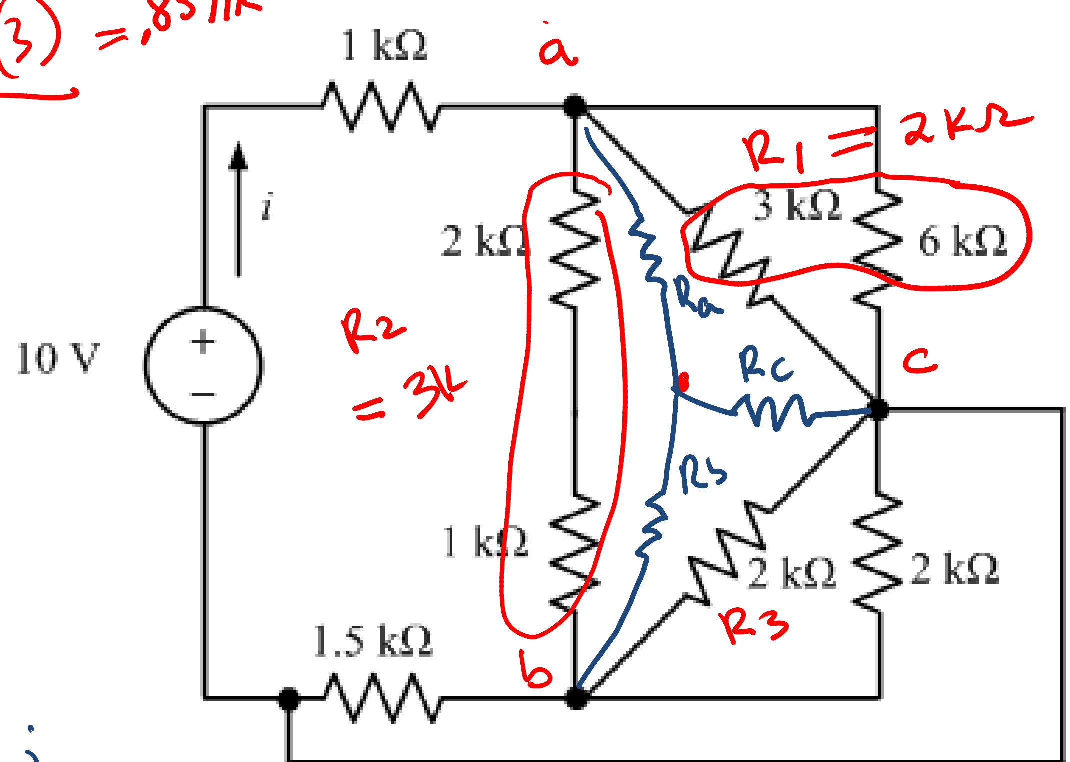
This answer checks with the delta to wye solution earlier.

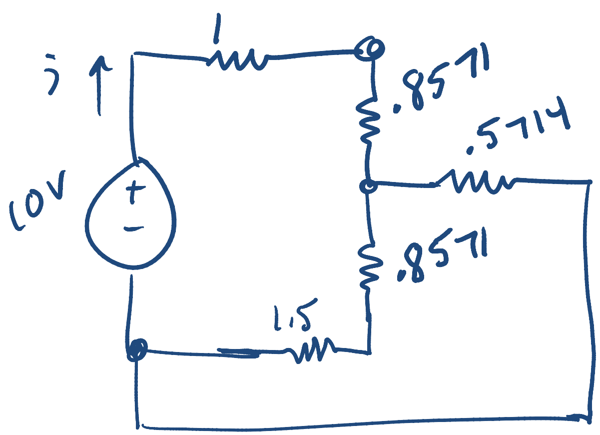
Cleo: Problem 3 Find the current i in the circuit. (First find equivalent R seen by 10V) by converting the Delta to a Wye

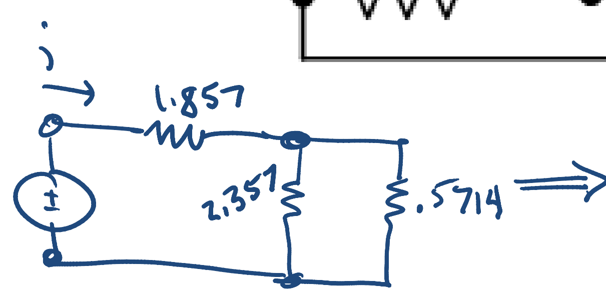
$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3} = \frac{(2)(3)}{7} = .8571k$

$R_b = \frac{(2)(3)}{7} = .8571k$

$R_c = \frac{(2)(2)}{7} = .5714$







$R_{eq} = 2.317$

$i = \frac{10}{R_{eq}} = 4.31\ mA$

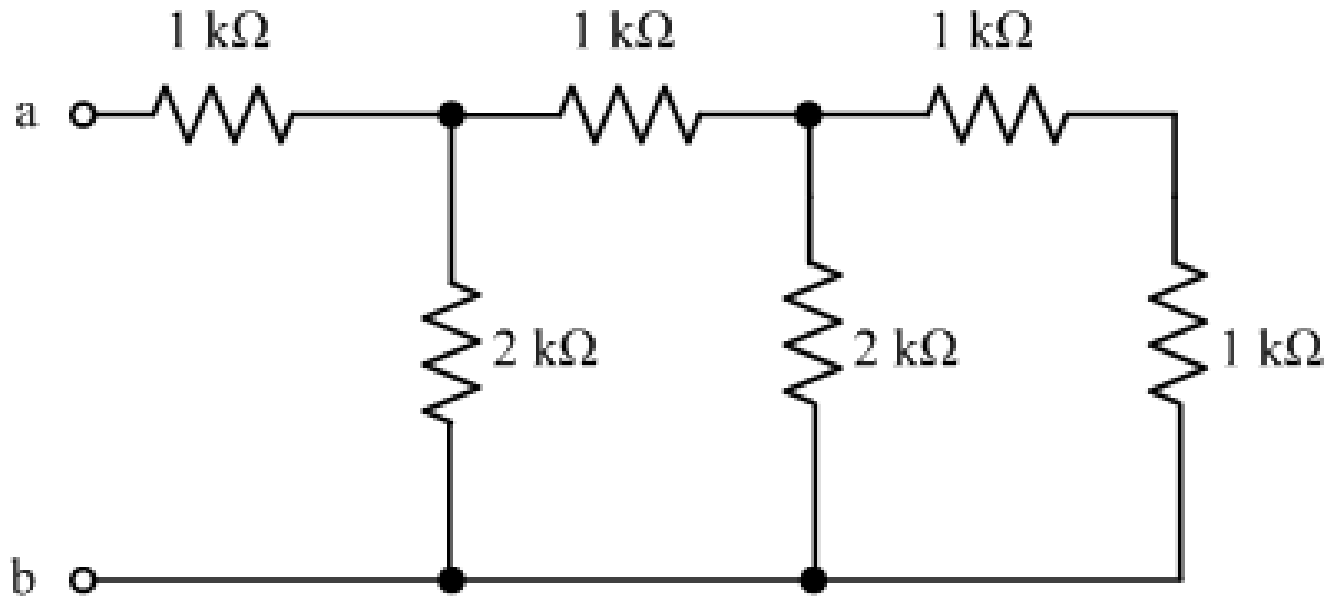
4.38 mA

Basic Laws of Circuits

End of Lesson 5

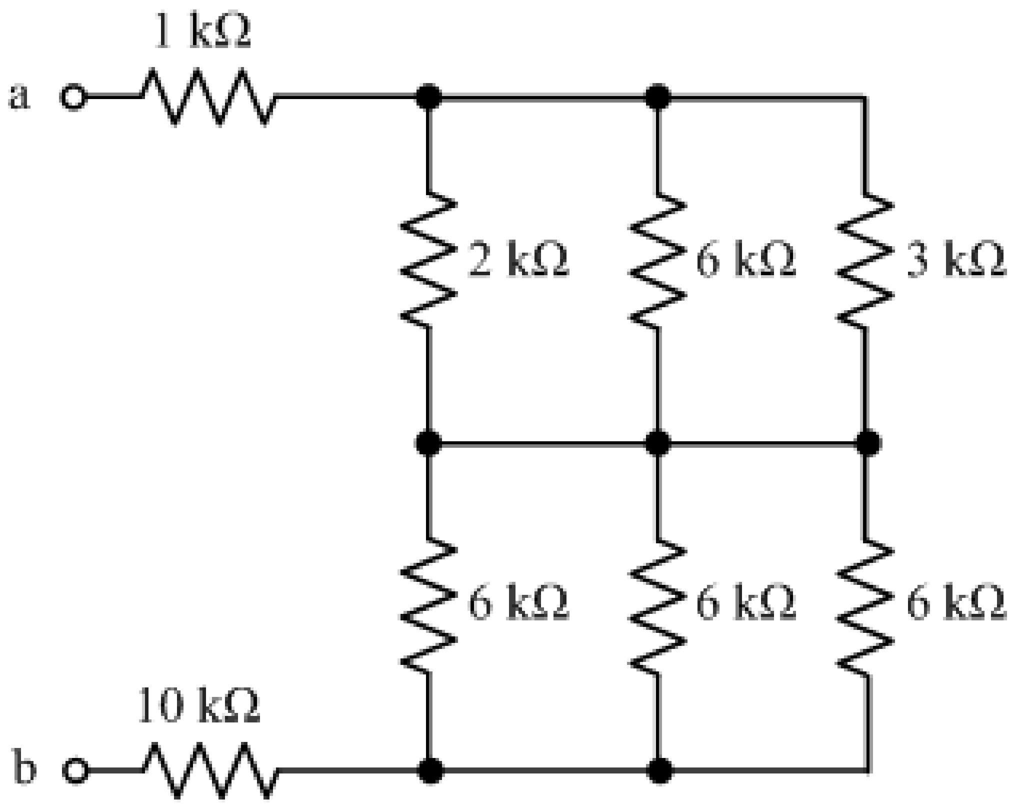
Equivalent Resistance

Cleo: Problem 1 Find the equivalent resistance at terminals a and b.



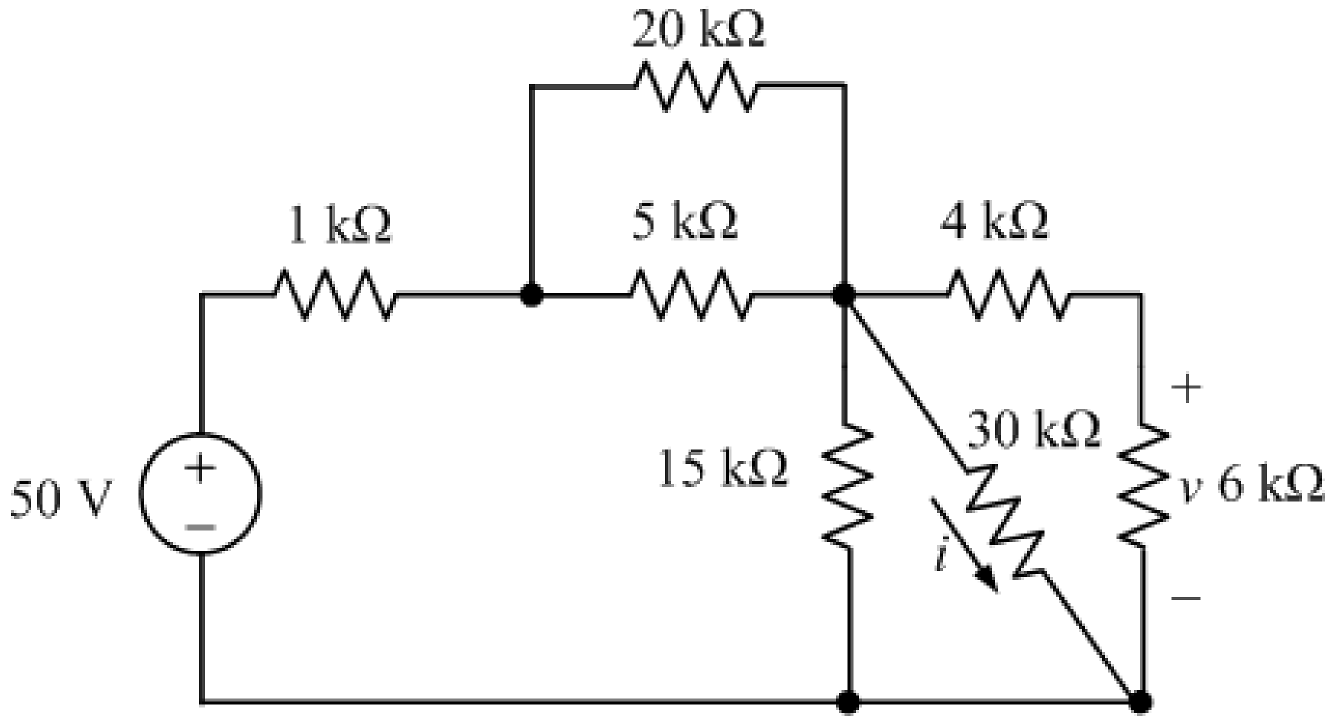
(2k-ohm)

Cleo: Problem 2 Reduce the circuit to a single resistor at terminals a and b.

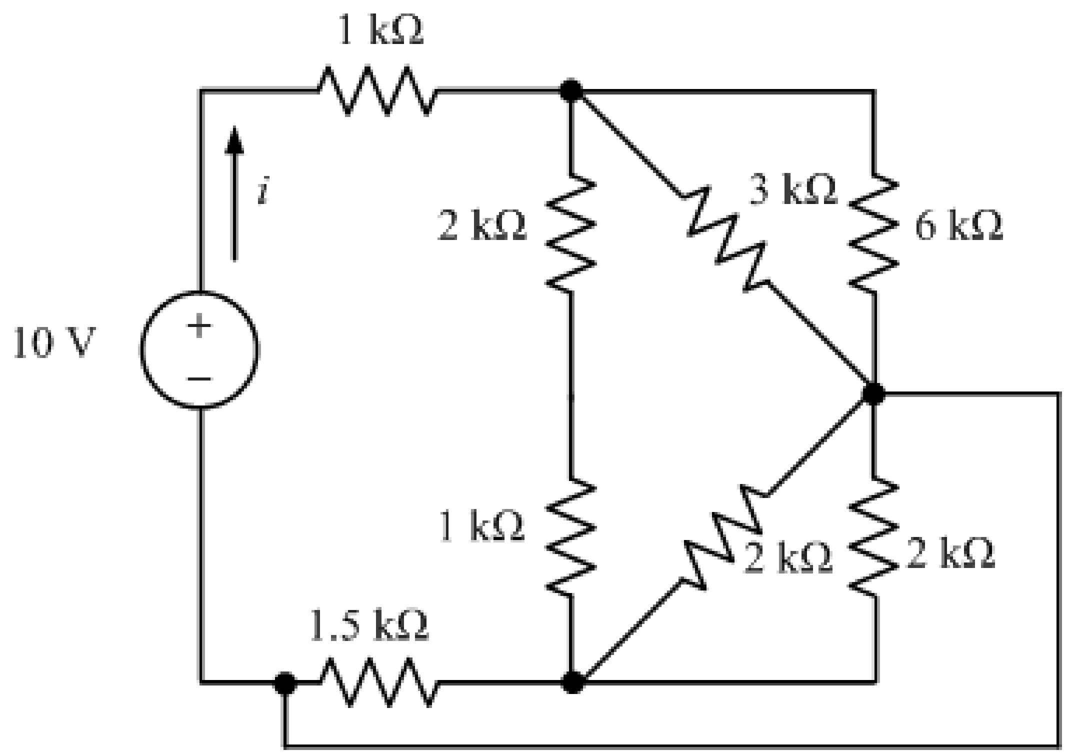


(14 k-ohm)

Find the equivalent resistance seen by the 50V source



Cleo: Problem 3 Find the current i in the circuit. (First find equivalent R seen by 10V)



4.38 mA