ENGR12

Chapter 2

Basic Laws

Basic Laws - Chapter 2

2.1 Ohm's Law.

2.2 Nodes, Branches, and Loops.

2.3 Kirchhoff's Laws.

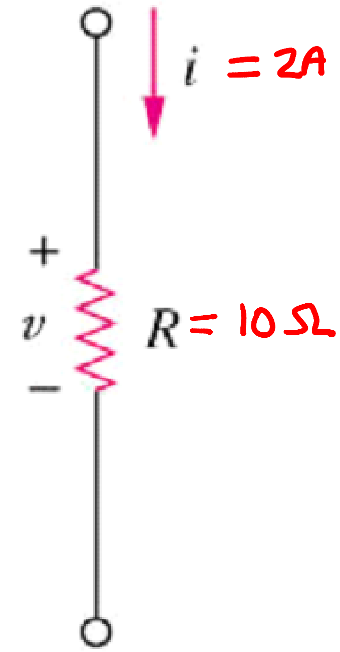
2.4 Series Resistors and Voltage Division.

2.5 Parallel Resistors and Current Division.

2.6 Wye-Delta Transformations.

2.1 Ohms Law (1)

- Ohm's law states that the voltage across a resistor is directly proportional to the current I flowing through the resistor.



- Mathematical expression for Ohm's Law is as follows:

$$v = iR = (2)(10) = 20V$$

- Two extreme possible values of R: **0 (zero) and ∞ (infinite)** are related with two basic circuit concepts: short circuit and open circuit.

if $R = 0$, $v = iR = 0$ for all i (short)

if $R = \infty$, $i = \frac{v}{R} = 0$ for all v (open)

2.1 Ohms Law (2)

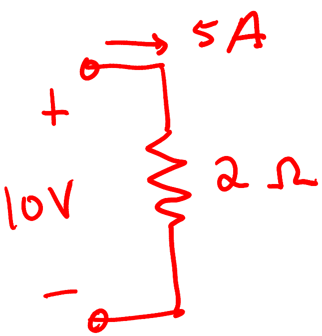
- *Conductance* is the ability of an element to conduct electric current; it is the reciprocal of resistance R and is measured in mhos or siemens.

if $R = 10\,k\Omega$

$$G = \frac{1}{R} = \frac{i}{v}$$

$G = \frac{1}{R} = .0001$ ℧ or .0001 S

- The power dissipated by a resistor:



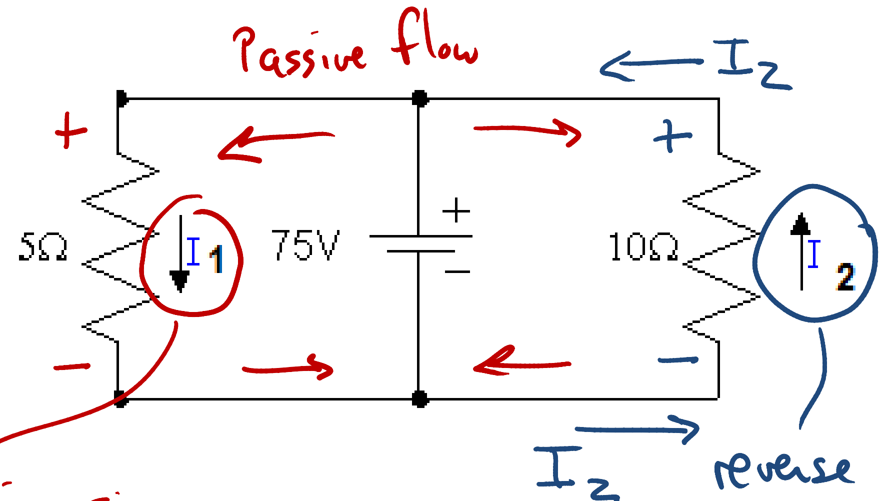
iR

$$p = vi = i^2R = \frac{v^2}{R}$$

$= (10)5 = (5^2)(2) = \frac{10^2}{2} = 50\,W$

Always Positive for Resistors

Find Currents I1 and I2



Passive sign

$v = iR$

$I_1 = \frac{75}{5} = 15A$

reverse of Passive sign this is "Active Sign" convention

Ohm's Law for ASC: $v = -iR$

$I_2 = -\frac{75}{10} = -7.5A$

TIP whenever you label i, v on a Resistor use PSC, then $v = iR$

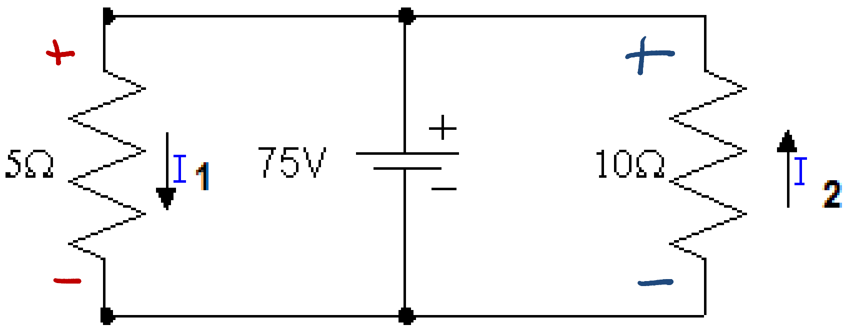
$i1 = 15A \quad i2 = -7.5A$

Find Power dissipated in each resistor

Passive Sign

$P_1 = iV = 15(75) = 1125W$

$P_1 > 0 \rightarrow$ absorbing



Active Sign

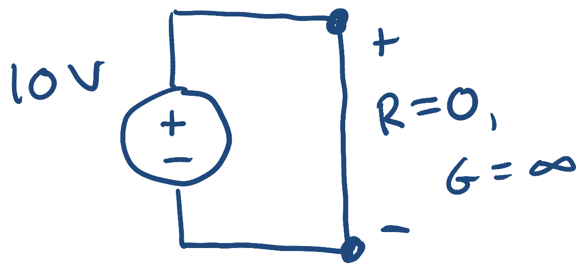
$P_2 = -iV = -(-7.5)(75) = 562.5W$

$P_2 > 0 \rightarrow$ absorbing

P1 = 1125 W P2 = 562.5 W

Shorts and Open Circuits

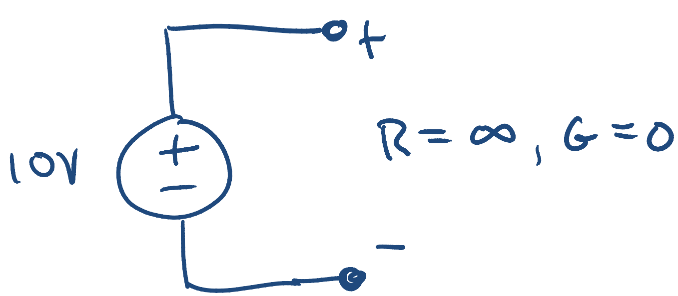
- Short Circuit: R = 0 Ohm



$$I = \frac{V}{R} = \frac{10}{0} = \infty \;!!$$

In practice, wire gets very hot
may melt – how fuses work

- Open Circuit: R = ∞ Ohm



$$I = \frac{V}{R} = \frac{10}{\infty} = 0 \;!!$$

no current will flow across an
open circuit

2.2 Nodes, Branches and Loops (1)

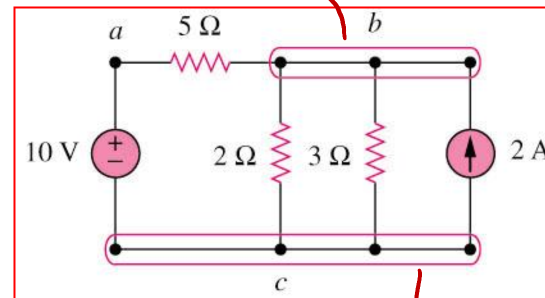
- A branch represents a single element such as a voltage source or a resistor.
- A node is the point of connection between two or more branches.
- A loop is any closed path in a circuit.

- A network with b branches, n nodes, and l independent loops will satisfy the fundamental theorem of network topology:

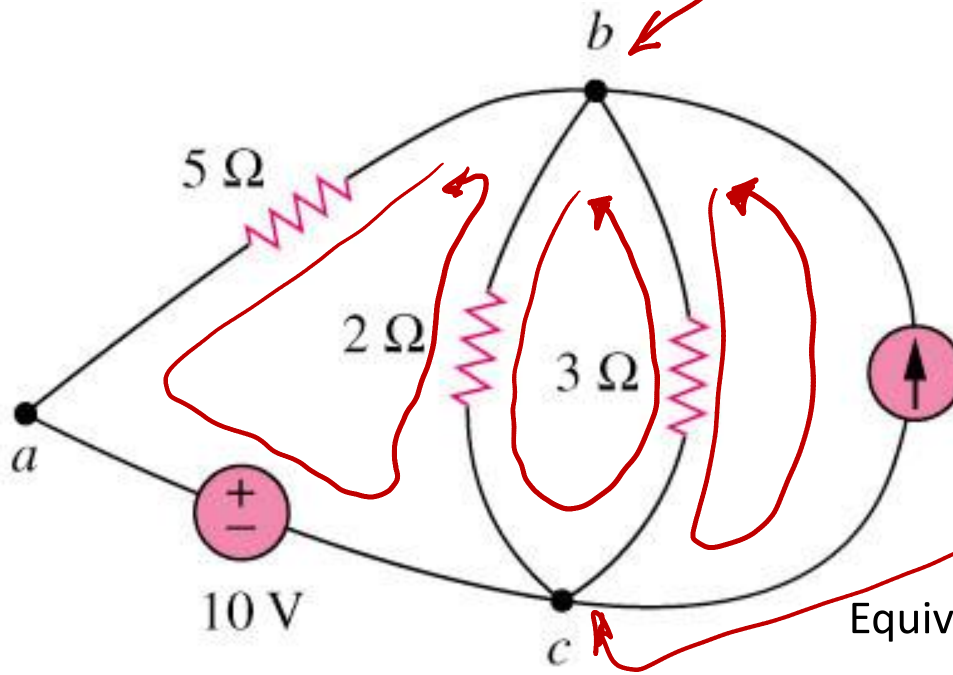
$$b = l + n - 1$$

2.2 Nodes, Branches and Loops (2)

Example 1



Original circuit



nodes connected by wire can be combined

Equivalent circuit

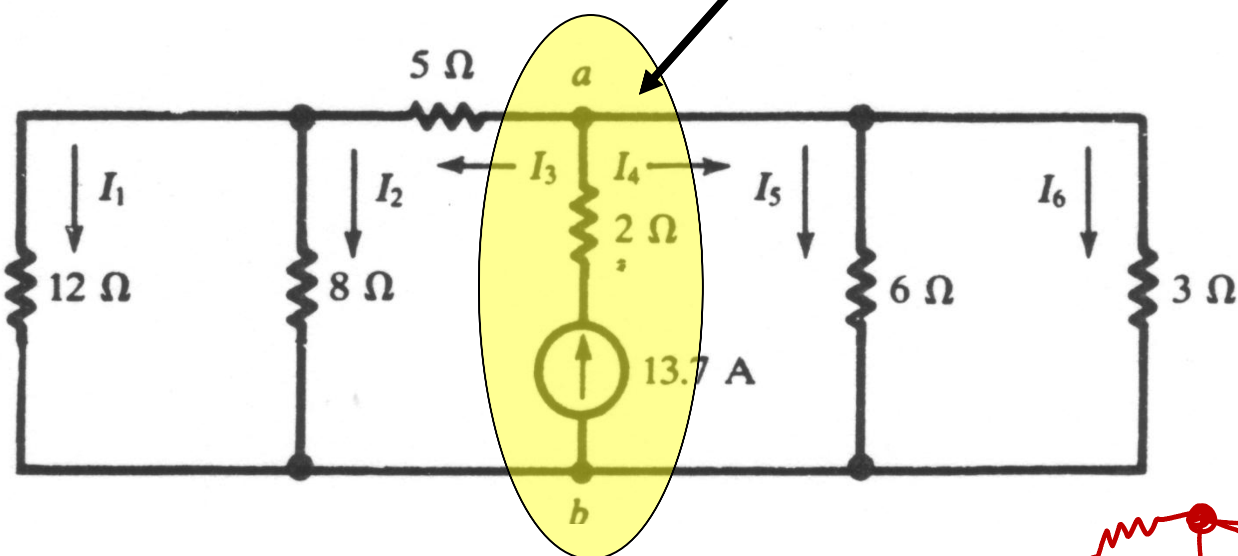
How many branches, nodes and loops are there?

5 = 3 + 3 − 1

2.2 Nodes, Branches and Loops (3)

Example 2

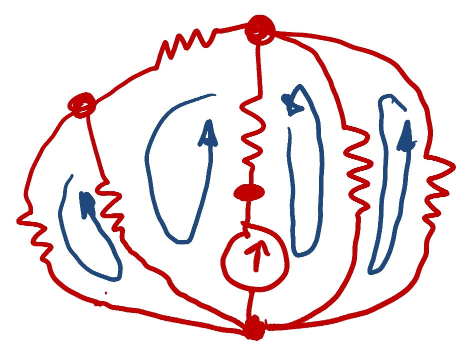
Should we consider it as one branch or two branches? either way is fine



How many branches, nodes and loops are there?

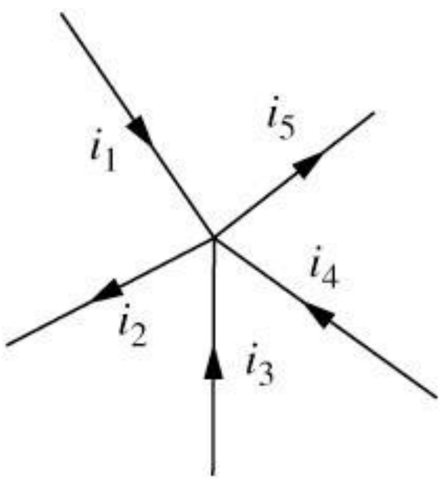
not combined : $7 = 4 + 4 - 1$

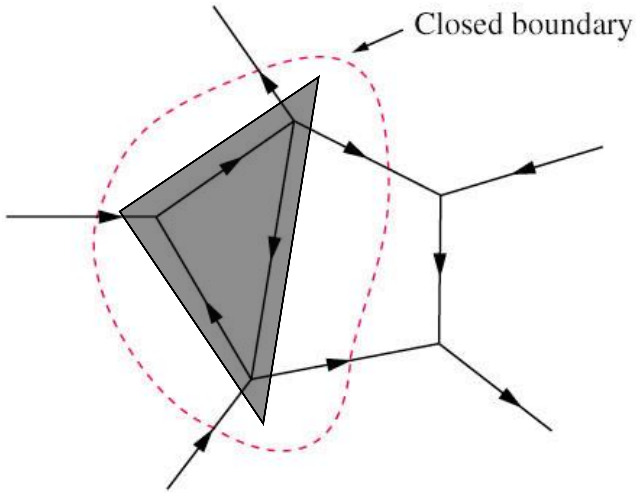
combined : $6 = 3 + 4 - 1$



2.3 Kirchhoff's Laws (1)

- Kirchhoff's current law (KCL) states that the algebraic sum of currents entering a node (or a closed boundary) is zero.





Mathematically,

$$\sum_{n=1}^{N} i_n = 0$$

2.3 Kirchhoff’s Laws (2)

Example 4

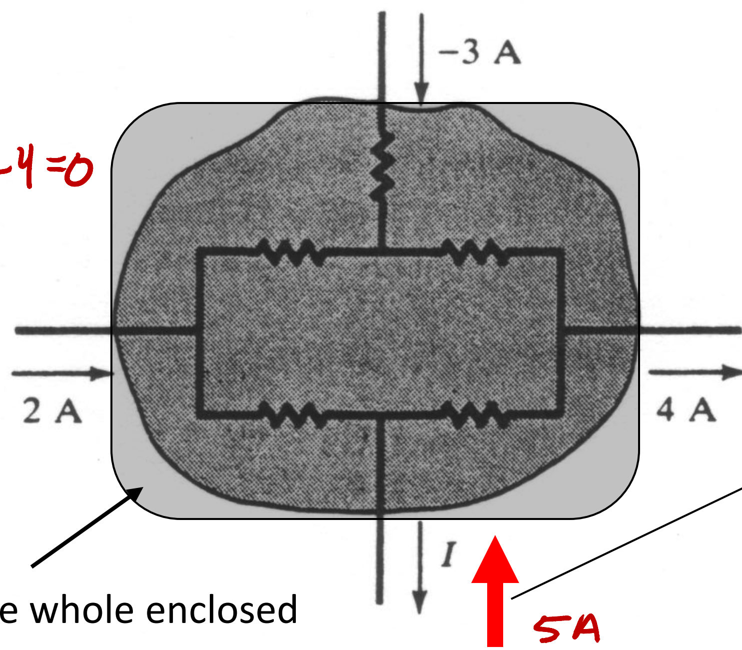
- Determine the current I for the circuit shown in the figure below.
  (Here we choose to use use positive for outbound currents)

KCL

$\Sigma i_n = 0$

$I - 2 - (-3) + 4 = 0$

$I = -5A$



This indicates that the actual current for I is flowing in the opposite direction.

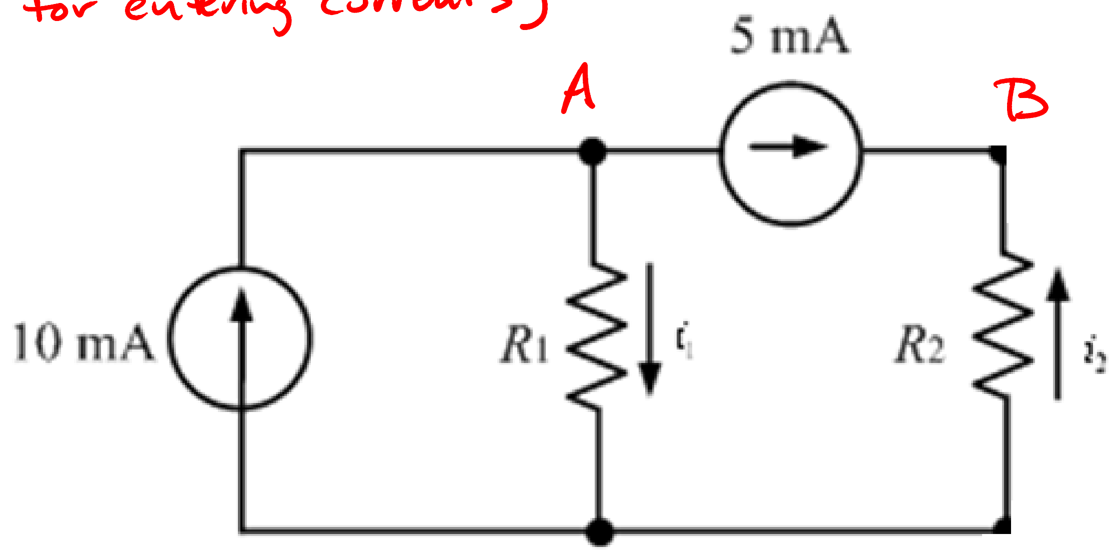
We can consider the whole enclosed area as one “node”.

Determine the current through each of the resistors in this circuit.

KCL at A (using + for entering currents)

$10 - i_1 - 5 = 0$

$i_1 = 5\text{ mA}$



KCL at B

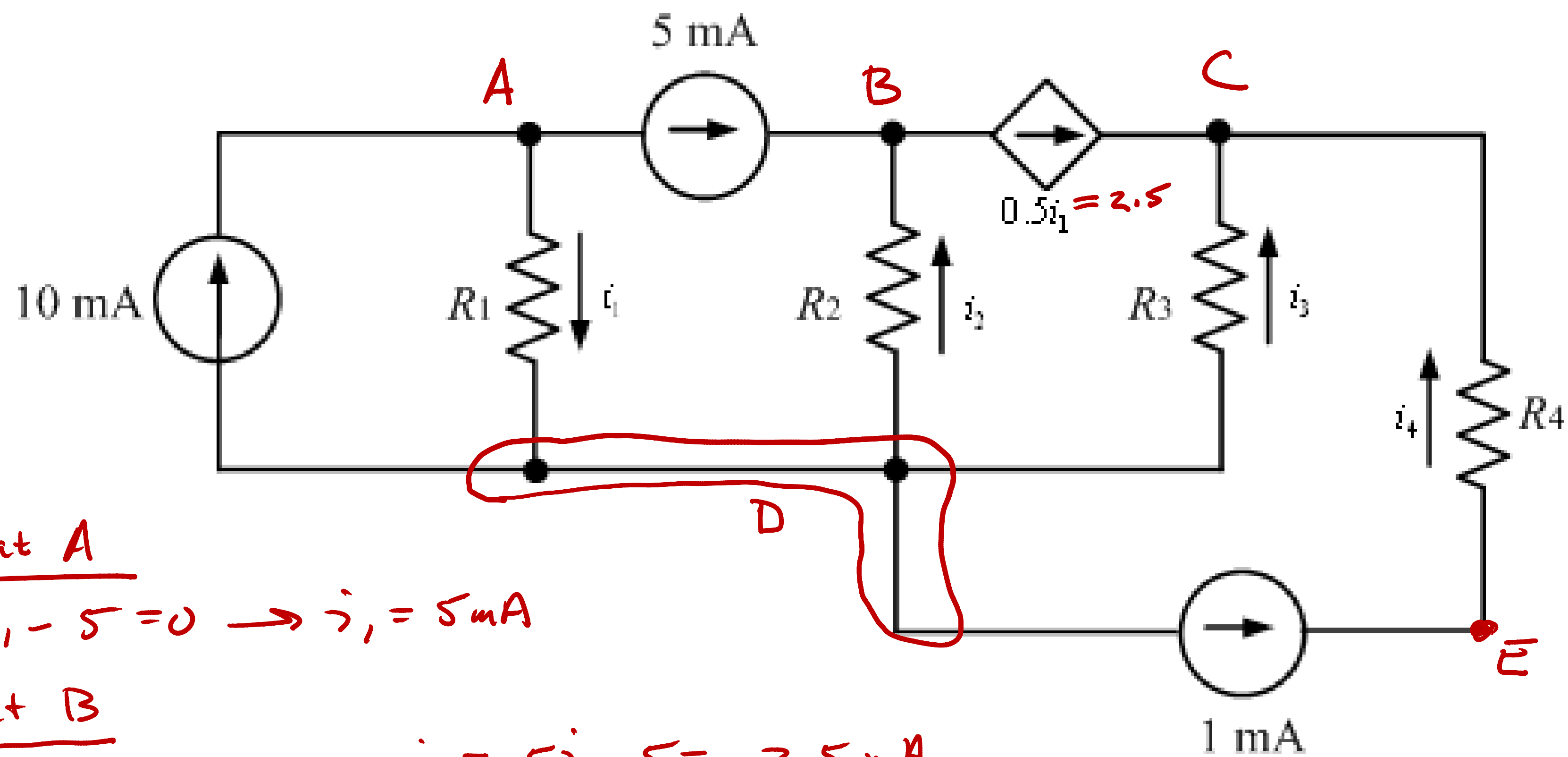
$5 + i_2 = 0$

$i_2 = -5\text{ mA}$

→ All components in a branch have same current

i1 = 5mA   i2 = -5 mA

Problem 1 Determine the current through each of the resistors in this circuit.



KCL at A

$10 - i_1 - 5 = 0 \rightarrow i_1 = 5\,mA$

KCL at B

$5 + i_2 - .5 i_1 = 0 \rightarrow i_2 = .5 i_1 - 5 = -2.5\,mA$

KCL at C

$2.5 + i_3 + i_4 = 0$

$i_3 = -3.5\,mA$

KCL at E

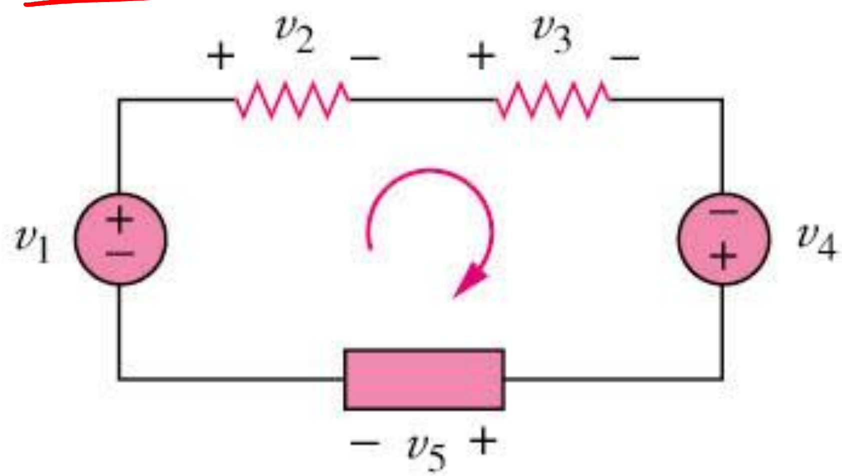
$-i_4 + 1 = 0 \rightarrow i_4 = 1\,mA$

i1 = 5mA, i2 = -2.5 mA, i3 = -3.5 mA, i4 = 1 mA

2.3 Kirchhoff's Laws (3)

- Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Convention: use first sign you encounter for each element: use clockwise path (ok to change but be consistent!)



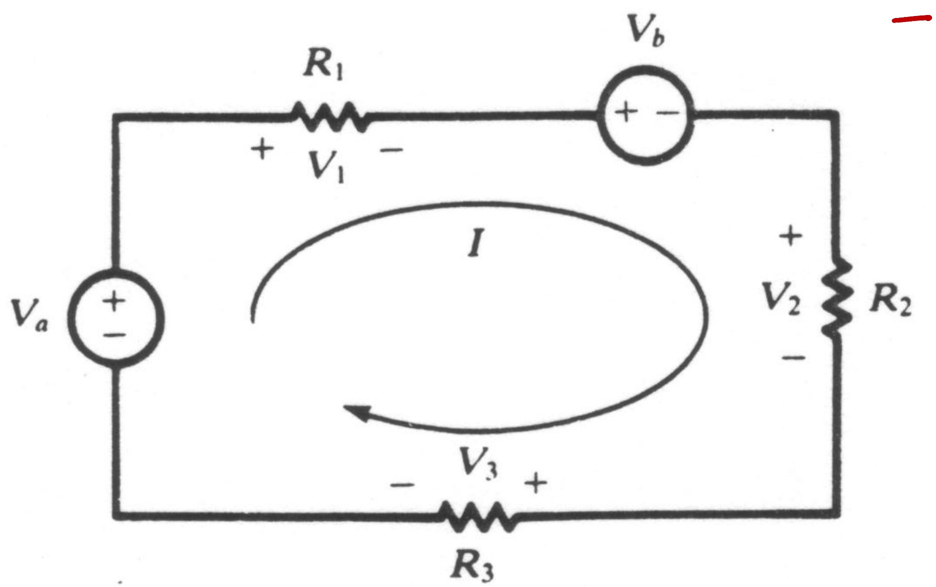
Mathematically, $\sum_{m=1}^{M} v_n = 0 = -v_1 + v_2 + v_3 - v_4 + v_5$

2.3 Kirchhoff's Laws (4)

Example 5

- Applying the KVL equation for the circuit of the figure below. (travelling clockwise, and using the 1st sign on each component we encounter along the path)

$$-V_a + V_1 + V_b + V_2 + V_3 = 0$$

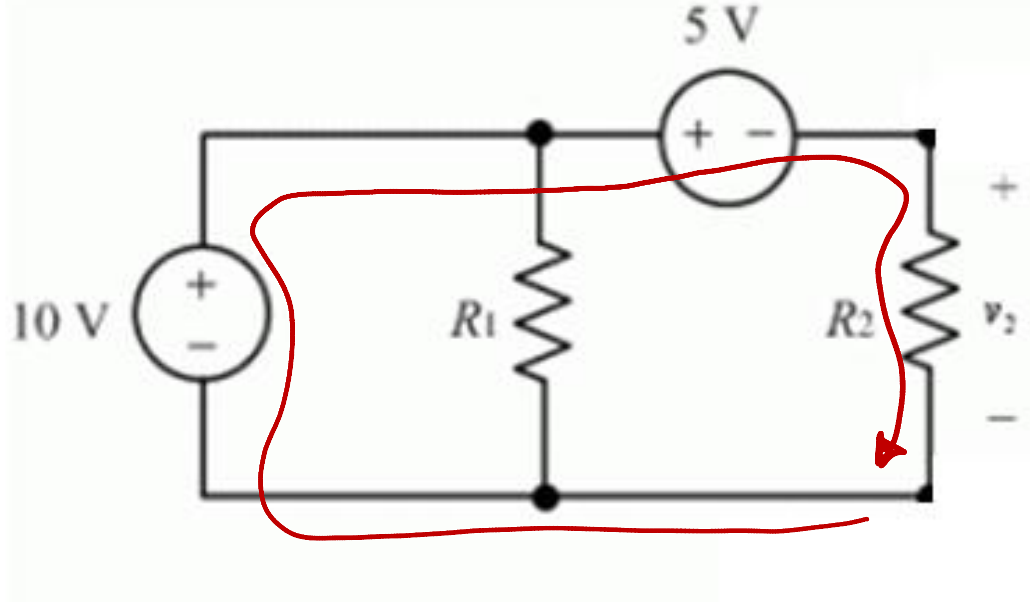


Find the voltage across resistor R2.

KVL on outer loop:

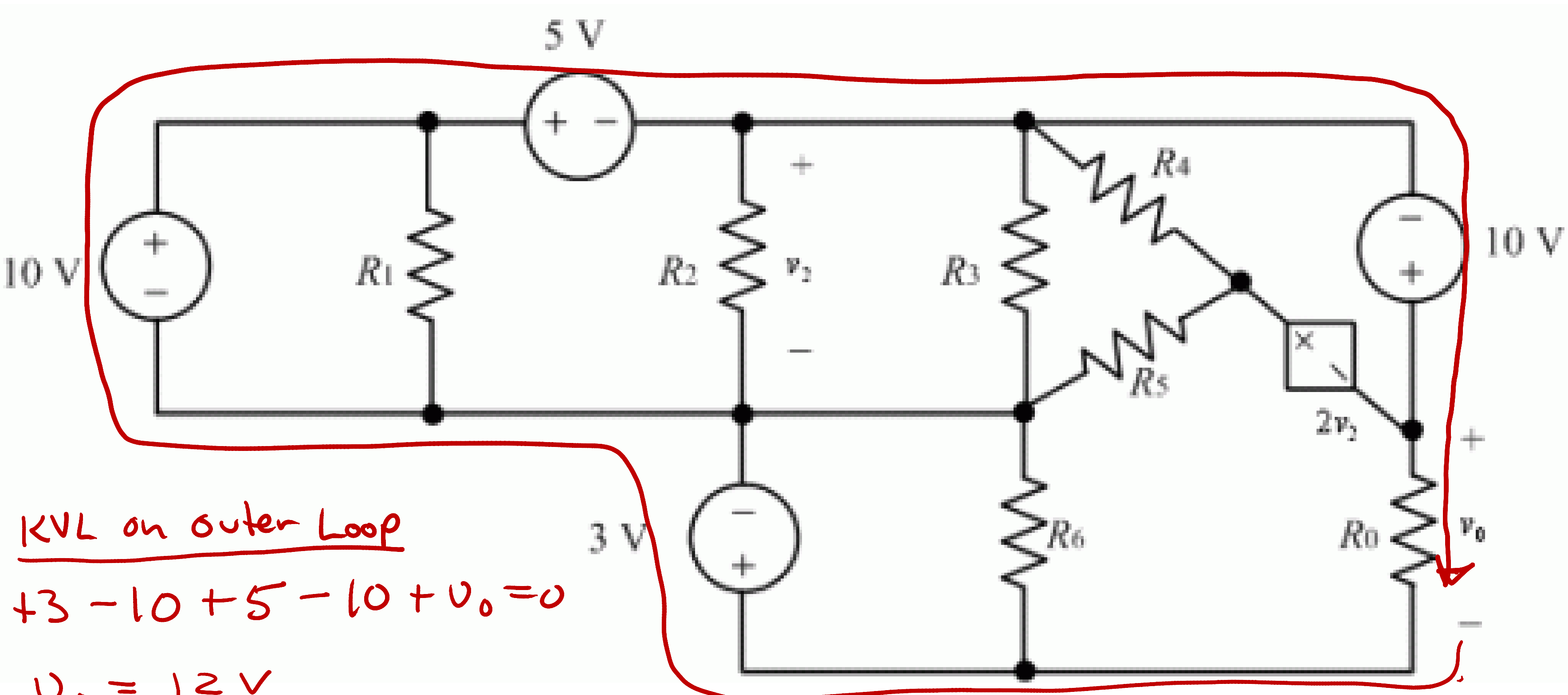
$$-10 + 5 + V_2 = 0$$

$$V_2 = 5\ V$$



v2 = 5 V

Problem 1 Find the voltage across resistor R0.



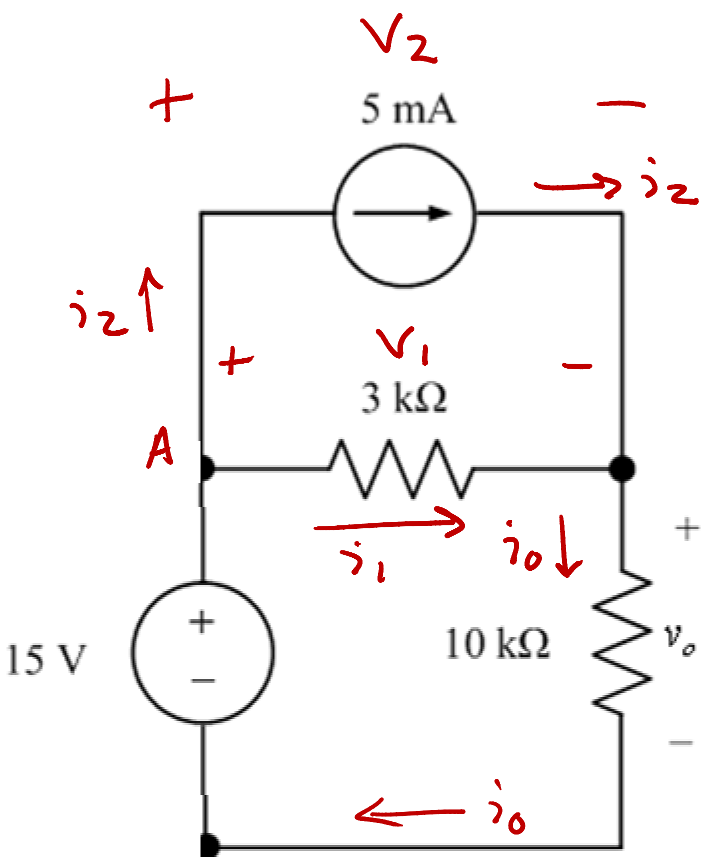
KVL on outer Loop

$+3 - 10 + 5 - 10 + v_0 = 0$

$v_0 = 12\,V$

v0 = 12V

KCL and KVL in one circuit
Find the value of V0.



Plan

1) Label all unknowns (use PSC!)
2) Write all OL, KVL, KCL equations
3) Solve system of eqns

① OL

a) $V_1 = 3 i_1$

b) $V_0 = 10 i_0$

② KVL

a) $-V_1 + V_2 = 0 \rightarrow V_1 = V_2$

b) $-15 + V_1 + V_0 = 0$

c) $-15 + V_2 + V_0 = 0$

③ KCL

a) $i_0 - i_1 - i_2 = 0 \rightarrow i_0 - i_1 = 5$

b) $i_2 - 5 = 0 \rightarrow i_2 = 5$

④ Substitute 1a + 1b into 2b

$10 i_0 + 3 i_1 = 15$

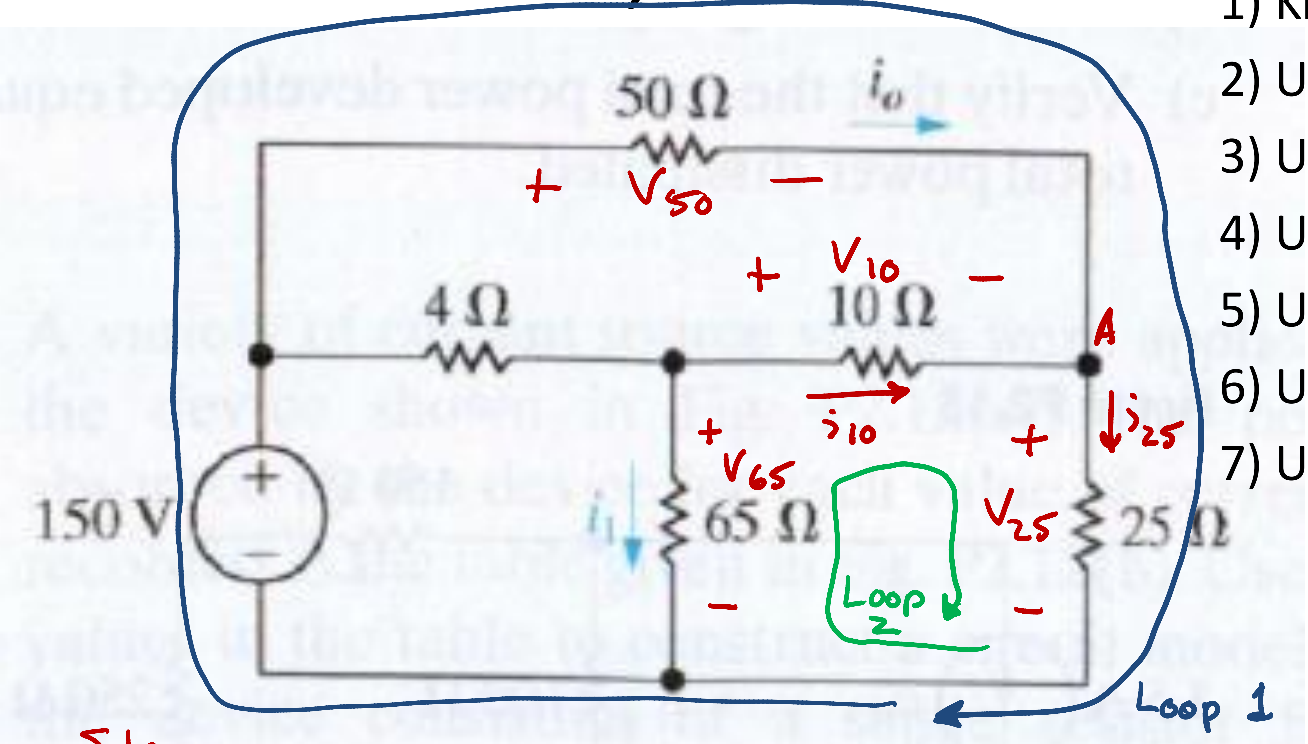
(3a) $i_0 - i_1 = 5$

$\xrightarrow{3 \times (3a) + 4}$ $13 i_0 = 30$, $i_0 = 2.308$

$V_0 = 10 i_0 = \underline{23.1 V}$

v0 = 23.1 V

Given io is 1A, Find i1



Plan

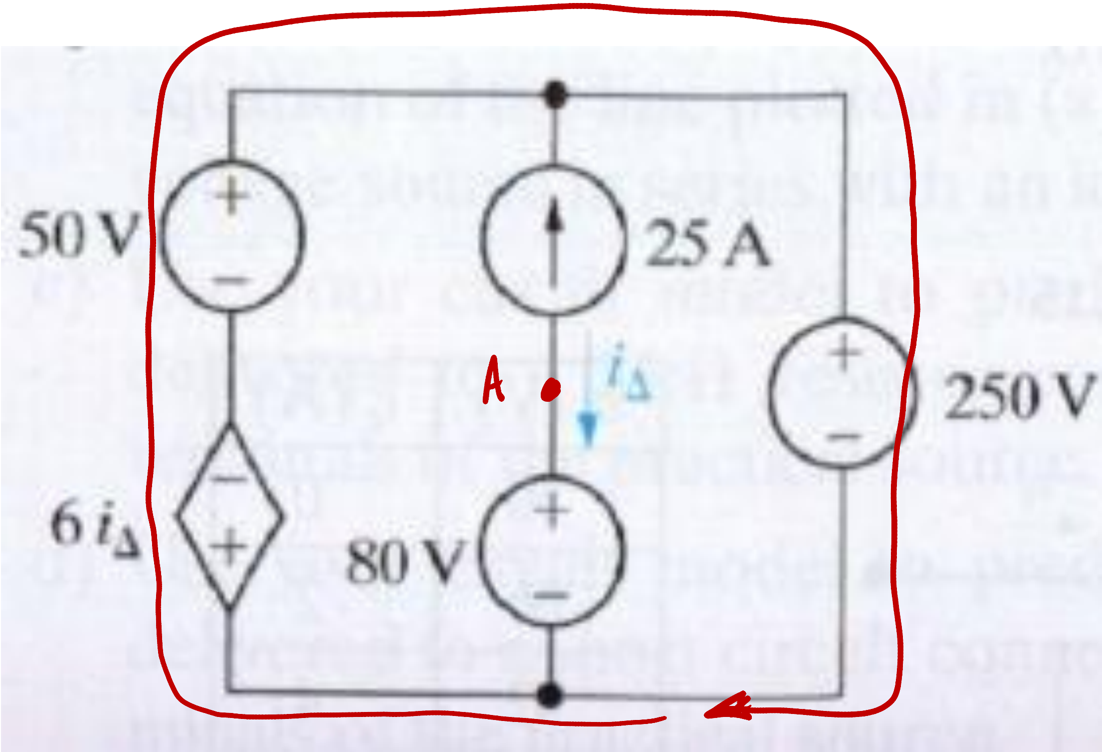
1) Knowing io use OL to find V_{50}
2) Use KVL to find V_{25} (Loop 1)
3) Use OL to find i25
4) Use KCL to find i10 (Node A)
5) Use OL to find V10
6) Use KVL to find V65 (Loop 2)
7) Use OL to find i1

Step

1) $V_{50} = 50 i_o = 50V$
2) $-150 + V_{50} + V_{25} = 0 \rightarrow V_{25} = 100V$
3) $i_{25} = V_{25}/25 = 4A$
4) $i_o + i_{10} - i_{25} = 0 \rightarrow i_{10} = i_{25} - i_o = 3A$
5) $V_{10} = i_{10}(10) = 30V$
6) $-V_{65} + V_{10} + V_{25} = 0 \rightarrow V_{65} = 30 + 100 = 130V$
7) $i_1 = V_{65}/65\Omega = 2A$ ✓

i1 = 2A

Is the interconnection valid?



KCL at A

$i_\Delta + 25A = 0 \rightarrow i_\Delta = -25A$

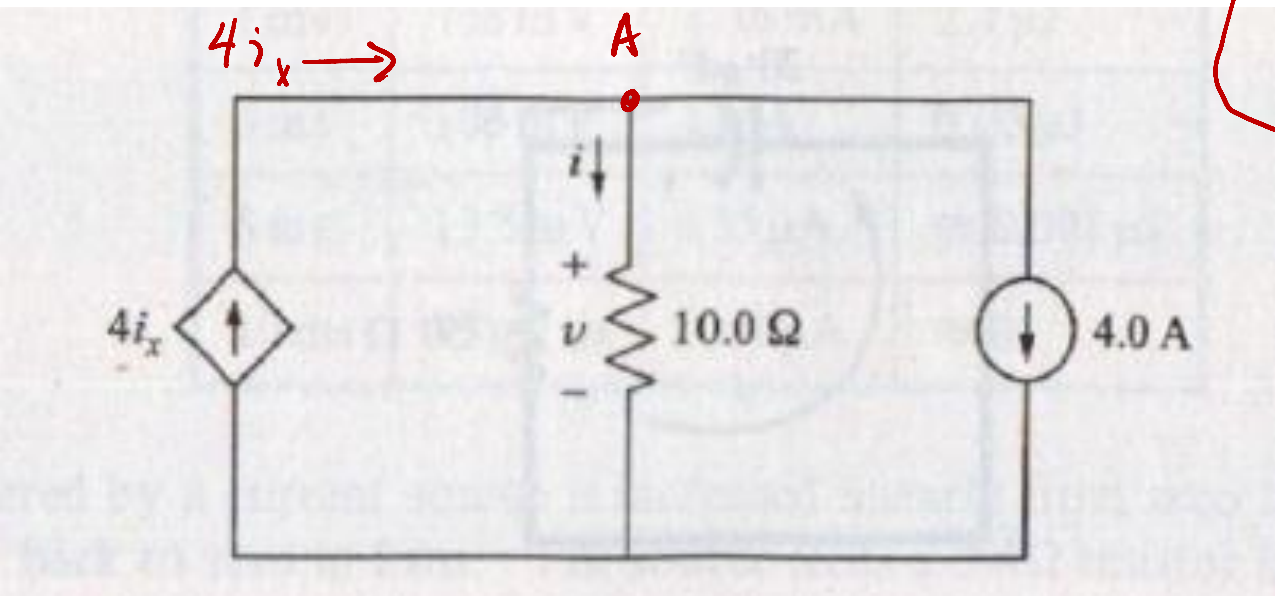
KVL outer loop

$6 i_\Delta - 50 + 250 = 0$

$i_\Delta = \frac{-200}{6} = -33.3 \neq -25$

∴ interconnect **NOT** valid

Find v if ix = - 3A



KCL at A

$4i_x = i + 4$

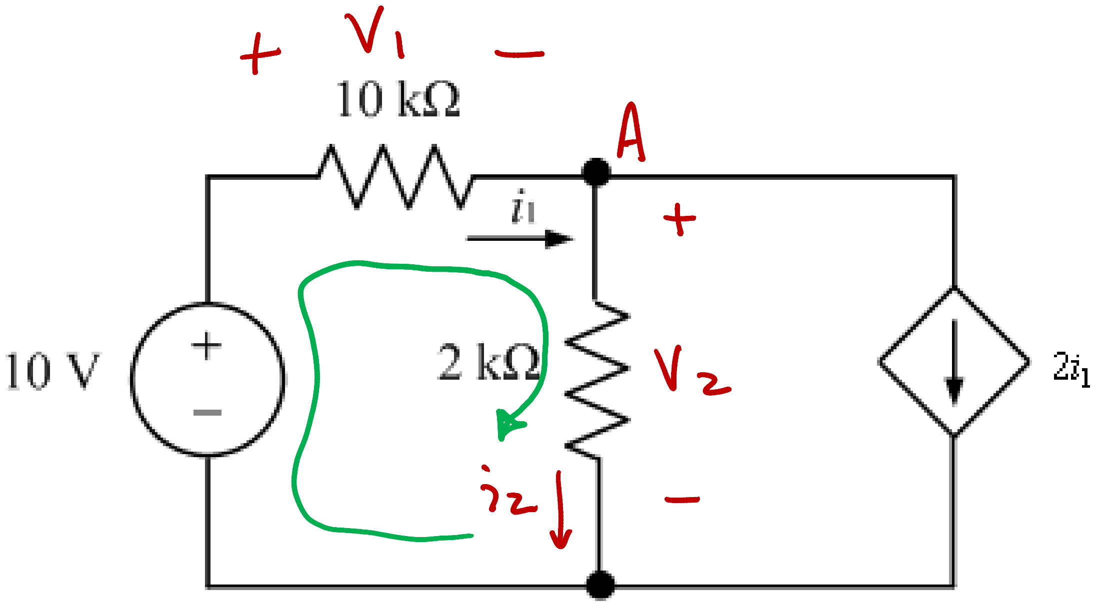
$-12 = i + 4$

$i = -16\,A$

OL

$V = 10\,i = -160\,V$

Problem 2 Find the current through the 10 kΩ resistor.



KCL at A

$i_1 = i_2 + 2i_1 \rightarrow i_2 = -i_1$

KVL

$-10 + V_1 + V_2 = 0$

OL

$V_1 = 10\, i_1$

$V_2 = 2\, i_2 = -2 i_1$

Solving

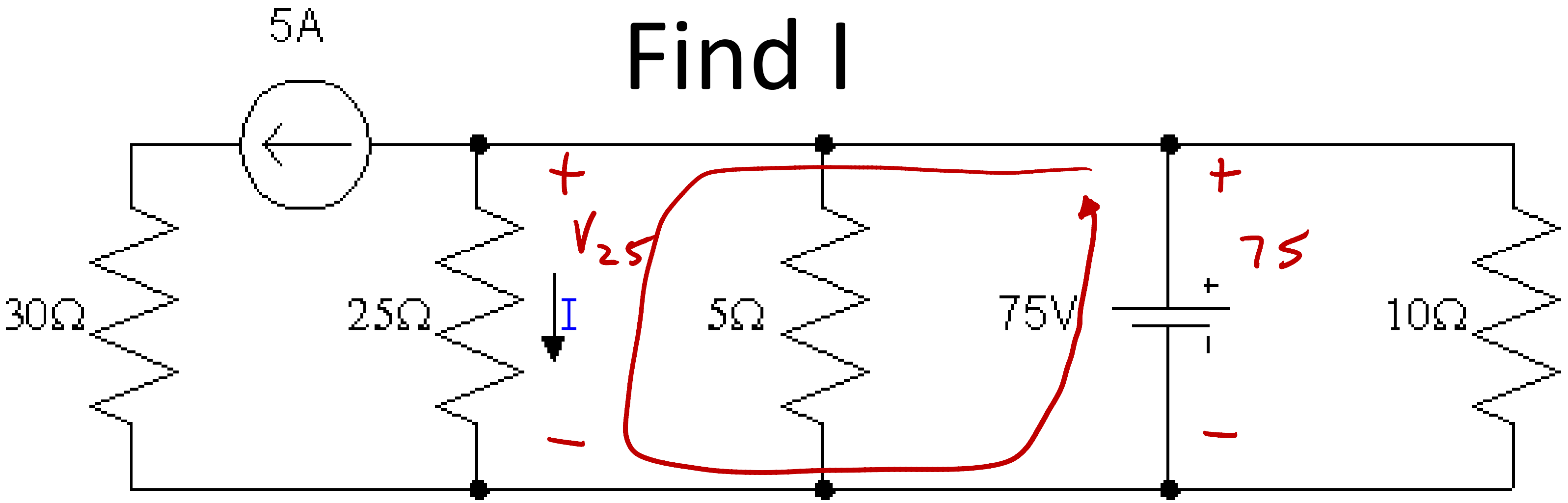
$V_1 + V_2 = 10$

$10 i_1 - 2 i_1 = 10$

$i_1 = \frac{10}{12} = .833 \text{ mA}$

Further Drills on KCL/KVL

Find I



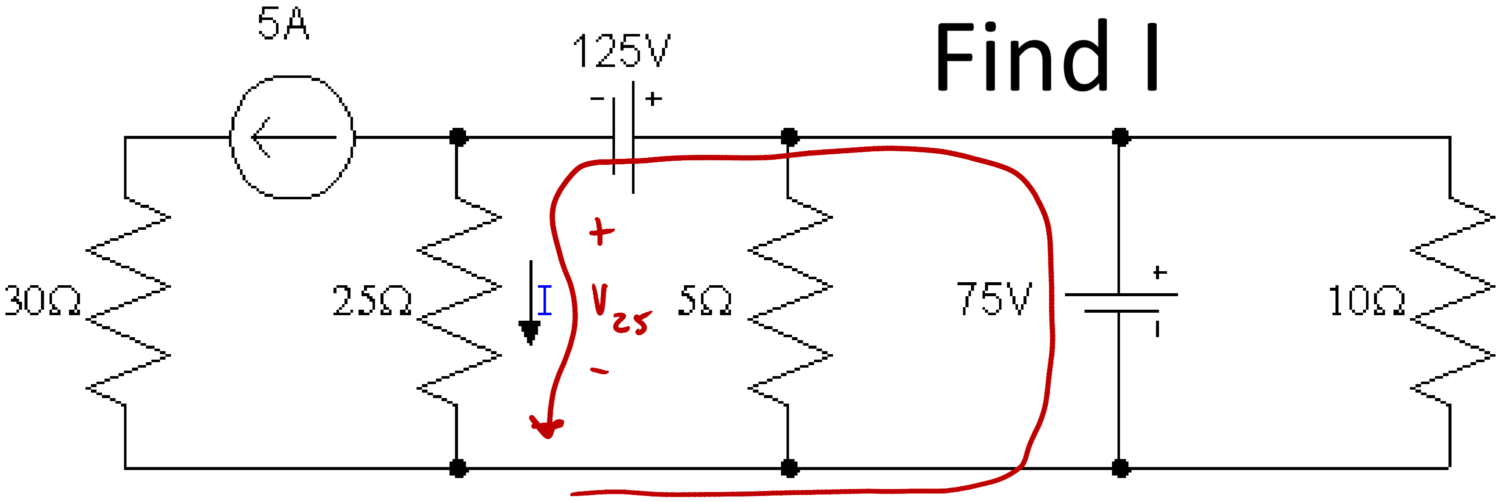
KVL

$$-75 + V_{25} = 0 \rightarrow V_{25} = 75V$$

OL

$$I = \frac{V_{25}}{25} = 3A$$

Find I



KVL

$$-75 + 125 + V_{25} = 0 \rightarrow V_{25} = -50$$

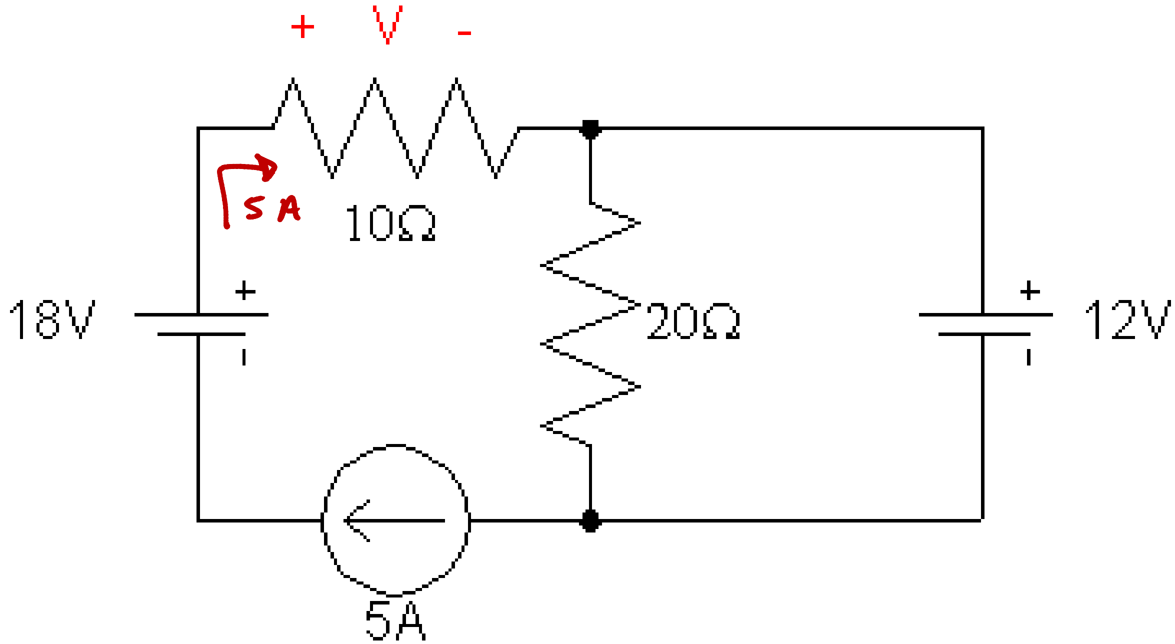
OL

$$I = \frac{-50}{25} = -2A$$

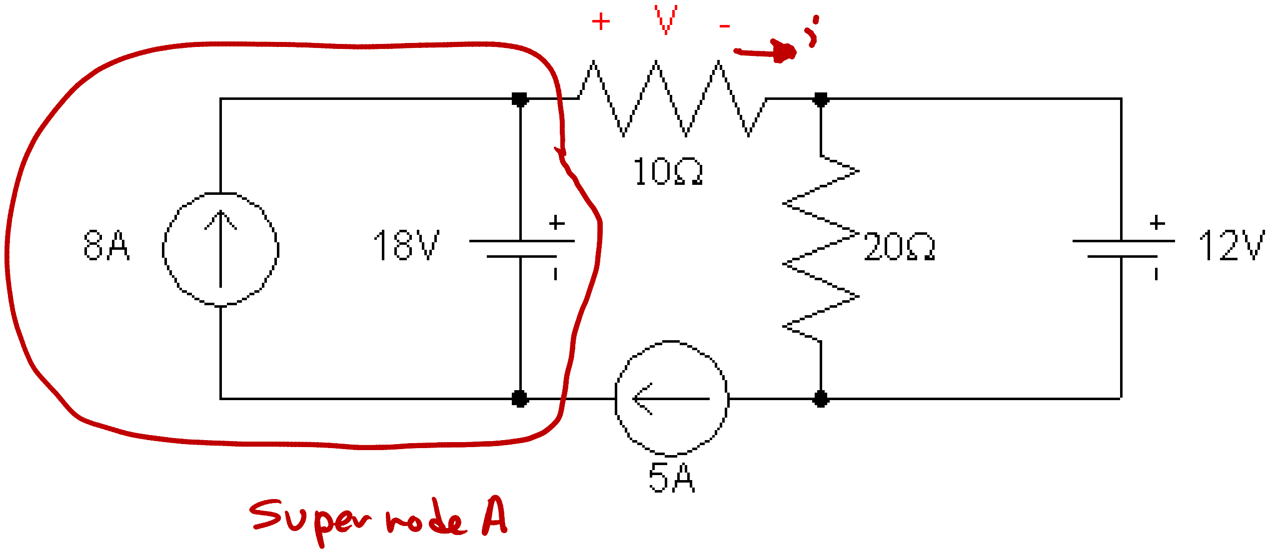
Find V

OC

$V = 10(5) = 50V$



Find V



KCL at Super Node A

$-5 + i = 0$

$i = 5\,A$

$V = 5(10) = 50\,V$

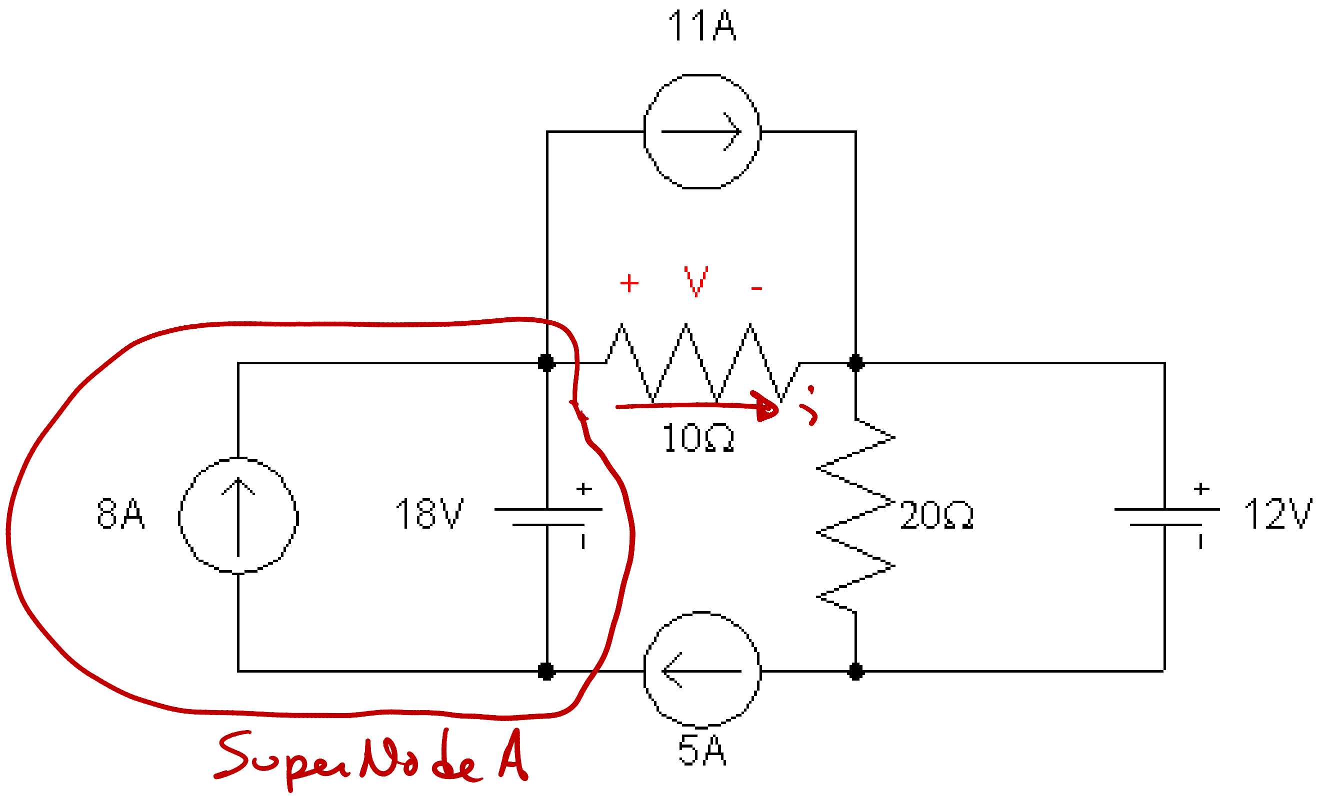
Find V

KCL S.N.A

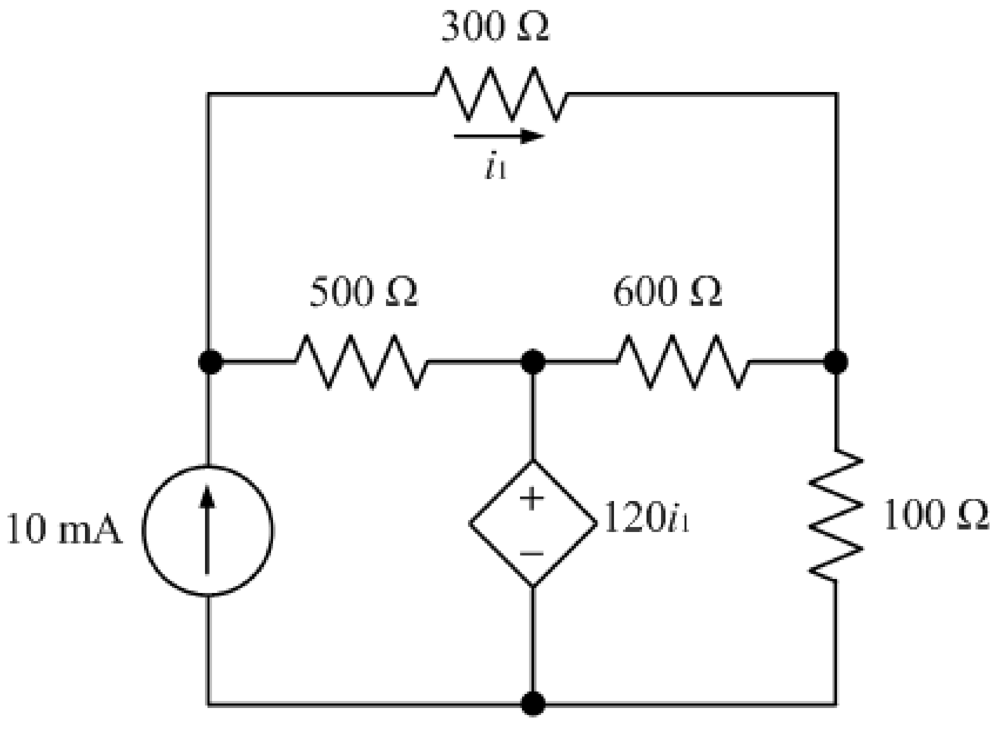
$5 - 11 - i = 0$

$i = -6A$

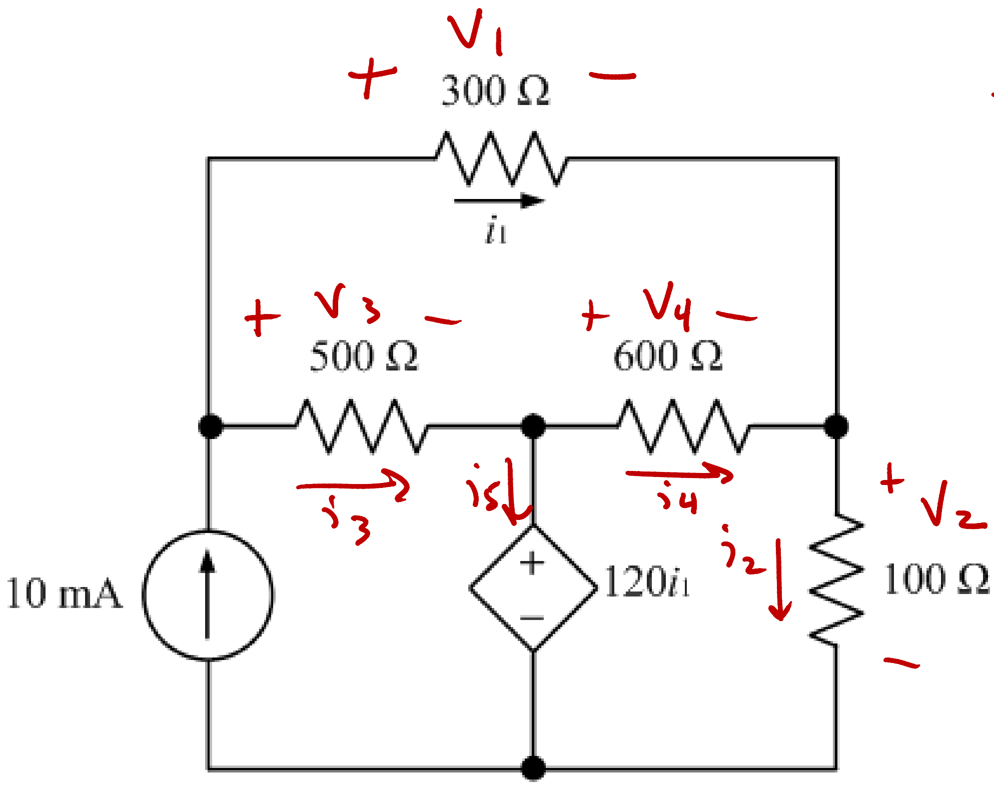
$V = 10(-6) = -60V$



Problem 3 Find the current through the 300 Ω resistor. HARD!!



Problem 3 Find the current through the 300 Ω resistor. HARD!!



KCL

$i_1 + i_3 = 10\text{ mA}$

$i_1 + i_4 = i_2$

$i_4 + i_5 = i_3$

KVL/OL

$-120 i_1 - 500 i_3 + 300 i_1 + 100 i_2 = 0$

$-120 i_1 + 600 i_4 + 100 i_2 = 0$

System of Eqs

$$-120 i_1 + 100 i_2 + 600 i_4 = 0$$
$$180 i_1 + 100 i_2 - 500 i_3 = 0$$
$$i_1 + i_3 = 10$$
$$i_1 - i_2 + i_4 = 0$$
$$-i_3 + i_4 + i_5 = 0$$

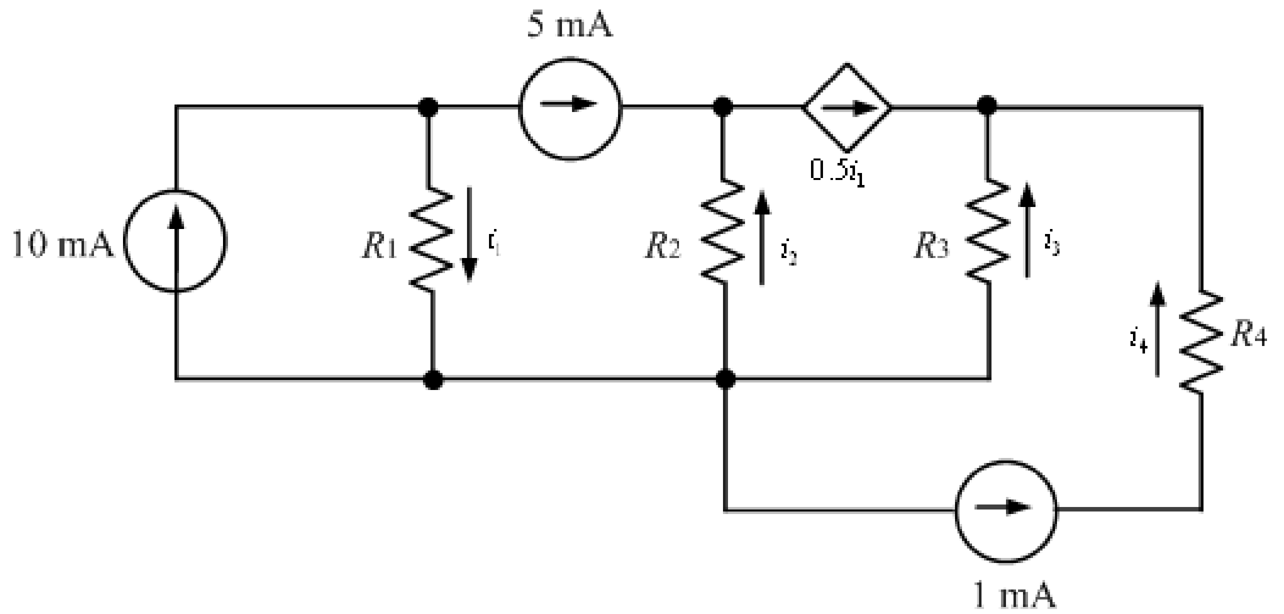
Matrix Form

$$A = \begin{bmatrix} -120 & 100 & 0 & 600 & 0 \\ 180 & 100 & -500 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

$$I = A \backslash b = \begin{bmatrix} 6.39 \\ 6.57 \\ 3.61 \\ 0.18 \\ 3.43 \end{bmatrix} \text{ mA}$$

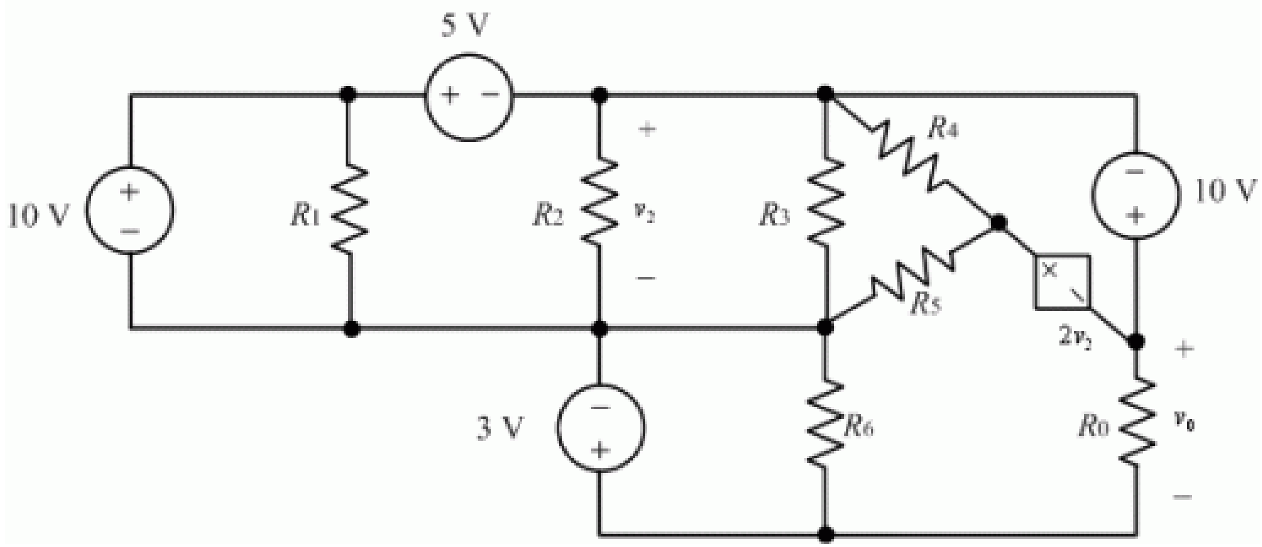
Handouts Day 1

1) Problem 1 Determine the current through each of the resistors in this circuit.

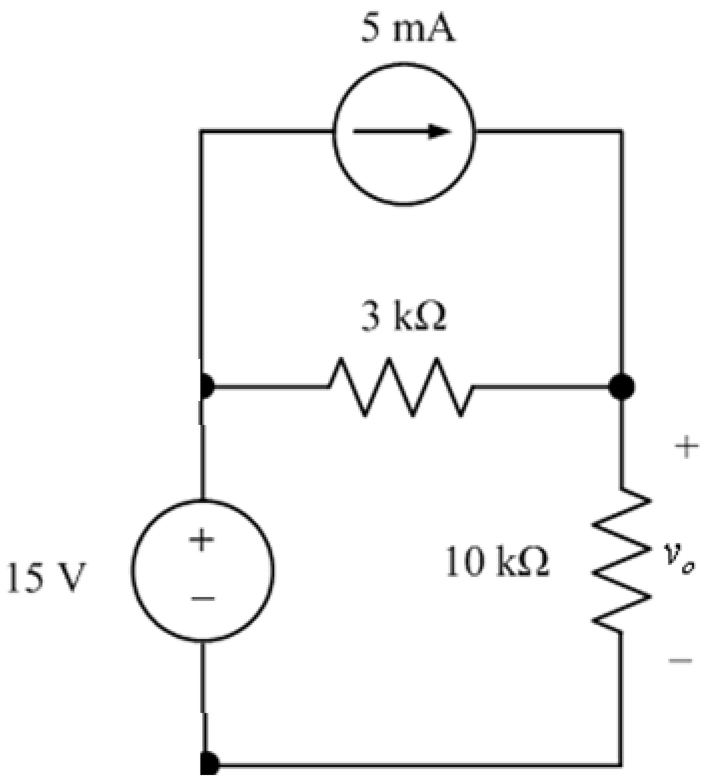


i1 = 5mA, i2 = -2.5 mA, i3 = -3.5 mA, i4 = 1 mA

2) Problem 1 Find the voltage across resistor R0. (12V)

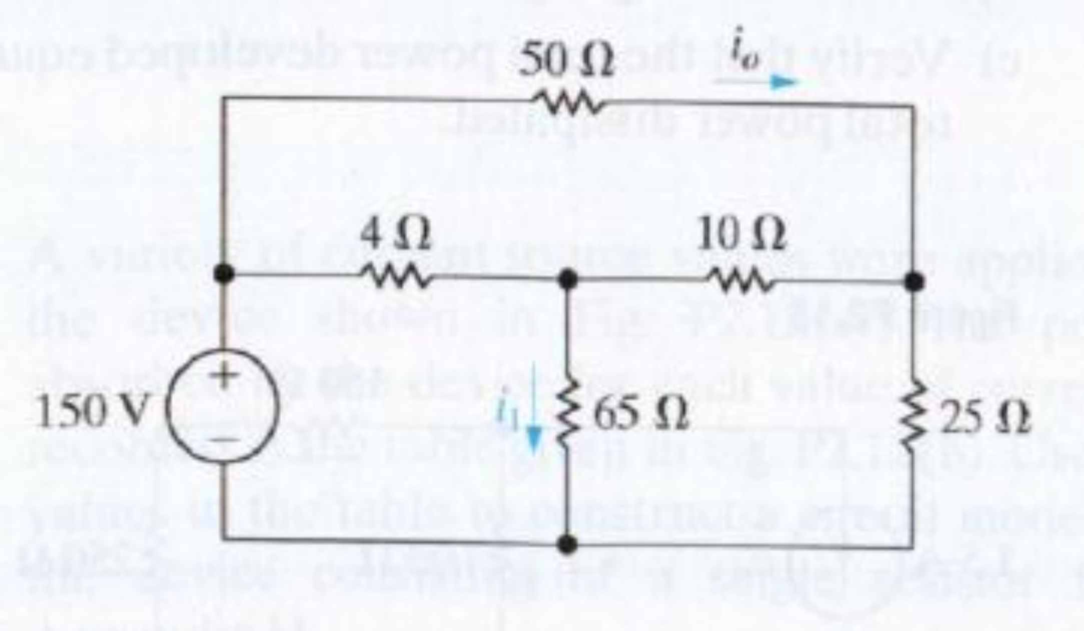


3) KCL and KVL in one circuit
Find the value of V0.



v0 = 23.1 V

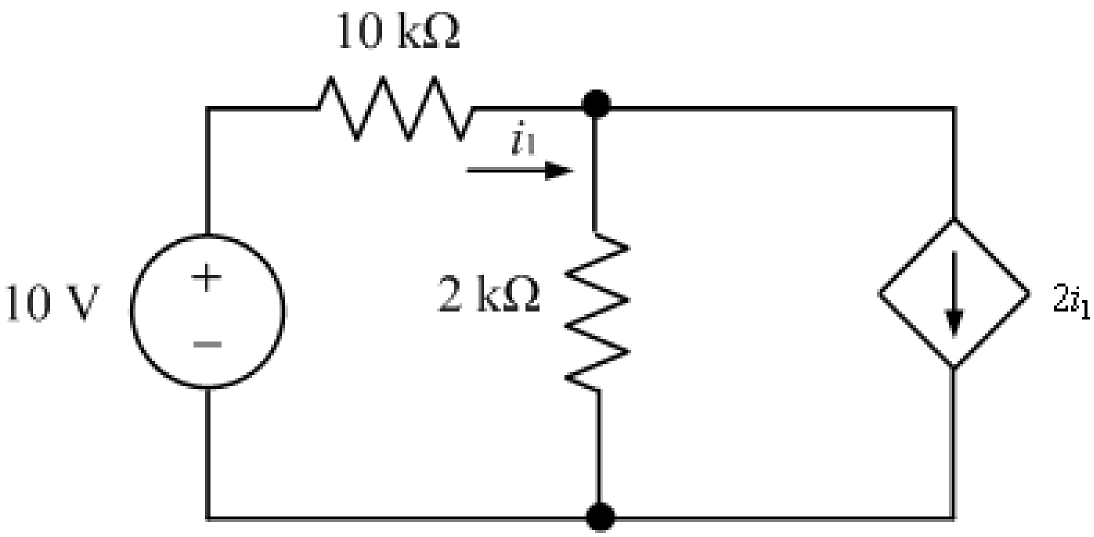
4) If io is 1A, Find i1



i1 = 2A

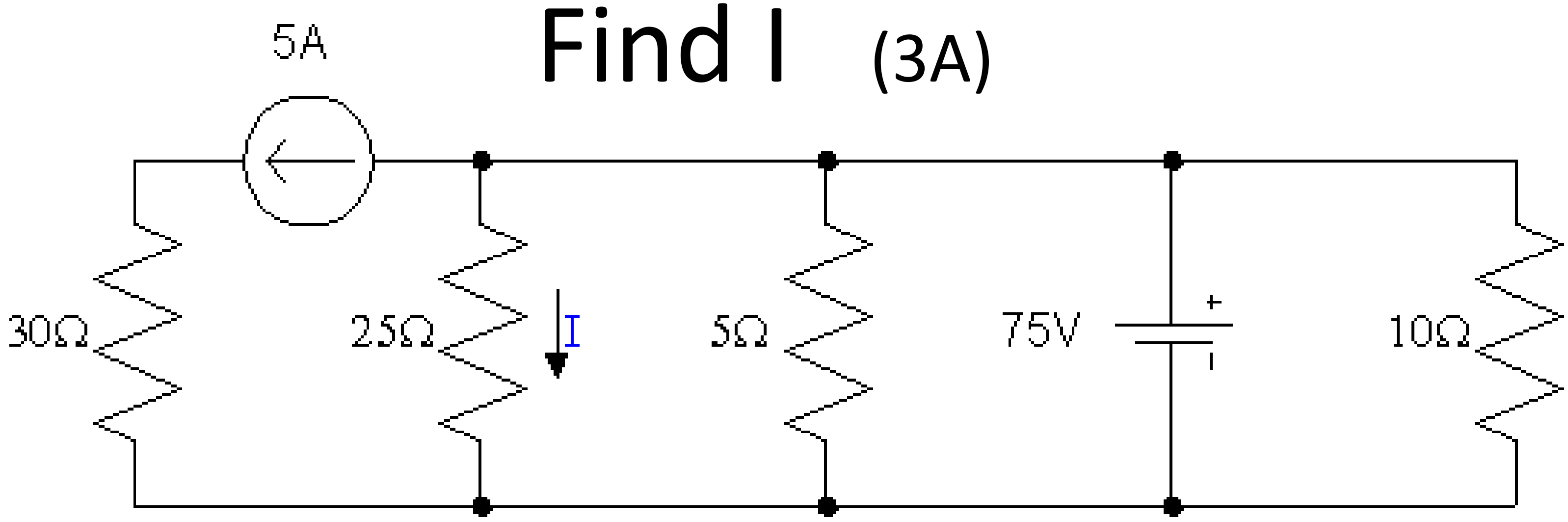
Handouts Day 2

1) Problem 2 Find the current through the 10 kΩ resistor.



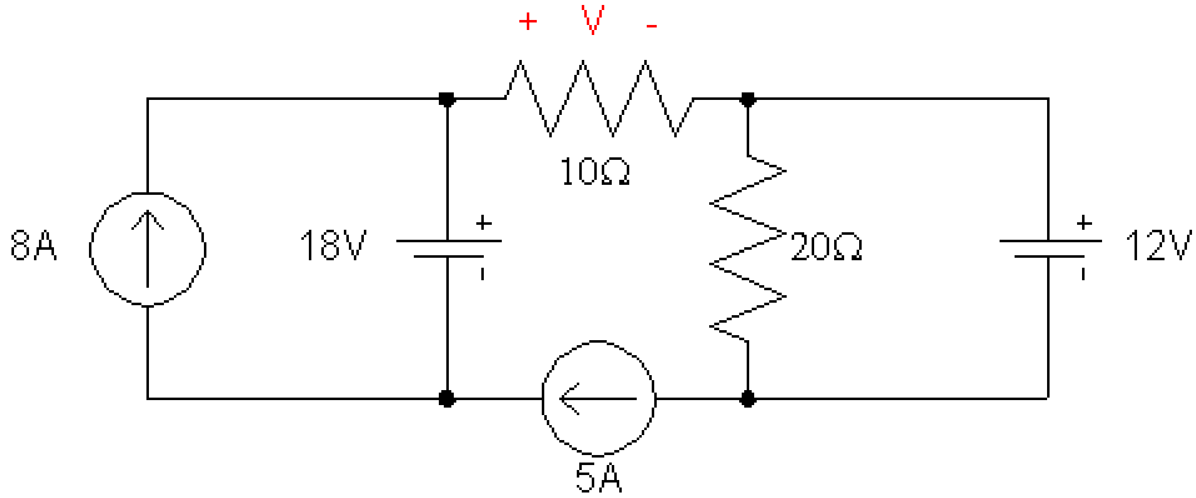
2)

Find I (3A)



3)

Find V



4)

Find V