EE2003
Circuit Theory

Chapter 13

Magnetically Coupled Circuits

Magnetically Coupled Circuit
Chapter 13

13.1 What is a transformer?

13.2 Mutual Inductance

13.3 Energy in a Coupled Circuit

13.4 Linear Transformers

13.5 Ideal Transformers

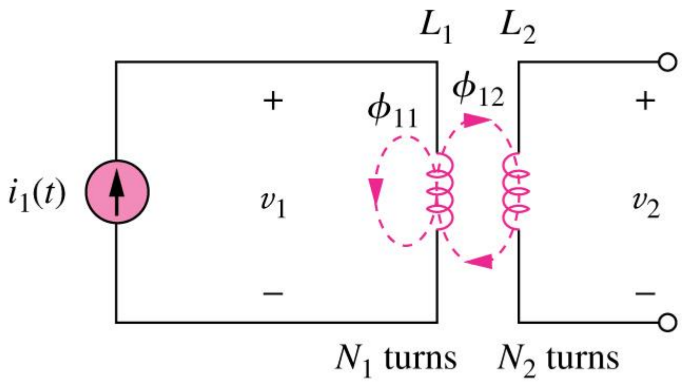
13.6 Applications

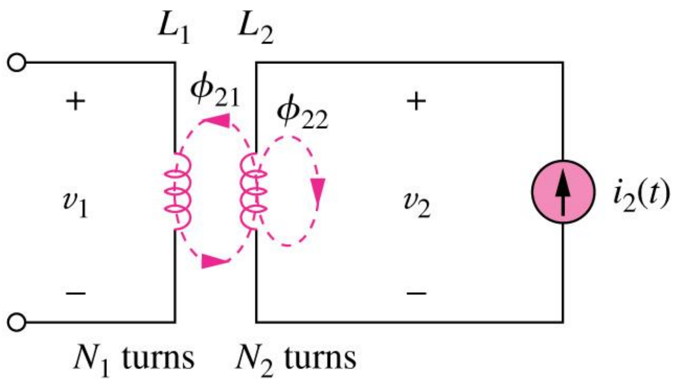
13.1 What is a transformer? (1)

- It is an electrical device designed on the basis of the concept of magnetic coupling
- It uses magnetically coupled coils to transfer energy from one circuit to another
- It is the key circuit elements for stepping up or stepping down ac voltages or currents, impedance matching, isolation, etc.

13.2 Mutual Inductance (1)

- It is the ability of one inductor to induce a voltage across a neighboring inductor, measured in henrys (H).





$$v_2 = M_{21}\frac{di_1}{dt}$$

$$v_1 = M_{12}\frac{di_2}{dt}$$

The open-circuit mutual voltage across coil 2

The open-circuit mutual voltage across coil 1

13.2 Mutual Inductance (2)

- If a current enters the dotted terminal of one coil, the reference polarity of the mutual voltage in the second coil is positive at the dotted terminal of the second coil.

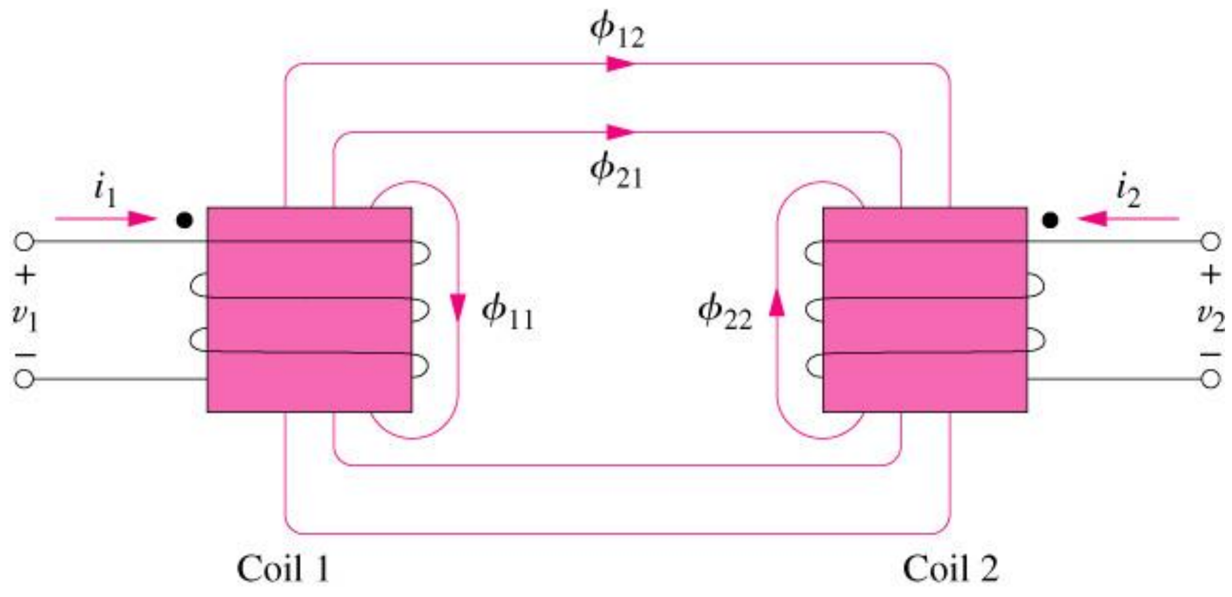
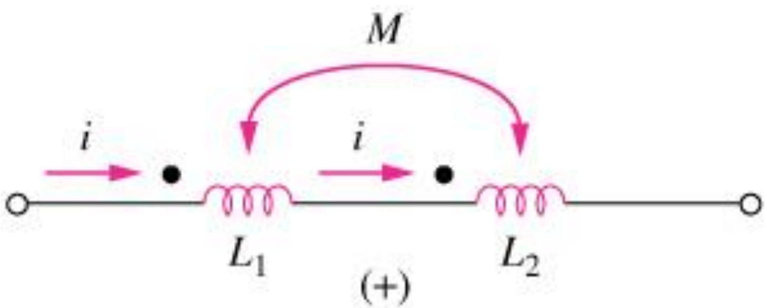


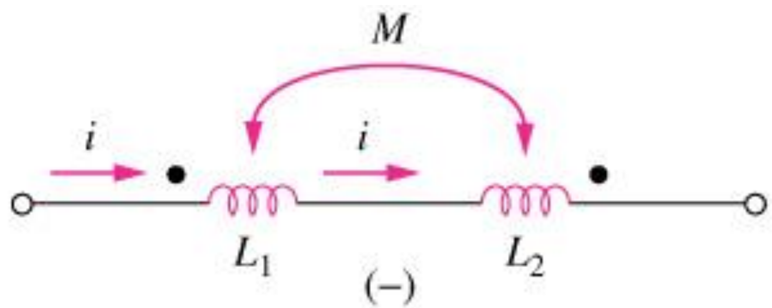
Illustration of the dot convention.

13.2 Mutual Inductance (3)

Dot convention for coils in series; the sign indicates the polarity of the mutual voltage; (a) series-aiding connection, (b) series-opposing connection.



(a)



(b)

$$L = L_1 + L_2 + 2M$$
(series - aiding connection)

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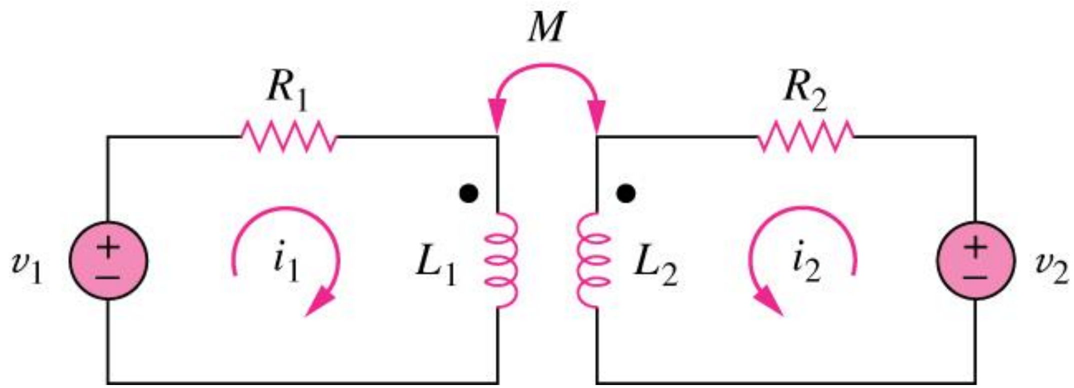
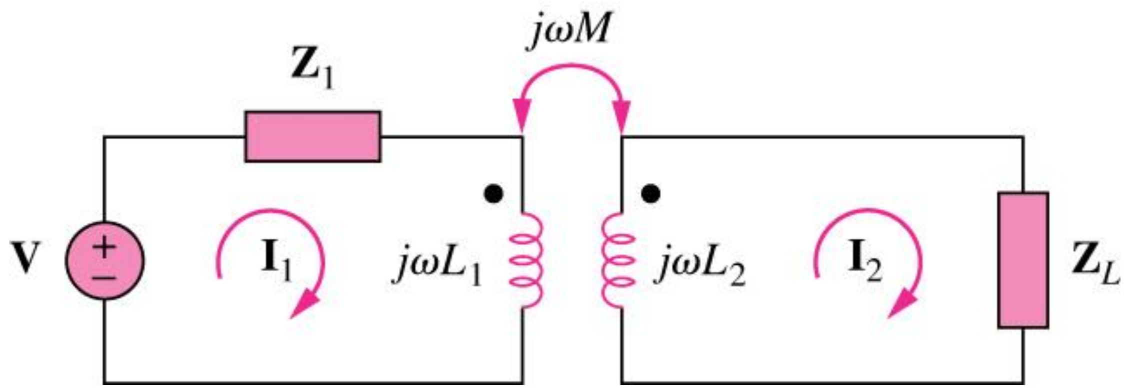
(series - aiding connection)

13.2 Mutual Inductance (4)



Time-domain
analysis of a circuit containing coupled coils.

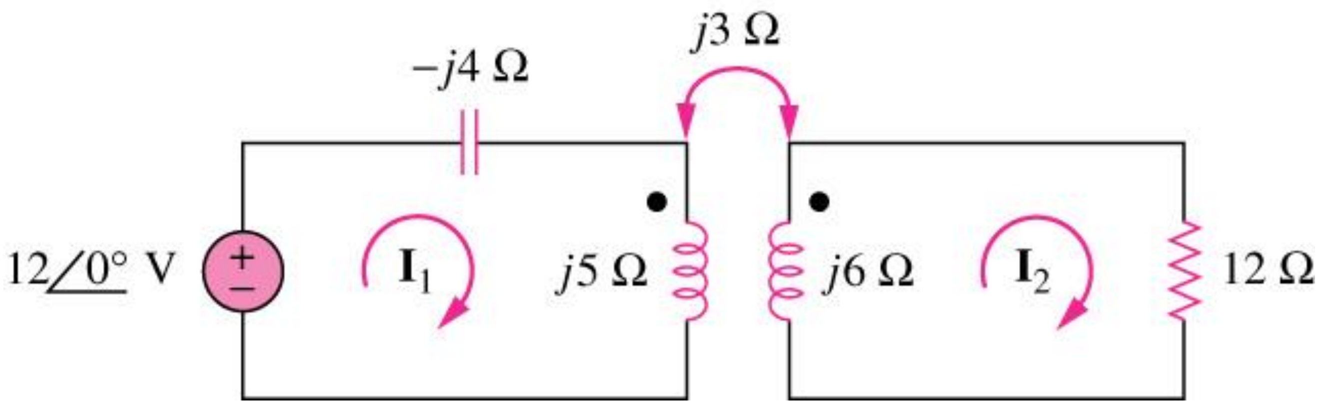


Frequency-domain
analysis of a circuit containing coupled coils

13.2 Mutual Inductance (5)

Example 1

Calculate the phasor currents I_1 and I_2 in the circuit shown below.

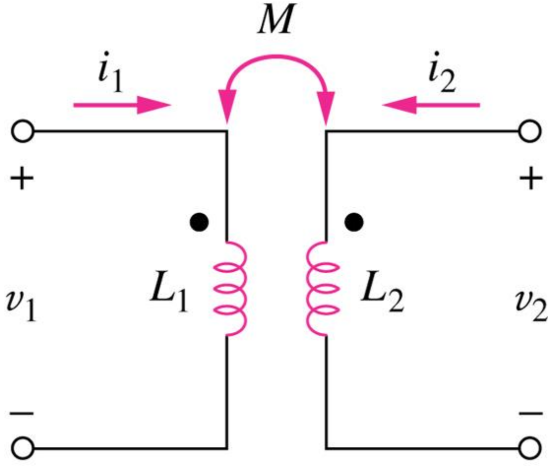


Ans: $I_1 = 13.01\angle -49.39°\text{A};\ \ I_2 = 2.91\angle 14.04°\text{A}$

*Refer to in-class illustration, textbook

13.3 Energy in a Coupled Circuit (1)

- The coupling coefficient, k, is a measure of the magnetic coupling between two coils; $0 \le k \le 1$.



$$M = k\sqrt{L_1 L_2}$$

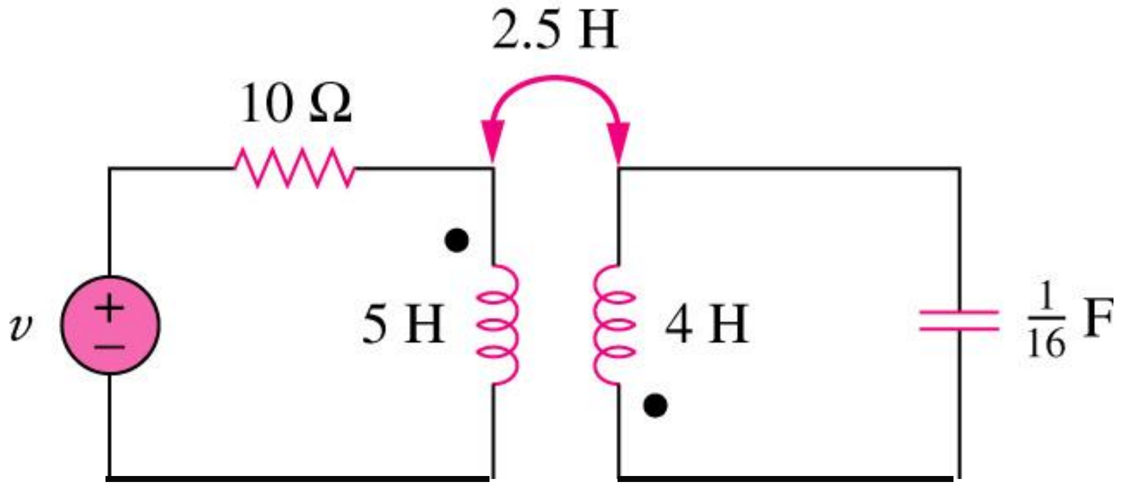
- The instantaneous energy stored in the circuit is given by

$$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M I_1 I_2$$

13.3 Energy in a Coupled Circuit (2)

Example 2

Consider the circuit below. Determine the coupling coefficient. Calculate the energy stored in the coupled inductors at time t = 1s if $v=60\cos(4t+30°)$ V.

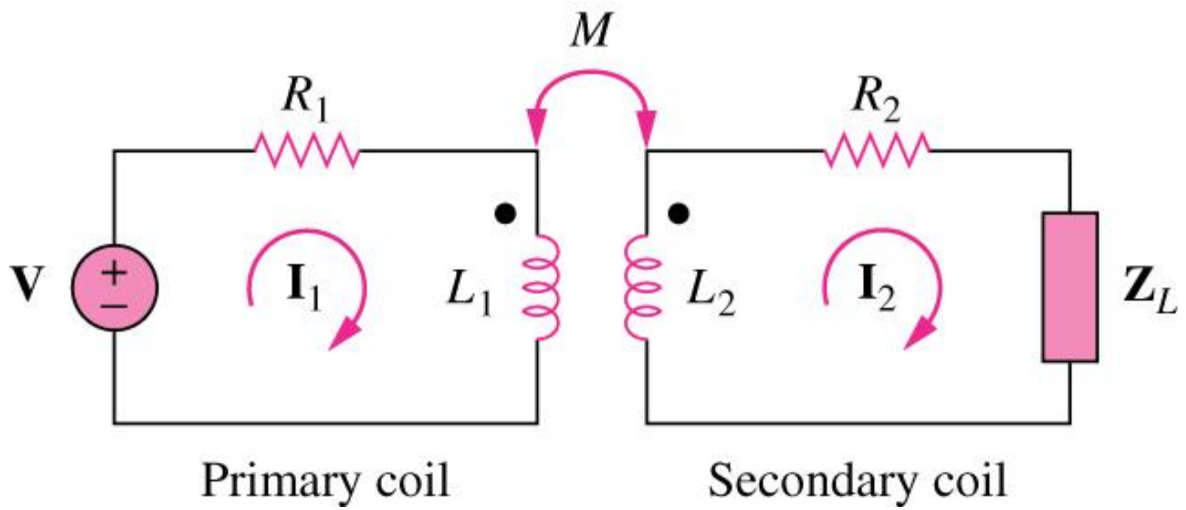


*Refer to in-class illustration, textbook

Ans: k=0.56; $w(1)$=20.73J

13.4 Linear Transformer (1)

- It is generally a four-terminal device comprising two (or more) magnetically coupled coils

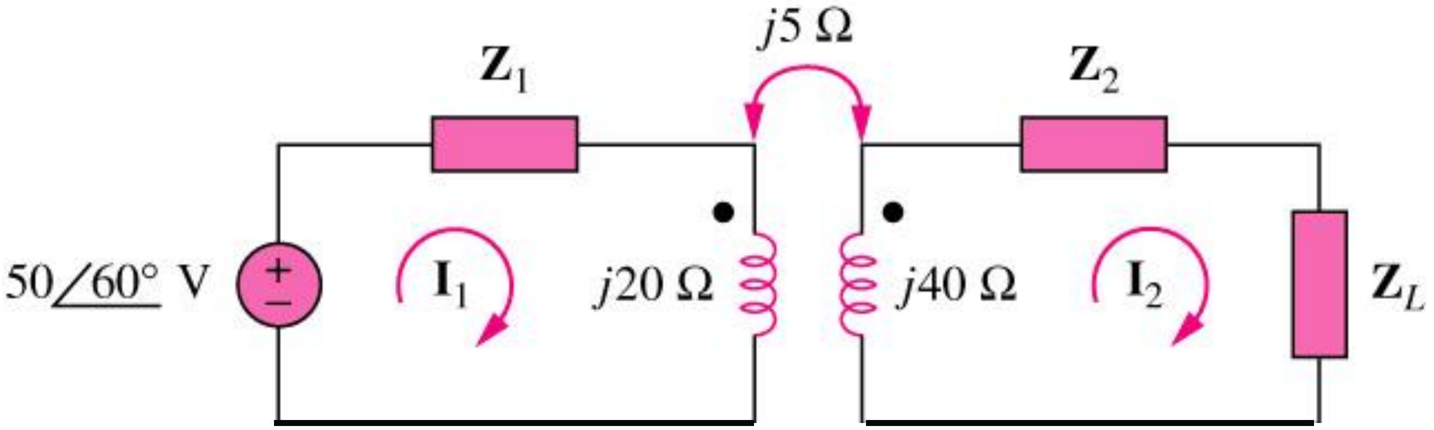


$$Z_{in} = \frac{V}{I_1} = R_1 + j\omega L_1 + Z_R \, , \; Z_R = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} \text{ is } \textit{reflected impedance}$$

13.4 Linear Transformer (2)

Example 3

In the circuit below, calculate the input impedance and current I_1. Take Z_1=60-*j*100Ω, Z_2=30+*j*40Ω, and Z_L=80+j60Ω.

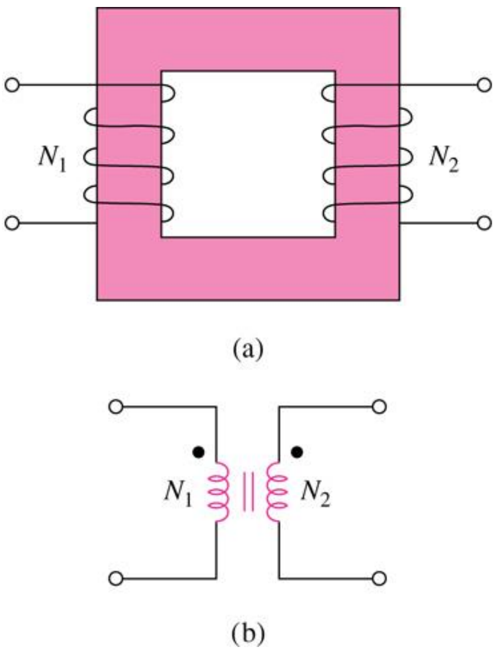


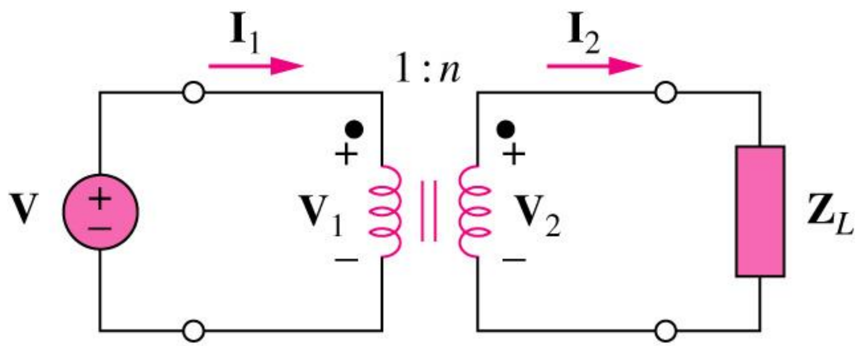
Ans: $Z_{in} = 100.14\angle -53.1°\Omega;\ I_1 = 0.5\angle 113.1°A$

*Refer to in-class illustration, textbook

13.5 Ideal Transformer (1)

- An ideal transformer is a unity-coupled, lossless transformer in which the primary and secondary coils have infinite self-inductances.





$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = n \qquad \frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{n}$$

V2>V1→ step-up transformer
V2<V1→ step-down transformer

(a) Ideal Transformer
(b) Circuit symbol

Example 4

An ideal transformer is rated at 2400/120V, 9.6 kVA, and has 50 turns on the secondary side.

Calculate:

(a) the turns ratio,

(b) the number of turns on the primary side, and

(c) the current ratings for the primary and secondary windings.

Ans:

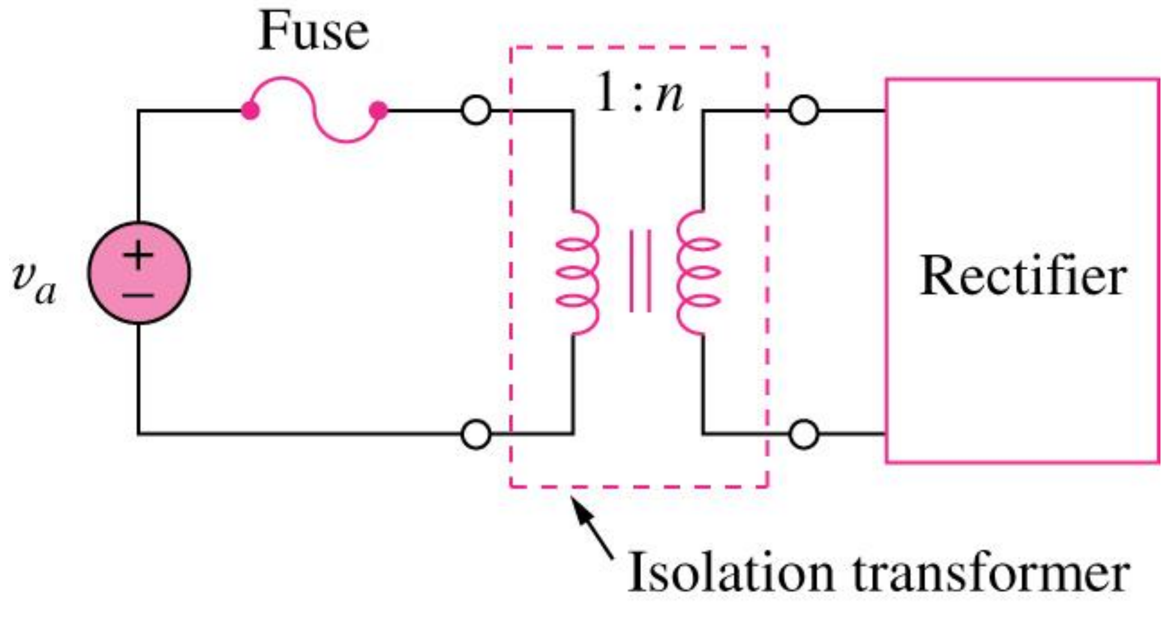
(a) This is a step-down transformer, n=0.05

(b) N1 = 1000 turns

(c) I1 = 4A and I2 = 80A

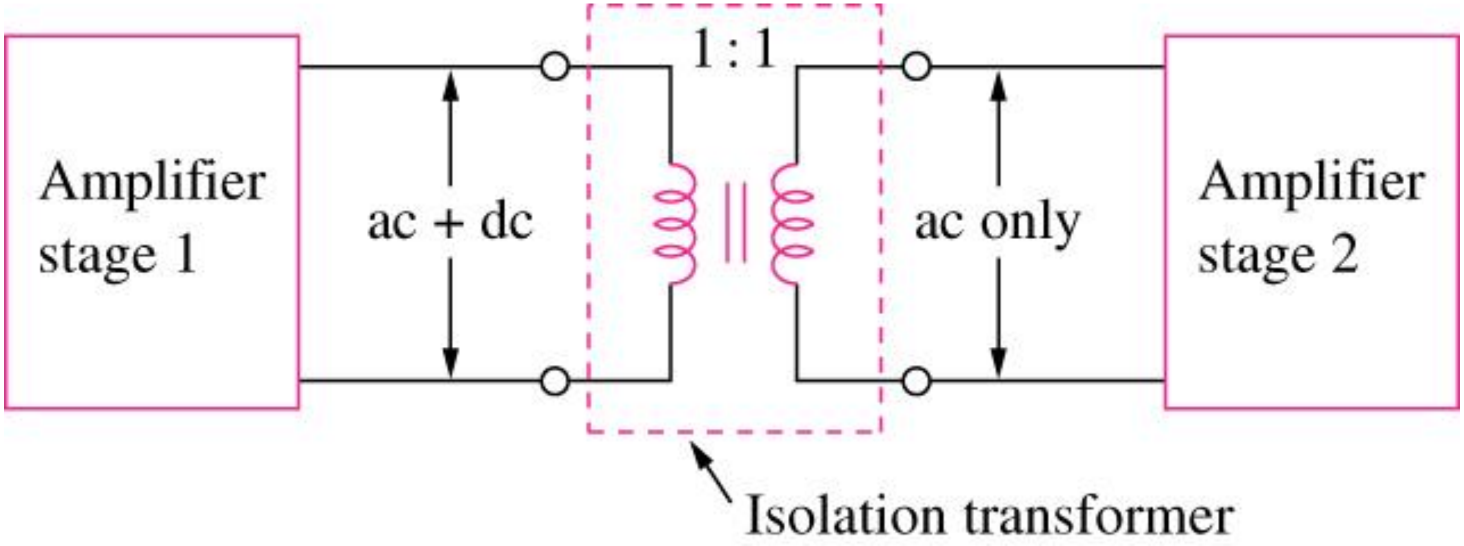
13.6 Applications (1)

- Transformer as an Isolation Device to isolate ac supply from a rectifier



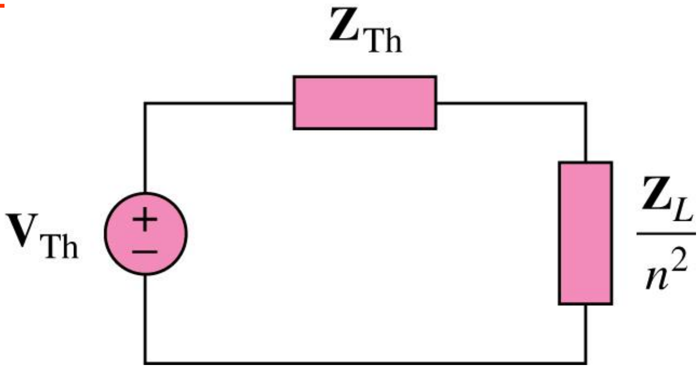
13.6 Applications (2)

- Transformer as an Isolation Device to isolate dc between two amplifier stages.



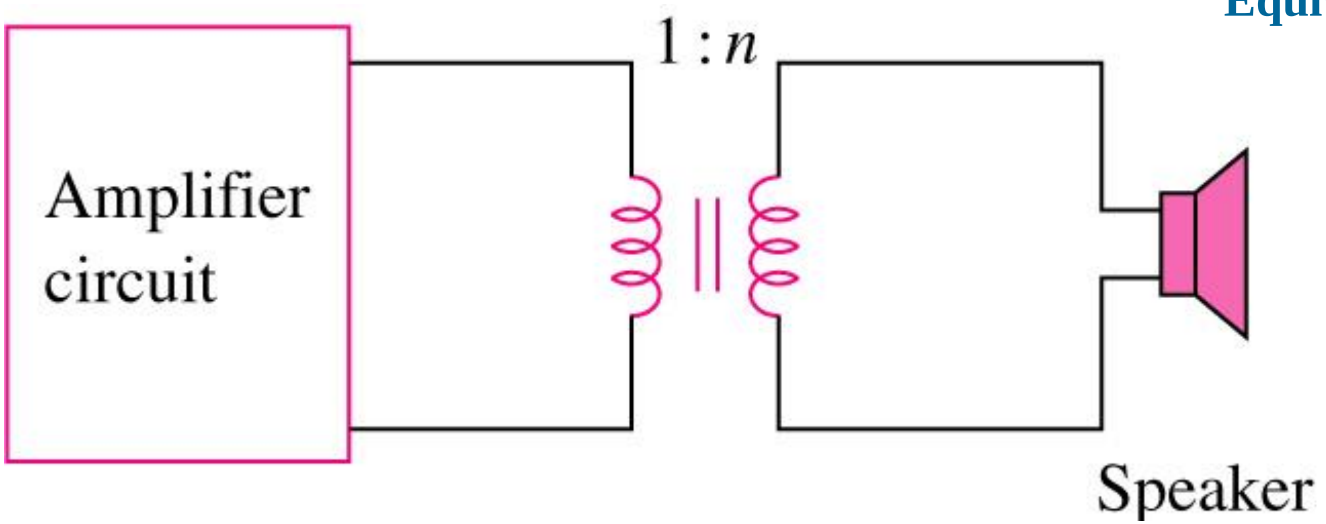
13.6 Applications (3)

- Transformer as a Matching Device



Equivalent circuit

Using an ideal transformer to match the speaker to the amplifier



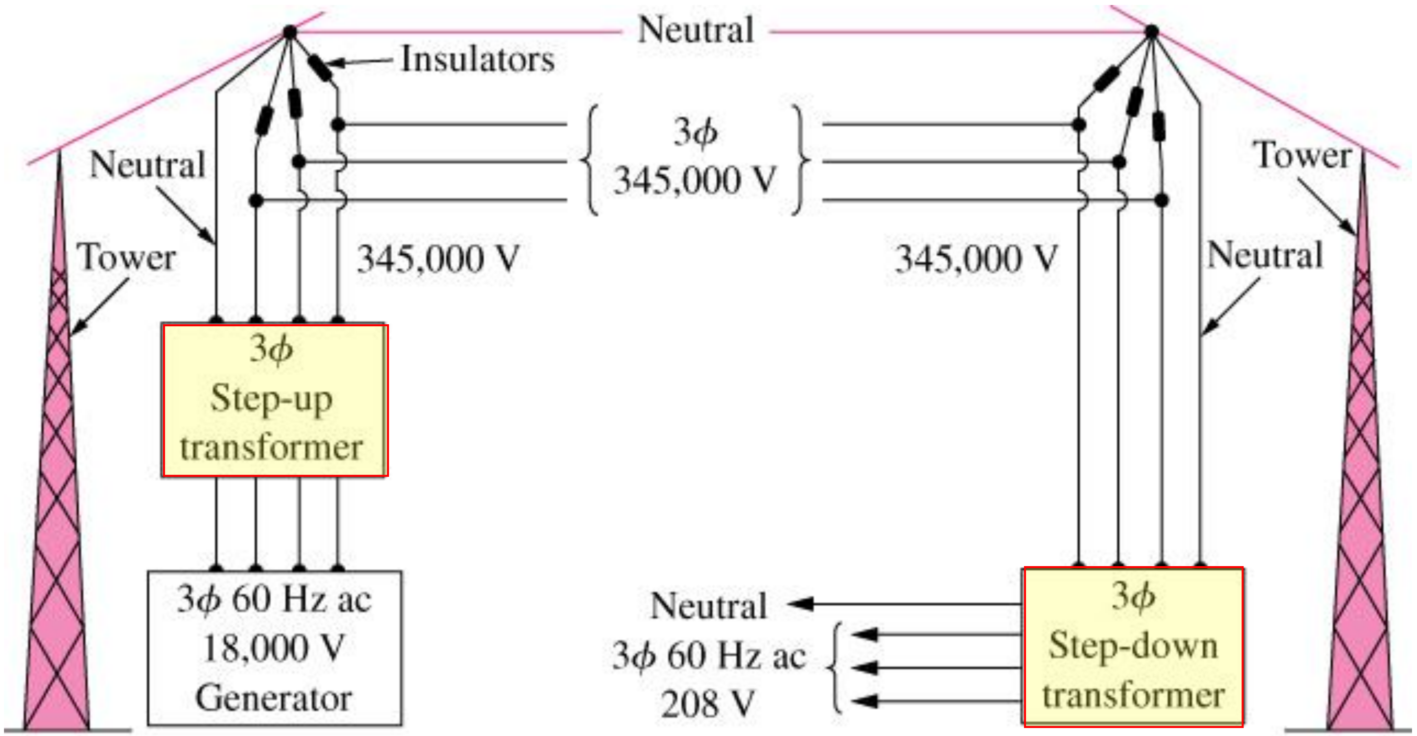
Example 5

Calculate the turns ratio of an ideal transformer required to match a 100Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.2$; $V_L = 3V$

13.6 Applications (5)

- A typical power distribution system



Handouts

Example 4

An ideal transformer is rated at 2400/120V, 9.6 kVA, and has 50 turns on the secondary side.

Calculate:

(a) the turns ratio,

(b) the number of turns on the primary side, and

(c) the current ratings for the primary and secondary windings.

Ans:

(a) This is a step-down transformer, n=0.05

(b) N1 = 1000 turns

(c) I1 = 4A and I2 = 80A

Example 5

Calculate the turns ratio of an ideal transformer required to match a 100Ω load to a source with internal impedance of 2.5kΩ. Find the load voltage when the source voltage is 30V.

Ans: $n = 0.2$; $V_L = 3V$