

EE2003

Circuit Theory

Chapter 12

Three-Phase Circuit

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MARATHON ELECTRIC

Mtr, 3 Ph, 1/2hp, 1725, 208-230/460, Eff 76.1

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General Purpose Motor, 3-Phase, 1/2 HP, Nameplate RPM 1725, Voltage 208-230/460, 56C Frame, Totally Enclosed Fan-Cooled, Number of Speeds 1, Full Load Amps 2.1-2.2/1.1, 60/50 Hz, Face Mounting, Thermal Protection None, Insulation Class B, Service Factor 1.25Max. Ambient Temp. 40 Degrees C, Rotation CW/CCW, Overall Length 10-15/16 In., Shaft Dia. 5/8 In., Shaft Length 1-7/8 In.

Grainger Item #	3N687
Price (ea.)	\$273.75
Brand	MARATHON ELECTRIC
Mfr. Model #	5K35MNB114A
UNSPSC #	26101612
Ship Qty. 	1
Sell Qty. (Will-Call) 	1
Ship Weight (lbs.)	18.4
Availability	Typically in Stock 
Catalog Page No.	18 
Country of Origin	Mexico
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Custom

Mtr, 3 P
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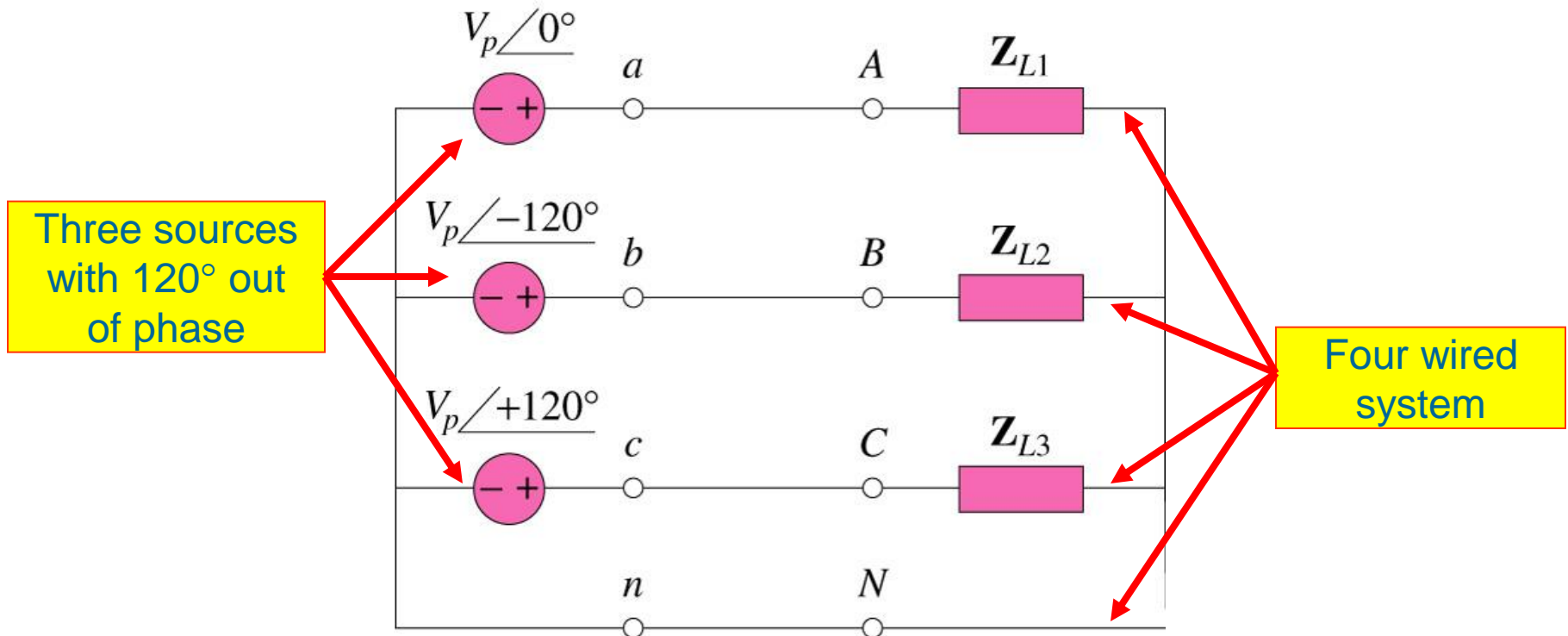
Three-Phase Circuits

Chapter 12

- 12.1 What is a Three-Phase Circuit?
- 12.2 Balance Three-Phase Voltages
- 12.3 Balance Three-Phase Connection
- 12.4 Power in a Balanced System
- 12.5 Unbalanced Three-Phase Systems
- 12.6 Application – Residential Wiring

12.1 What is a Three-Phase Circuit?(1)

- It is a system produced by a generator consisting of **three sources** having the same amplitude and frequency but **out of phase** with each other by 120° .



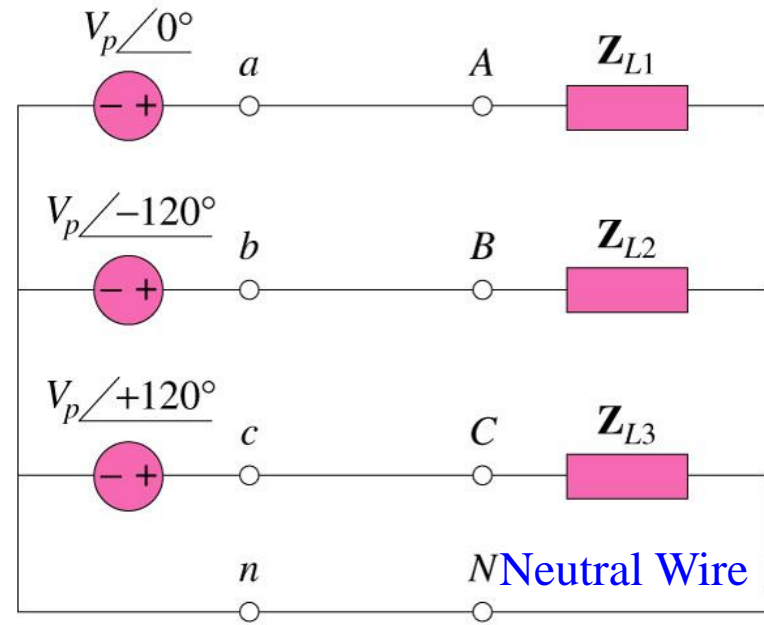
12.1 What is a Three-Phase Circuit?(2)

Advantages:

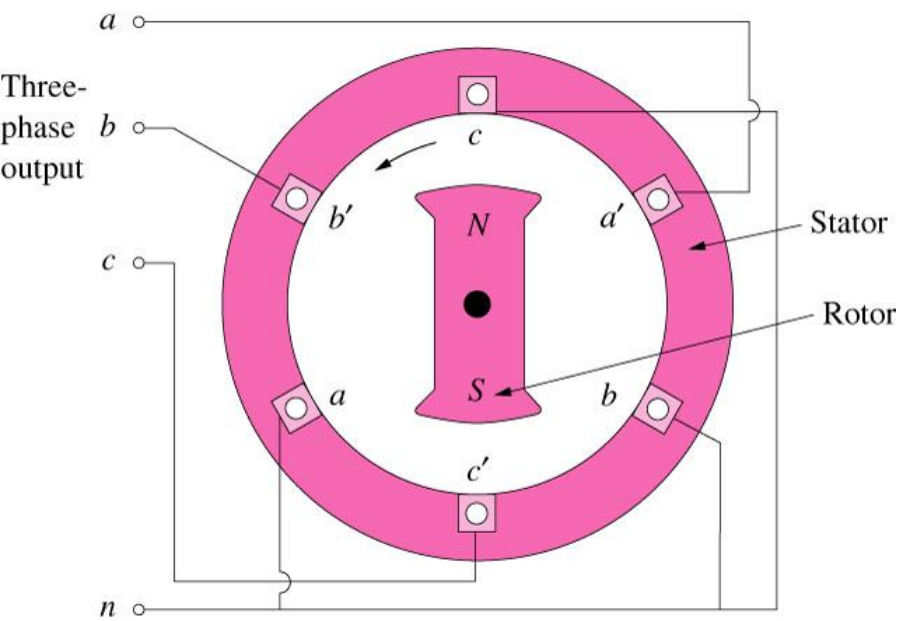
1. Most electric power is generated and distributed in three-phase.
2. The instantaneous power in a three-phase system can be constant.
3. Given the same amount of power, a three-phase system is more economical than single-phase.
4. In fact, the amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

Balanced Three-phase Voltages

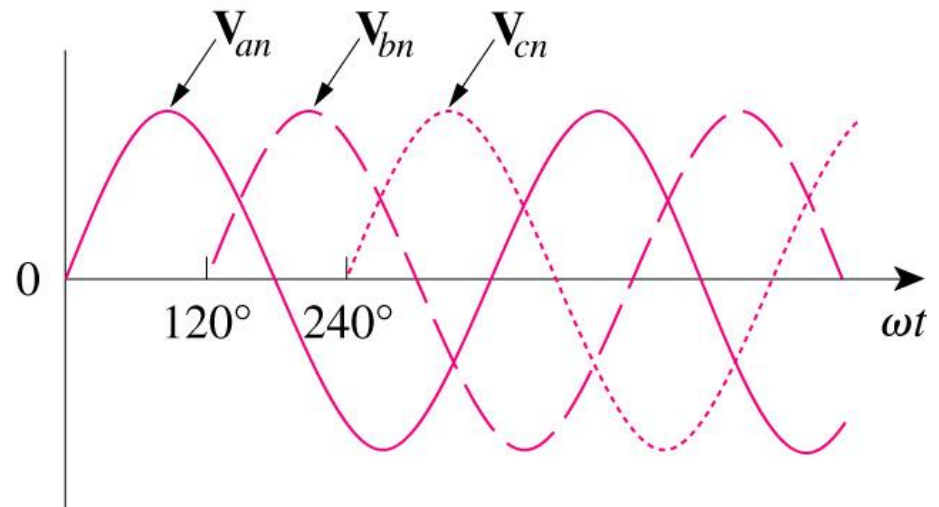
Three-phase four-wire system



A Three-phase Generator

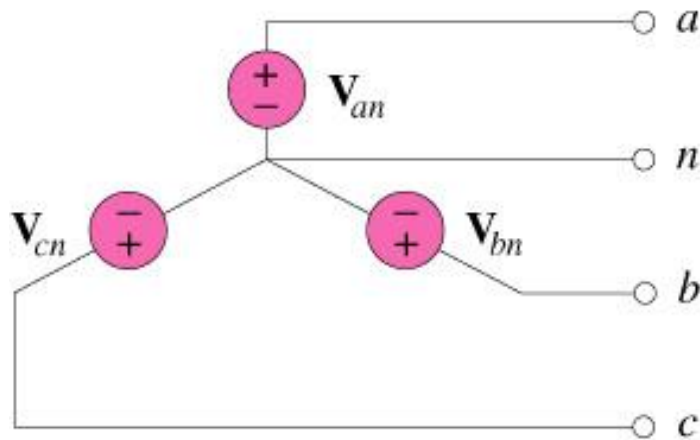


Voltages having 120° phase difference

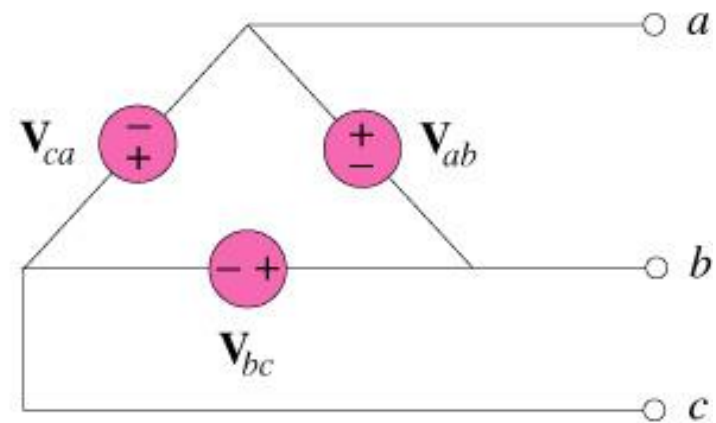


12.2 Balance Three-Phase Voltages (2)

- Two possible configurations:



(a)



(b)

Three-phase voltage sources: (a) Y-connected ; (b) Δ -connected

12.2 Balance Three-Phase Voltages (3)

- **Balanced phase voltages** are equal in magnitude and are out of phase with each other by 120° .
- The **phase sequence** is the time order in which the voltages pass through their respective maximum values.
- A **balanced load** is one in which the phase impedances are equal in magnitude and in phase

Example 1

Determine the phase sequence of the set of voltages.

$$v_{an} = 200 \cos(\omega t + 10^\circ)$$

$$v_{bn} = 200 \cos(\omega t - 230^\circ)$$

$$v_{cn} = 200 \cos(\omega t - 110^\circ)$$

12.2 Balance Three-Phase Voltages (5)

Solution:

The voltages can be expressed in phasor form as

$$\mathbf{V}_{an} = 200 \angle 10^\circ \text{ V}$$

$$\mathbf{V}_{bn} = 200 \angle -230^\circ \text{ V}$$

$$\mathbf{V}_{cn} = 200 \angle -110^\circ \text{ V}$$

We notice that \mathbf{V}_{an} leads \mathbf{V}_{cn} by 120° and \mathbf{V}_{cn} in turn leads \mathbf{V}_{bn} by 120° .

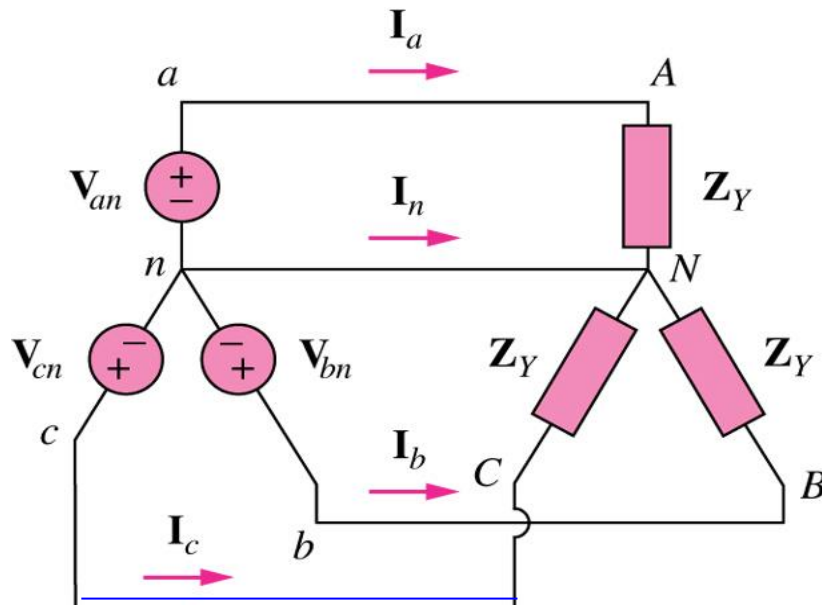
Hence, we have an acb sequence.

12.3 Balance Three-Phase Connection (1)

- Four possible connections
 1. Y-Y connection (Y-connected source with a Y-connected load)
 2. Y- Δ connection (Y-connected source with a Δ -connected load)
 3. Δ - Δ connection
 4. Δ -Y connection

12.3 Balance Three-Phase Connection (2)

- A **balanced Y-Y** system is a three-phase system with a balanced y-connected source and a balanced y-connected load.



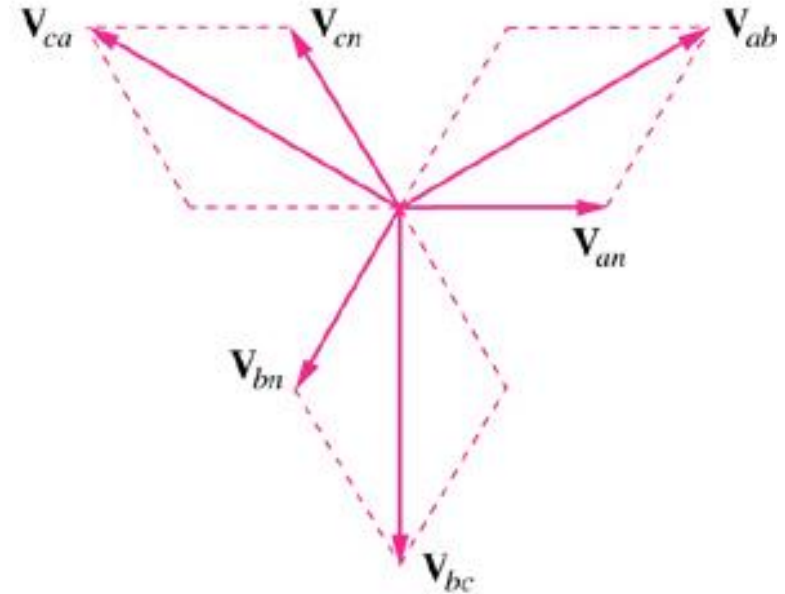
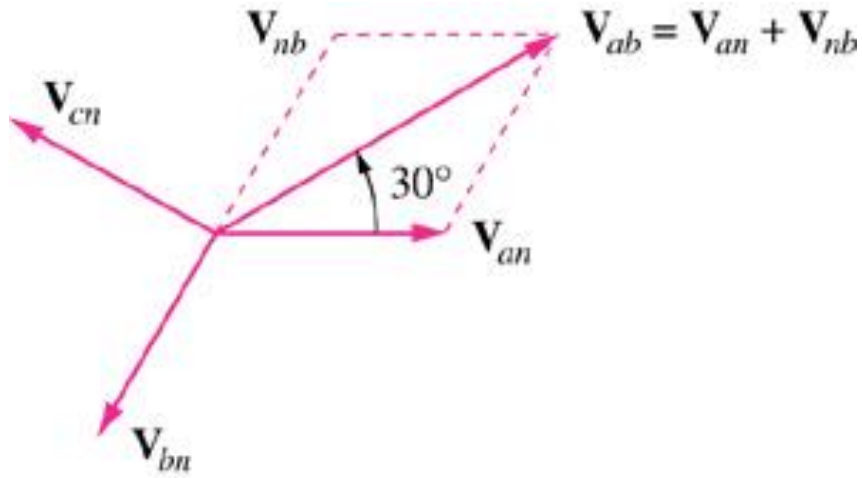
$$V_L = \sqrt{3}V_p, \text{ where}$$

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$$

Balanced Wye-wye Connection

- Phasor diagram of phase and line voltages

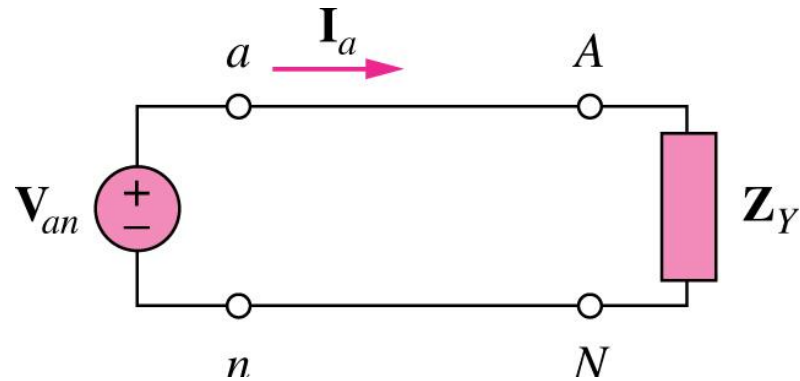


$$\begin{aligned} V_L &= |V_{ab}| = |V_{bc}| = |V_{ca}| \\ &= \sqrt{3} |V_{an}| = \sqrt{3} |V_{bn}| = \sqrt{3} |V_{cn}| = \sqrt{3} V_p \end{aligned}$$

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

Single Phase Equivalent of Balanced Y-Y Connection

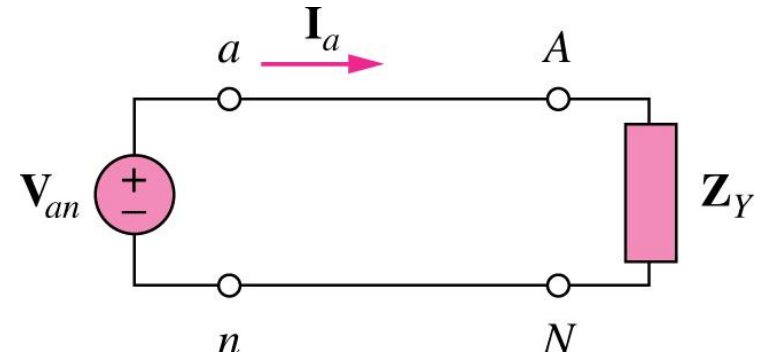
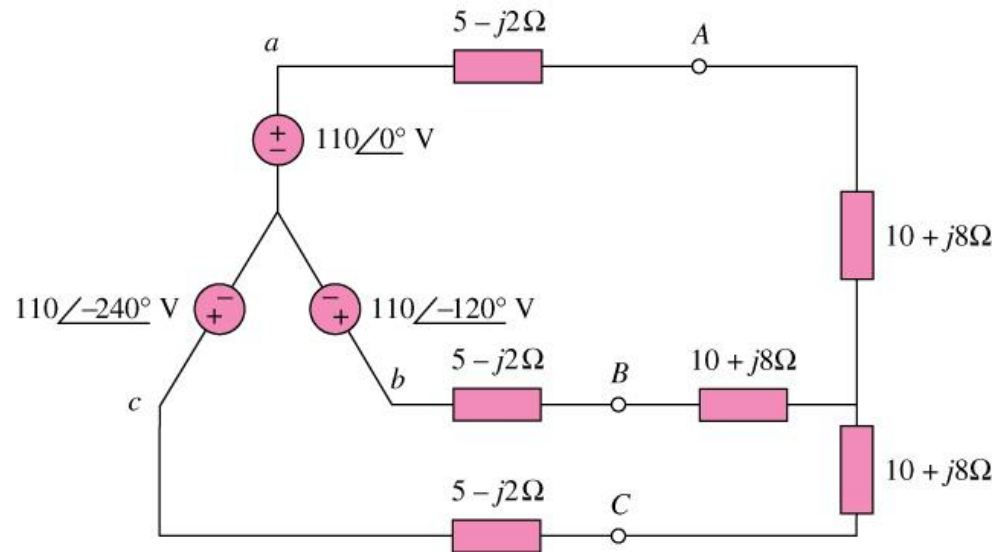
- Balanced three phase circuits can be analyzed on “per phase “ basis.
- We look at one phase, say phase a and analyze the single phase equivalent circuit.
- Because the circuit is balanced, we can easily obtain other phase values using their phase relationships.



$$I_a = \frac{V_{an}}{Z_Y}$$

Example 2

Calculate the line currents in the three-wire Y-Y system shown below:



$$110 \angle 0^\circ / (5 - j2 + 10 + j8) = 6.809 \angle -21.80^\circ$$

Ans

$$I_a = 6.81 \angle -21.8^\circ \text{ A}$$

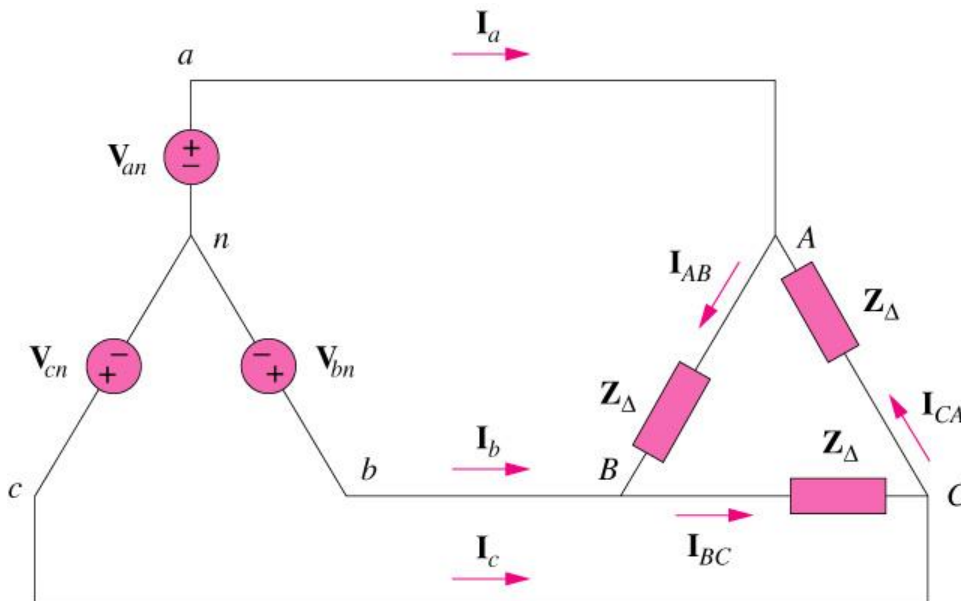
$$I_b = 6.81 \angle -141.8^\circ \text{ A}$$

$$I_c = 6.81 \angle 98.2^\circ \text{ A}$$

12.3 Balance Three-Phase Connection (4)

- A **balanced Y- Δ** system is a three-phase system with a balanced y-connected source and a balanced Δ -connected load.

$$I_L = \sqrt{3}I_p, \text{ where}$$
$$I_L = |I_a| = |I_b| = |I_c|$$
$$I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$$



Example 3

A balanced *abc*-sequence Y-connected source with ($V_{an} = 100\angle 10^\circ$) is connected to a Δ -connected load $(8+j4)\Omega$ per phase. Calculate the phase and line currents.

Solution

$$V_{an} = 100 \angle 10^\circ$$

$$V_{bn} = 100 \angle (10 - 120)^\circ$$

$$V_{cn} = 100 \angle (10 + 120)^\circ$$

$$V_{ab} = V_{an} - V_{bn} = 173.205 \angle 40.00^\circ$$

$$V_{bc} = V_{bn} - V_{cn} = 173.205 \angle -80.00^\circ$$

$$V_{ca} = V_{cn} - V_{an} = 173.205 \angle 160.00^\circ$$

$$I_{ab} = V_{ab} / (8 + j4) = 19.365 \angle 13.43^\circ$$

$$I_{ca} = V_{ca} / (8 + j4) = 19.365 \angle 133.43^\circ$$

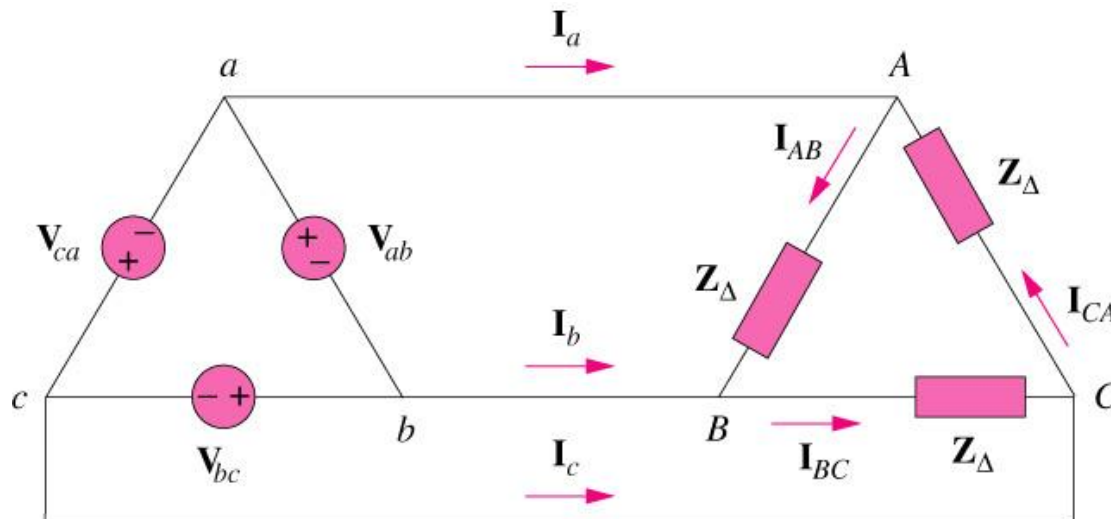
$$I_a = I_{ab} - I_{ca} = 33.541 \angle -16.57^\circ$$

$$I_a = \frac{V_{an}}{Z_{\Delta} / 3} = \frac{100 \angle 10^\circ}{2.981 \angle 26.57^\circ} = 33.54 \angle -16.57^\circ \text{ A}$$

Other line currents are obtained using the *abc* phase sequence

12.3 Balance Three-Phase Connection (6)

- A **balanced Δ - Δ** system is a three-phase system with a balanced Δ -connected source and a balanced Δ -connected load.



Example 4

A balanced Δ -connected load having an impedance $20 - j15 \Omega$ is connected to a Δ -connected positive-sequence generator having ($V_{ab} = 330 \angle 0^\circ \text{ V}$). Calculate the phase currents of the load and the line currents.

Ans:

$$I_{ab} = 110 \angle 0^\circ / (5 - 2j + 10 + 8j) = 13.200 \angle 36.87^\circ$$

$$I_{bc} = 110 \angle -120^\circ / (5 - 2j + 10 + 8j) = 13.200 \angle -81.13^\circ$$

$$I_{ca} = 110 \angle 120^\circ / (5 - 2j + 10 + 8j) = 13.200 \angle 156.87^\circ$$

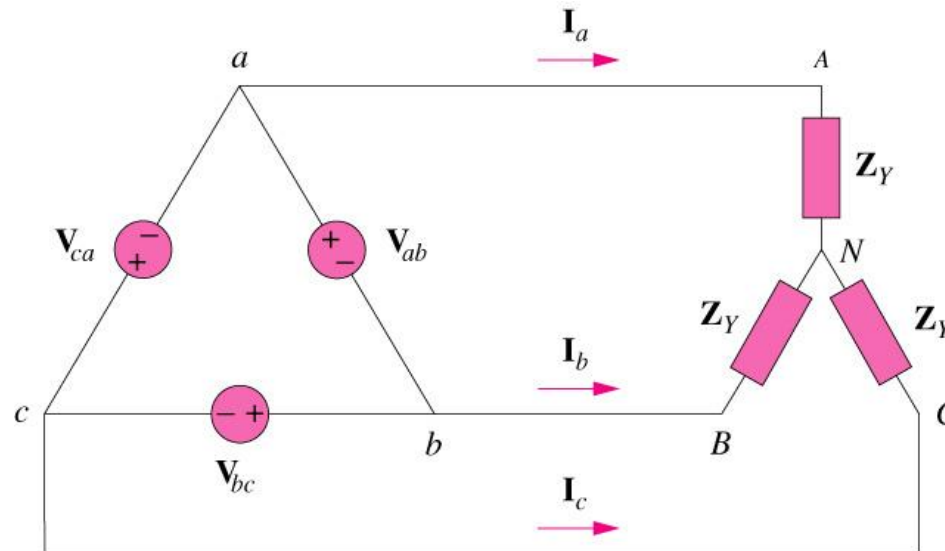
$$I_a = I_{ab} - I_{ca} = 22.86 \angle 6.87^\circ$$

$$I_{AB} = 13.2 \angle 36.87^\circ \text{ A}; I_{BC} = 13.2 \angle -81.13^\circ \text{ A}; I_{CA} = 13.2 \angle 156.87^\circ \text{ A}$$

$$I_a = 22.86 \angle 6.87^\circ \text{ A}; I_b = 22.86 \angle -113.13^\circ \text{ A}; I_c = 22.86 \angle 126.87^\circ \text{ A}$$

12.3 Balance Three-Phase Connection (8)

- A **balanced Δ -Y** system is a three-phase system with a balanced y-connected source and a balanced y-connected load.



12.3 Balanced Three-Phase Connection (9)

Example 5

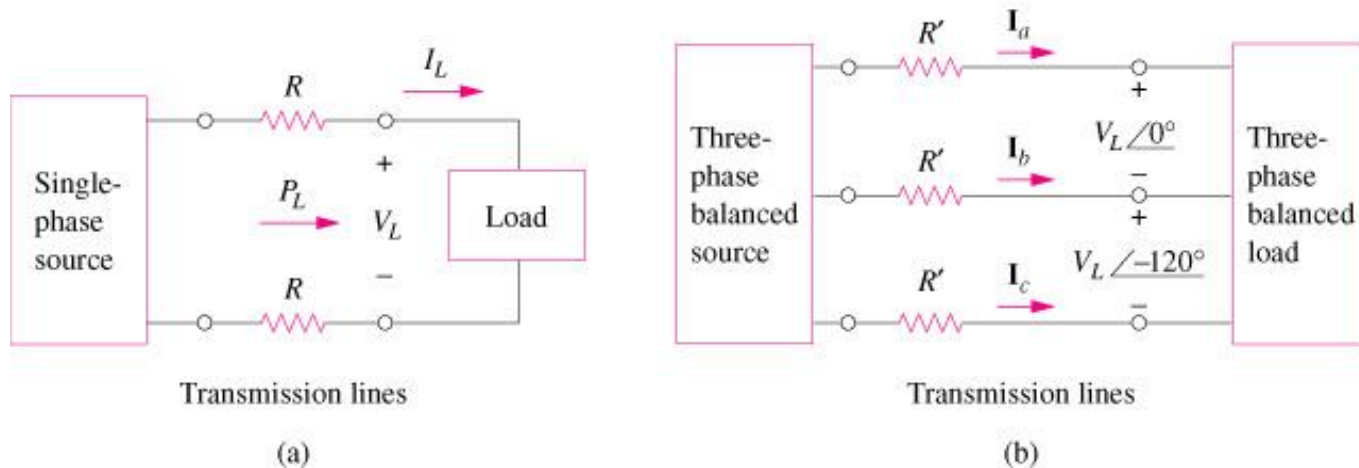
A balanced Y-connected load with a phase impedance $40+j25 \Omega$ is supplied by a balanced, positive-sequence Δ -connected source with a line voltage of 210V. Calculate the phase currents. Use V_{ab} as reference.

Answer

$$\begin{aligned} \text{The phase currents} \quad I_{AN} &= 2.57 \angle -62^\circ \text{ A}; \\ I_{BN} &= 2.57 \angle -178^\circ \text{ A}; \\ I_{CN} &= 2.57 \angle 58^\circ \text{ A}; \end{aligned}$$

12.4 Power in a Balanced System (1)

- Comparing the power loss in (a) a single-phase system, and (b) a three-phase system



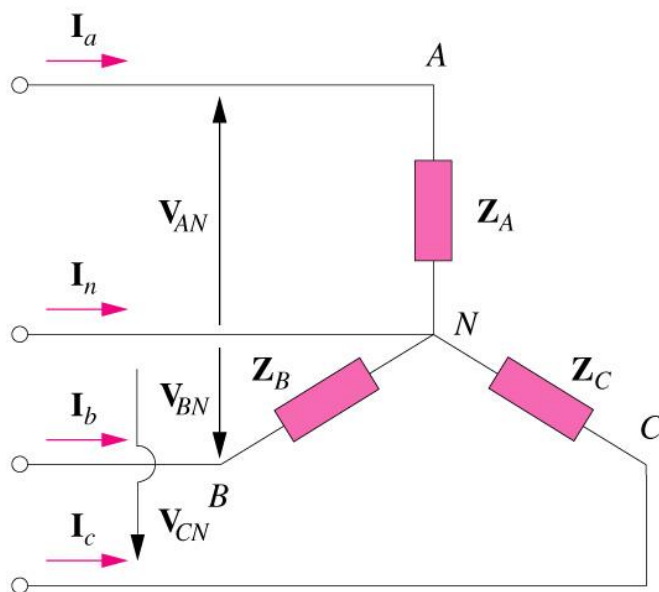
$$P'_{loss} = 2R \frac{P_L^2}{V_L^2}, \text{ single - phase}$$

$$P'_{loss} = R' \frac{P_L^2}{V_L^2}, \text{ three - phase}$$

- If same power loss is tolerated in both system, three-phase system use only 75% of materials of a single-phase system

12.5 Unbalanced Three-Phase Systems (1)

- An unbalanced system is due to unbalanced voltage sources or an unbalanced load.



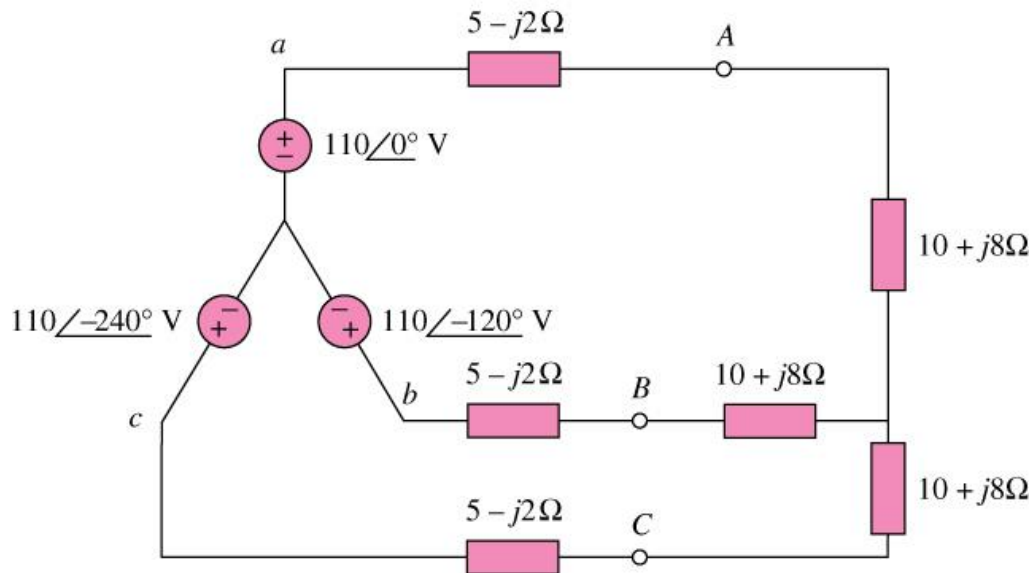
$$I_a = \frac{V_{AN}}{Z_A}, I_b = \frac{V_{BN}}{Z_B}, I_c = \frac{V_{CN}}{Z_C},$$
$$I_n = -(I_a + I_b + I_c)$$

- To calculate power in an unbalanced three-phase system requires that we find the power in each phase.
- The total power is not simply three times the power in one phase but the sum of the powers in the three phases.

12.3 Unbalanced Three-Phase Systems (2)

Example 6

Determine the total average power, reactive power, and complex power at the source and at the load



Ans

At the source:

$$S_s = -(2087 + j834.6) \text{ VA}$$

$$P_a = -2087 \text{ W}$$

$$P_r = -834.6 \text{ VAR}$$

At the load:

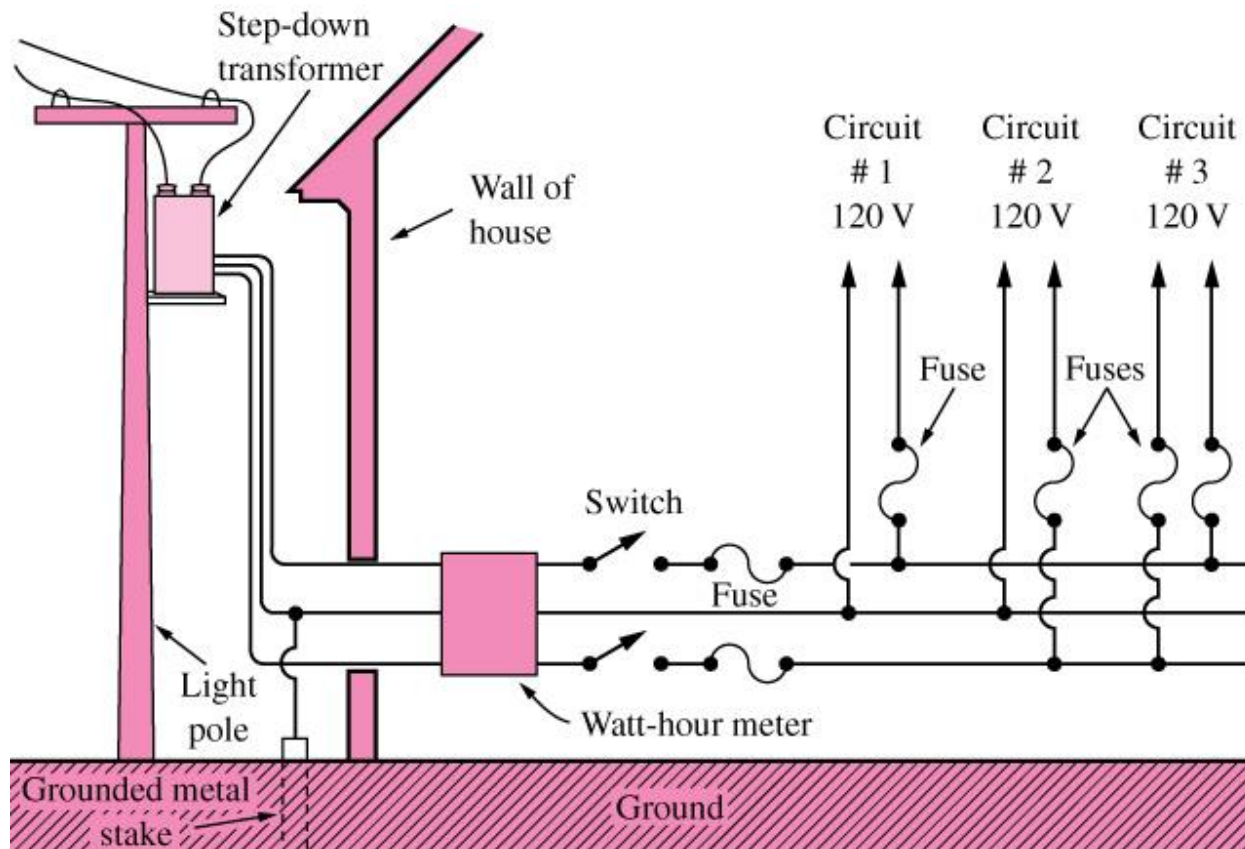
$$S_L = (1392 + j1113) \text{ VA}$$

$$P_a = 1392 \text{ W}$$

$$P_r = 1113 \text{ VAR}$$

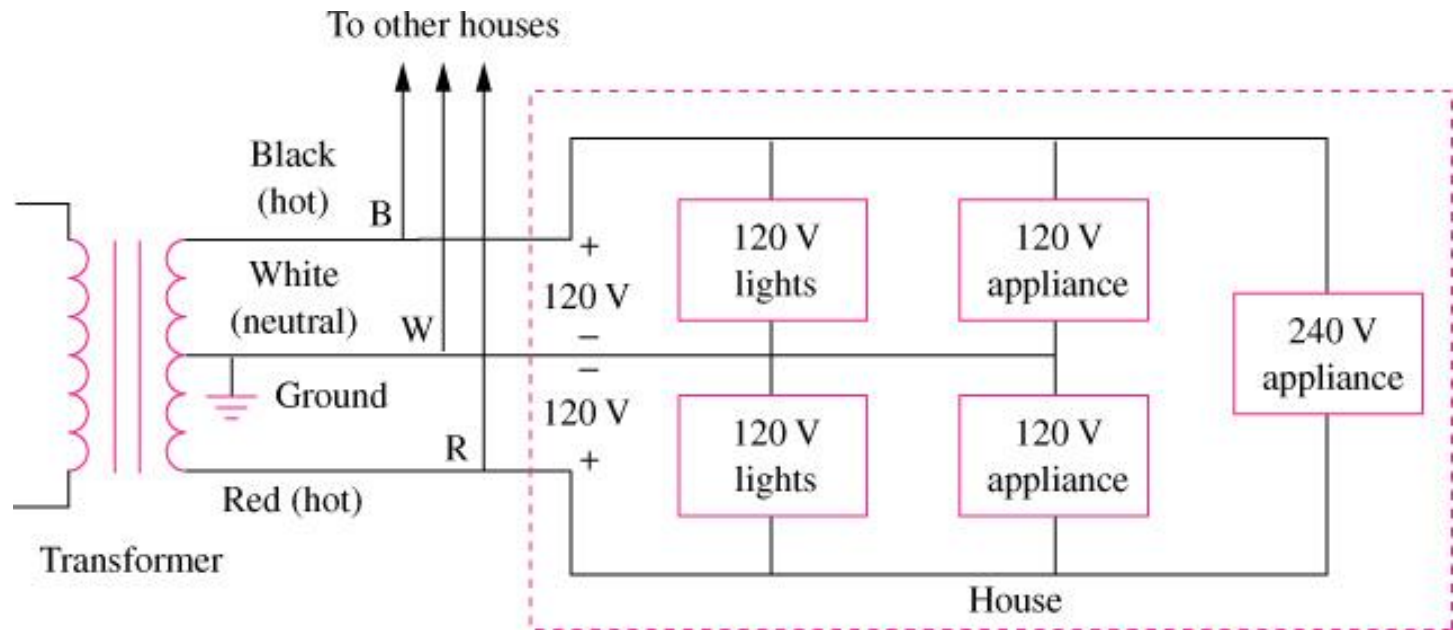
*Refer to in-class illustration, textbook

12.6 Application – Residential Wiring (1)



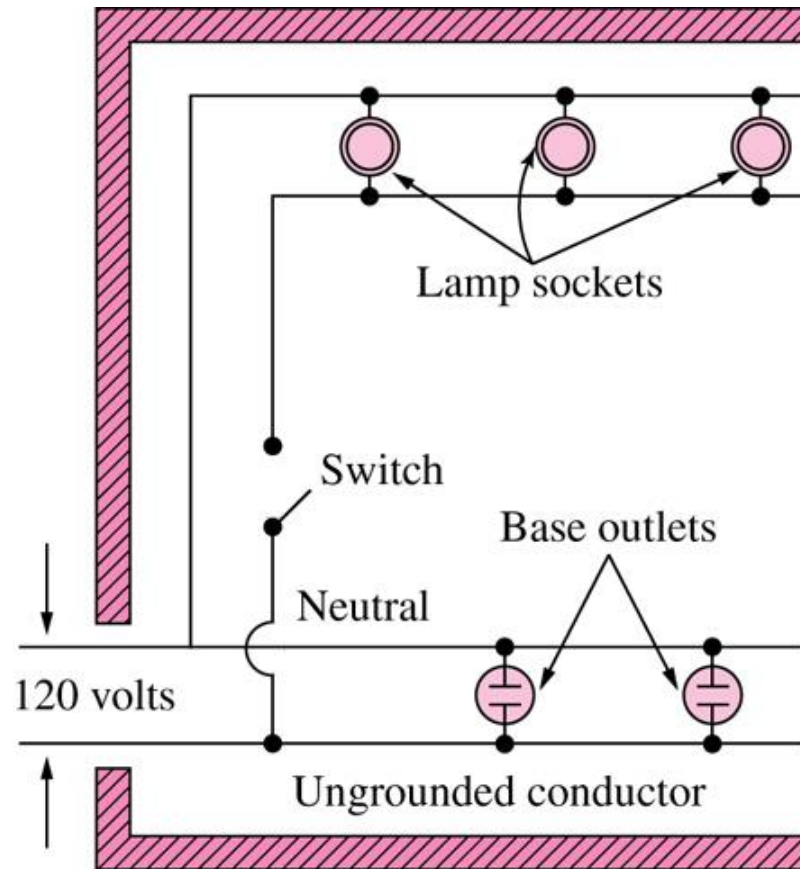
A 120/240 household power system

12.6 Application – Residential Wiring (2)



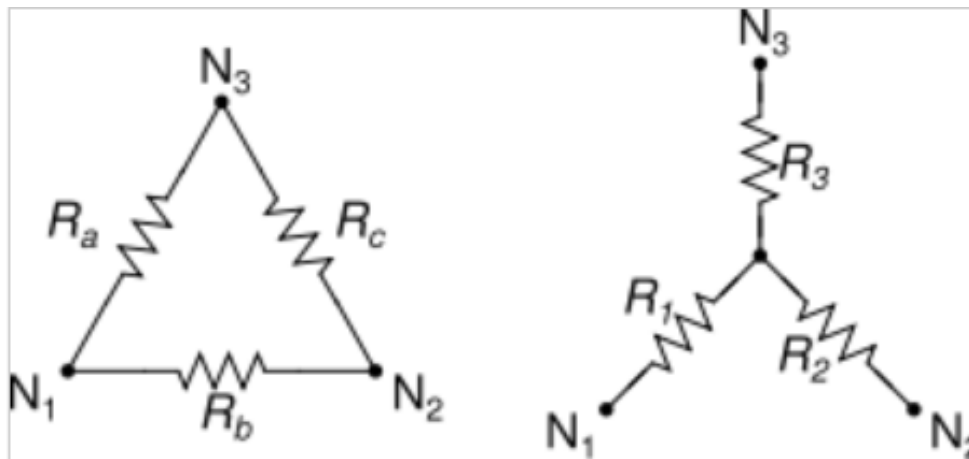
Single-phase three-wire residential wiring

12.6 Application – Residential Wiring (3)



A typical wiring diagram of a room

General Delta to Wye conversion



Delta to Wye

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_c}{R_a + R_b + R_c}$$

Wye to Delta

$$R_1 = \frac{R_b R_c}{R_T}$$

$$R_2 = \frac{R_a R_c}{R_T}$$

$$R_3 = \frac{R_a R_b}{R_T}$$

where $R_T = R_a + R_b + R_c$

works the same way for complex impedances

HANDOUTS

Example 1

Determine the phase sequence of the set of voltages.

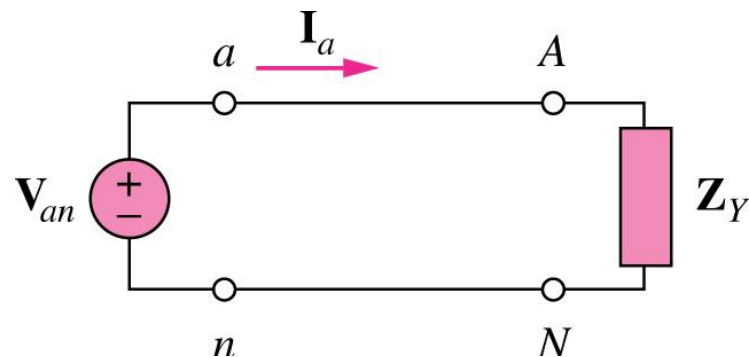
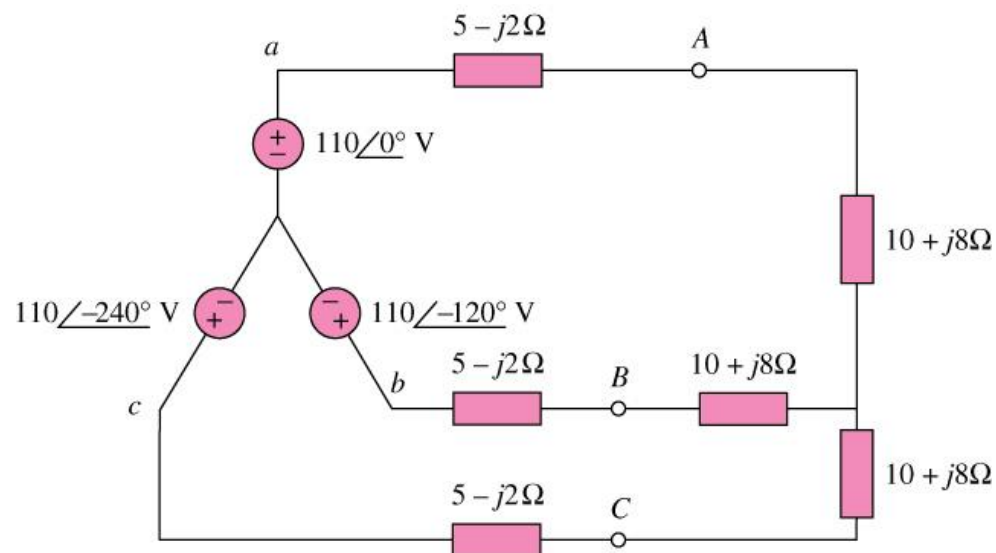
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$$v_{bn} = 200 \cos(\omega t - 230^\circ)$$

$$v_{cn} = 200 \cos(\omega t - 110^\circ)$$

Example 2

Calculate the line currents in the three-wire Y-Y system shown below:



Ans

$$I_a = 6.81 \angle -21.8^\circ \text{ A}$$

$$I_b = 6.81 \angle -141.8^\circ \text{ A}$$

$$I_c = 6.81 \angle 98.2^\circ \text{ A}$$

Example 3

A balanced *abc*-sequence Y-connected source with ($V_{an} = 100\angle 10^\circ$) is connected to a Δ -connected load $(8+j4)\Omega$ per phase. Calculate the phase and line currents.

Solution

$$I_a = \frac{V_{an}}{Z_{\Delta}/3} = \frac{100\angle 10^\circ}{2.981\angle 26.57^\circ} = 33.54\angle -16.57^\circ \text{ A}$$

Other line currents are obtained using the *abc* phase sequence

Example 4

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Ans:

$$I_{AB} = 13.2 \angle 36.87^\circ \text{ A}; I_{BC} = 13.2 \angle -81.13^\circ \text{ A}; I_{CA} = 13.2 \angle 156.87^\circ \text{ A}$$

$$I_a = 22.86 \angle 6.87^\circ \text{ A}; I_b = 22.86 \angle -113.13^\circ \text{ A}; I_c = 22.86 \angle 126.87^\circ \text{ A}$$