EE2003
Circuit Theory

Chapter 11

AC Power Analysis

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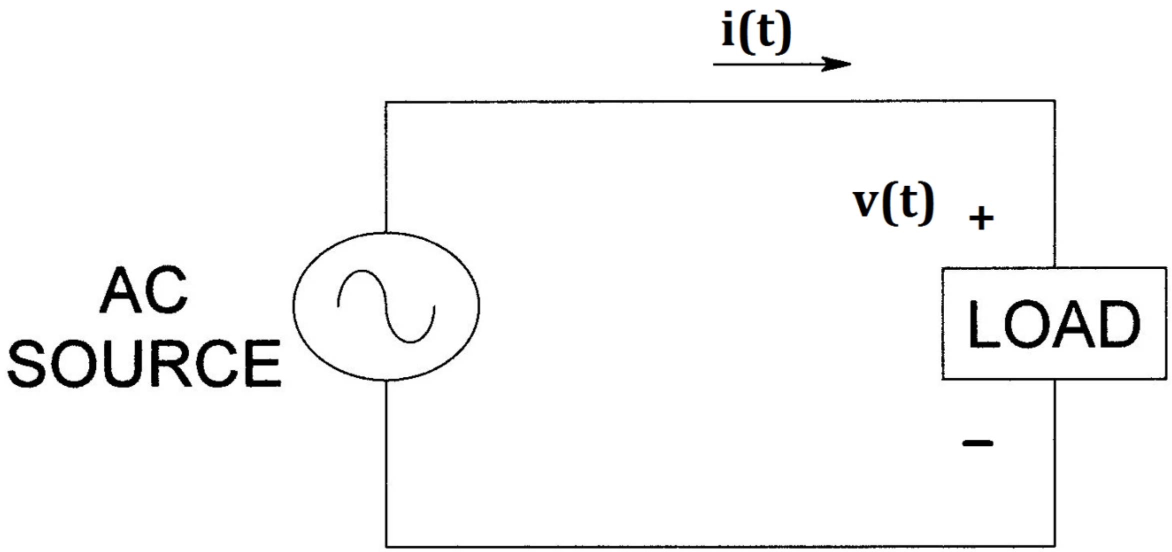
Chapter 11

Consider a typical AC circuit



- If we define

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

- Then we can compute

$$p(t) = v(t)\, i(t) = V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

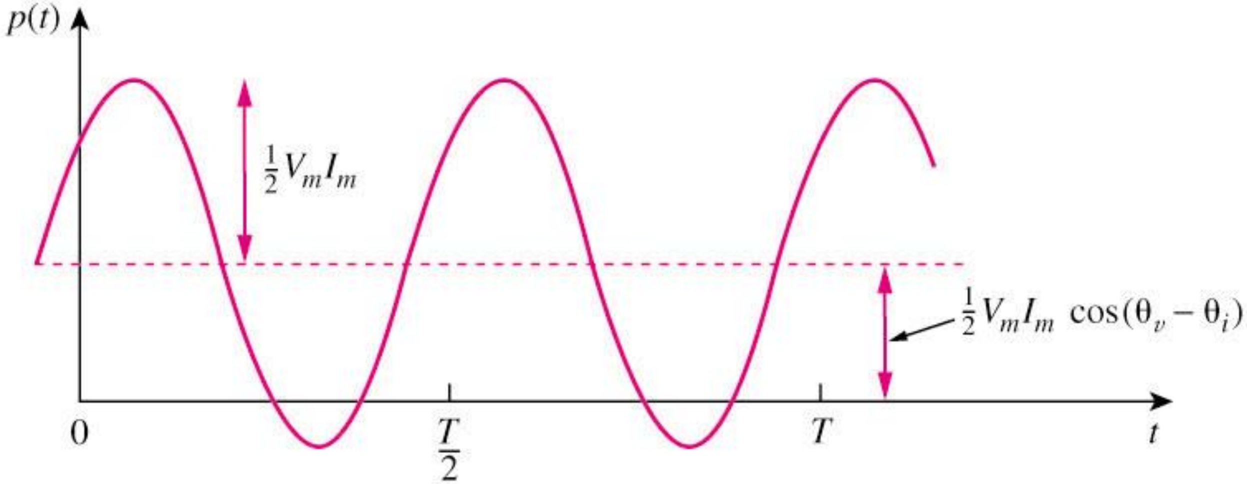
11.1 Instantaneous and Average Power (1)

- The instantaneous power, p(t)

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power **Sinusoidal power at 2ωt**

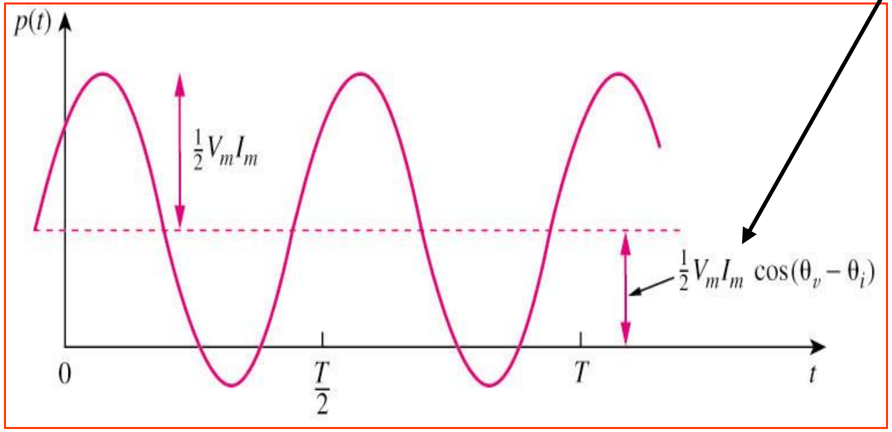


p(t) > 0: power is absorbed by the circuit; p(t) < 0: power is absorbed by the source.

11.1 Instantaneous and Average Power (2)

- The average power, P, *is the average of the instantaneous power over one period.*

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$



1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90o$, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

Example 1

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 80\cos(10t + 20°)$$
$$i(t) = 15\sin(10t + 60°)$$

$= 15\cos(10t + 60° - 90°)$

$= 15\cos(10t - 30°)$

$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$

$= \frac{1}{2}(80)(15)\cos(20 - -30)$

$= 600\cos(50°)$

$= 385.7$ W

$p(t) = 385.7 + 600\cos(20t + 20 + -30)$

$= 385.7 + 600\cos(20t - 10)$ W

Answer: $385.7 + 600\cos(20t - 10°)\text{W}, 387.5\text{W}$

Example 2

A current $\mathbf{I} = 10\angle 30°$ flows through an impedance $\mathbf{Z} = 20\angle -22°\,\Omega$. Find the average power delivered to the impedance.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

V_m? $\quad \mathbb{V} = \mathbb{I}\,Z = (10\angle 30)(20\angle -22)$

$$= 200\angle(30-22) = 200\angle 8^\circ$$

$$\therefore P = \frac{1}{2}(200)(10)\cos(8^\circ - 30^\circ)$$

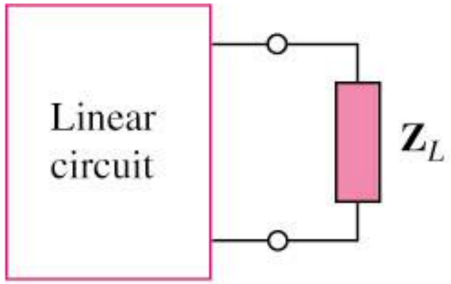
$$= 1000\cos(-22^\circ)$$

$$= 927.2\ W$$

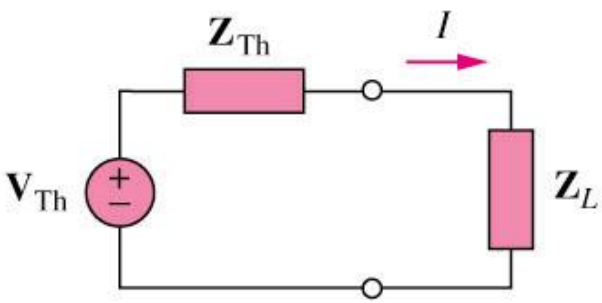
notice this is ϕ angle of Z!

Answer: 927.2W

11.2 Maximum Average Power Transfer (1)



(a)



(b)

$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

The maximum average power can be transferred to the load if

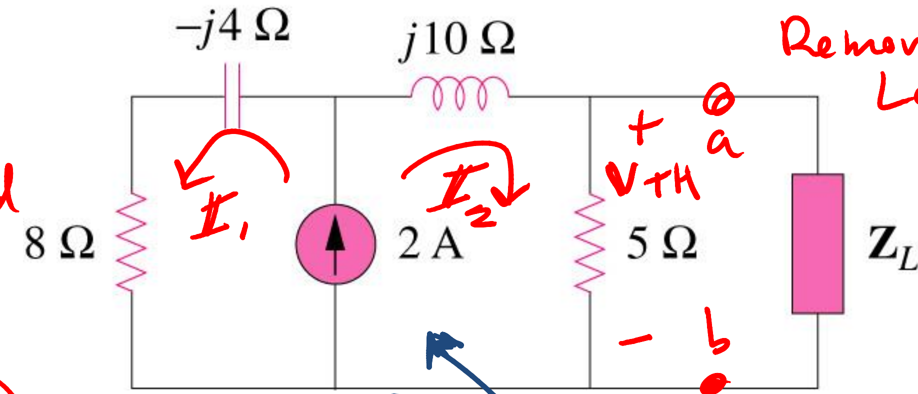
$X_L = -X_{TH}$ and $R_L = R_{TH}$

$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

If the load is purely real, then $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

Example 3

For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate that maximum average power.



current divider

1) FIND V_{TH} – Remove Z_L

$V_{TH} = V_{ab}$ w/ load removed

$= I_2 (5\,\Omega)$

$= \dfrac{(2A)(8-j4)}{(8-4j)(5+10j)} (5\,\Omega)$

$= \dfrac{160-200j}{41} = 6.247\angle -51.34$

2) FIND Z_{TH} – Turn off 2A src

$Z_{TH} = 5 \,//\, (8-4j+10j) = \dfrac{5(8+6j)}{5+8+6j}$

$= \dfrac{140+30j}{41} = 3.41 + .7317j$

3) MAX Power:

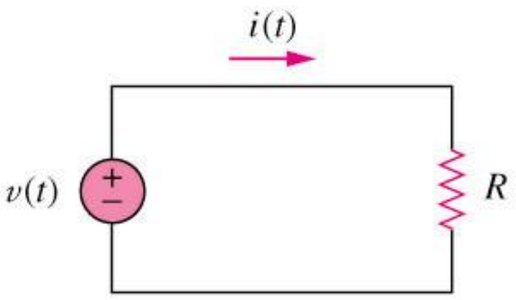
Choose $Z_L = R_{TH} - jX_{TH}$

$Z_L = 3.41 - j\,0.737$

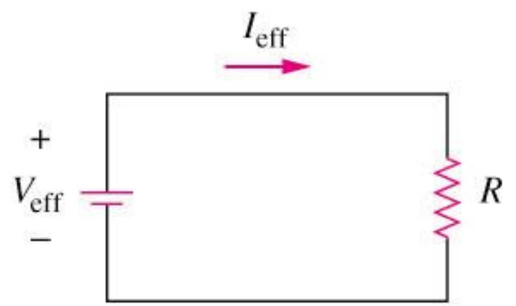
$P_{max} = \dfrac{|V_{TH}|^2}{8 R_{TH}} = \dfrac{(6.247)^2}{8(3.41)} = 1.43W$

Answer: **3.415 – j0.7317Ω, 1.429W**

11.3 Effective or RMS Value (1)



(a)



(b)

The total power dissipated by R is given by:

$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

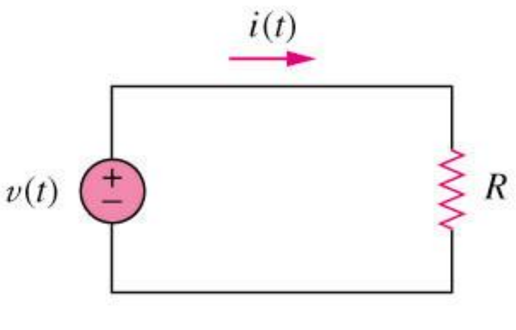
Hence, I_{eff} is equal to:

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}$$

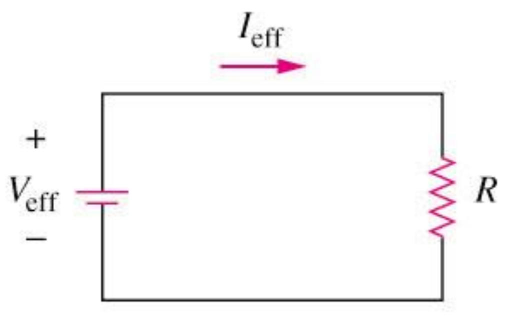
The rms value is a constant itself which depending on the shape of the function i(t).

The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.3 Effective or RMS Value (2)



(a)



(b)

The rms value of a sinusoid $i(t) = I_m\cos(\omega t)$ is given by:

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

The average power can be written in terms of the rms values:

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Note: If you express amplitude of a phasor source(s) in rms, then all the answers as a result of this phasor source(s) must also be in rms value.

Example 4

A sinusoidal voltage having a max amplitude of 625 V is applied to the terminals of a 50Ω resistor. Find a) RMS voltage. b) RMS current. c) Average power

a) $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{625}{\sqrt{2}} = 441.9\ V$

b) $I_{rms} = \frac{V_{rms}}{50\Omega} = \frac{441.9}{50} = 8.839\ A$

c) $P = V_{rms} I_{rms} \cos(\theta_v - \theta_i) = (441.9)(8.839) = \underline{3906\ W}$

$\theta_v - \theta_i$ = 0 for resistor only load

Also $P = \frac{V_{rms}^2}{R} = \frac{441.9^2}{50} = \underline{3906\ W}$

$= I_{rms}^2 (R) = \underline{3906\ W}$

a) Vrms = 441.9V, b) Irms = 8.839A, c)P =3906W

11.4 Apparent Power and Power Factor (1)

- Apparent Power, S, is the product of the r.m.s. values of voltage and current (independent of their phase)
- It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

$$= \frac{V_m I_m}{2} \cos(\theta_v - \theta_i)$$

Apparent Power, S

Power Factor, pf

$$S = V_{rms} I_{rms}$$
$$= V_m I_m \left(\tfrac{1}{2}\right)$$

- Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

$$Z = \frac{V}{I} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle (\theta_v - \theta_i)$$

11.4 Apparent Power and Power Factor (2)

Purely resistive load (R)	$\theta_v - \theta_i = 0$, Pf = 1	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, pf = 0	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

Example 5 Obtain the power factor and the apparent power of a load whose impedance is Z = 60 + j40 when the applied voltage is 160cos(377t+10°)

$Z = 60 + j40 = 72.11\angle 33.69^\circ$

↑ power factor angle

Power Factor $= \cos(33.69) = \underline{\underline{0.8321}}$ $\underline{\text{lagging}}$

$I = \frac{V}{Z} = \frac{160\angle 10^\circ}{72.11\angle 33.69} = 2.21\angle -23.69^\circ$

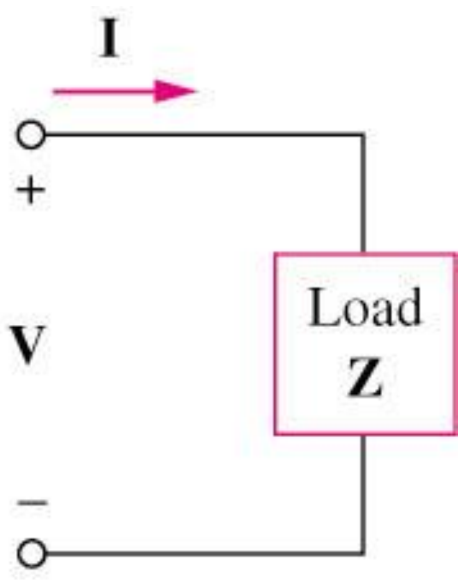
$S = \frac{V_m I_m}{Z} = \frac{(160)(2.21)}{Z} = \underline{\underline{176.8\ VA}}$ Apparent Power

$P = S\cos(\theta_v - \theta_i) = S\cos(10 - {-23.69}) = 176.8\cos(33.69^\circ)$
$= 147.1\ W$ Avg Pwr

a) Power factor = 0.8321 lagging, b) S = 176.8VA, c) P = 147.1 W

11.5 Complex Power (1)

Complex power **S** is the product of the voltage and the complex conjugate of the current:

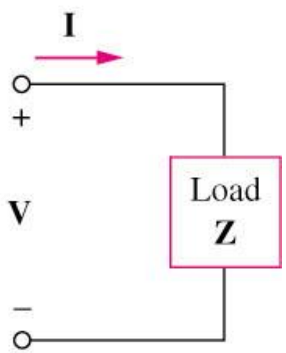


$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$S = \frac{1}{2} \mathbf{V}\,\mathbf{I}^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

$$S = \frac{1}{2}(V_m \angle \theta_v)(I_m \angle \theta_i)^* = \frac{1}{2} V_m I_m \angle(\theta_v - \theta_i)$$

11.5 Complex Power (2)



$$S = \frac{1}{2} V\, I^* = V_{rms}\, I_{rms}\, \angle \theta_v - \theta_i$$

$$\Rightarrow S = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

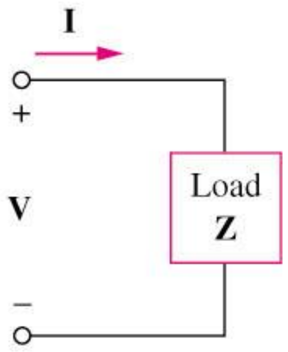
$$S = P + jQ$$

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

11.5 Complex Power (3)



$$\Rightarrow S = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = \underbrace{P}_{} + j\, \underbrace{Q}_{}$$

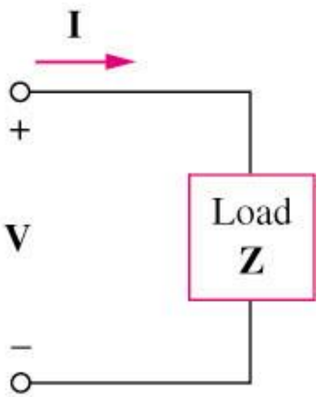
Apparent Power, $S = |\mathbf{S}| = Vrms*Irms = \sqrt{P^2 + Q^2}$

Real (average) power, $P = Re(\mathbf{S}) = S \cos(\theta_v - \theta_i)$

Reactive Power, $Q = Im(\mathbf{S}) = S \sin(\theta_v - \theta_i)$

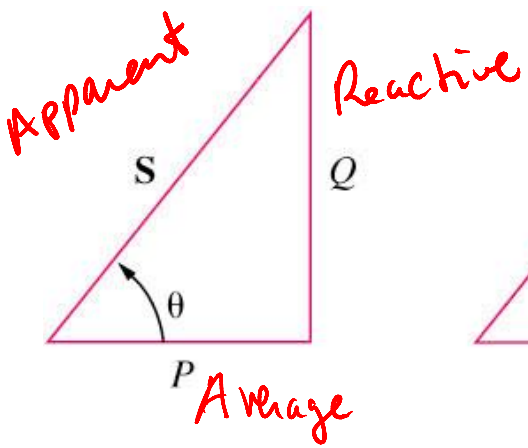
Power factor, $pf = P/S = \cos(\theta_v - \theta_i)$

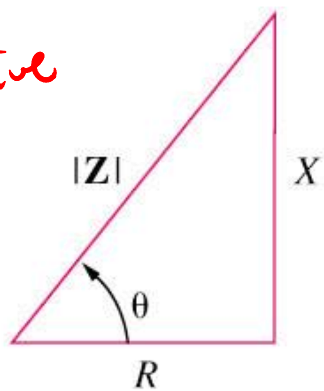
11.5 Complex Power (4)

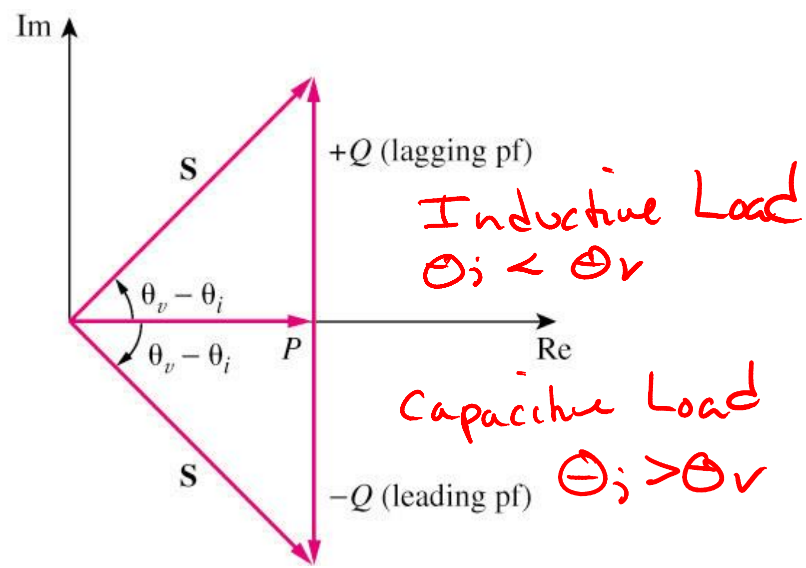


$$\Rightarrow S = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

$$S = P + j\, Q$$

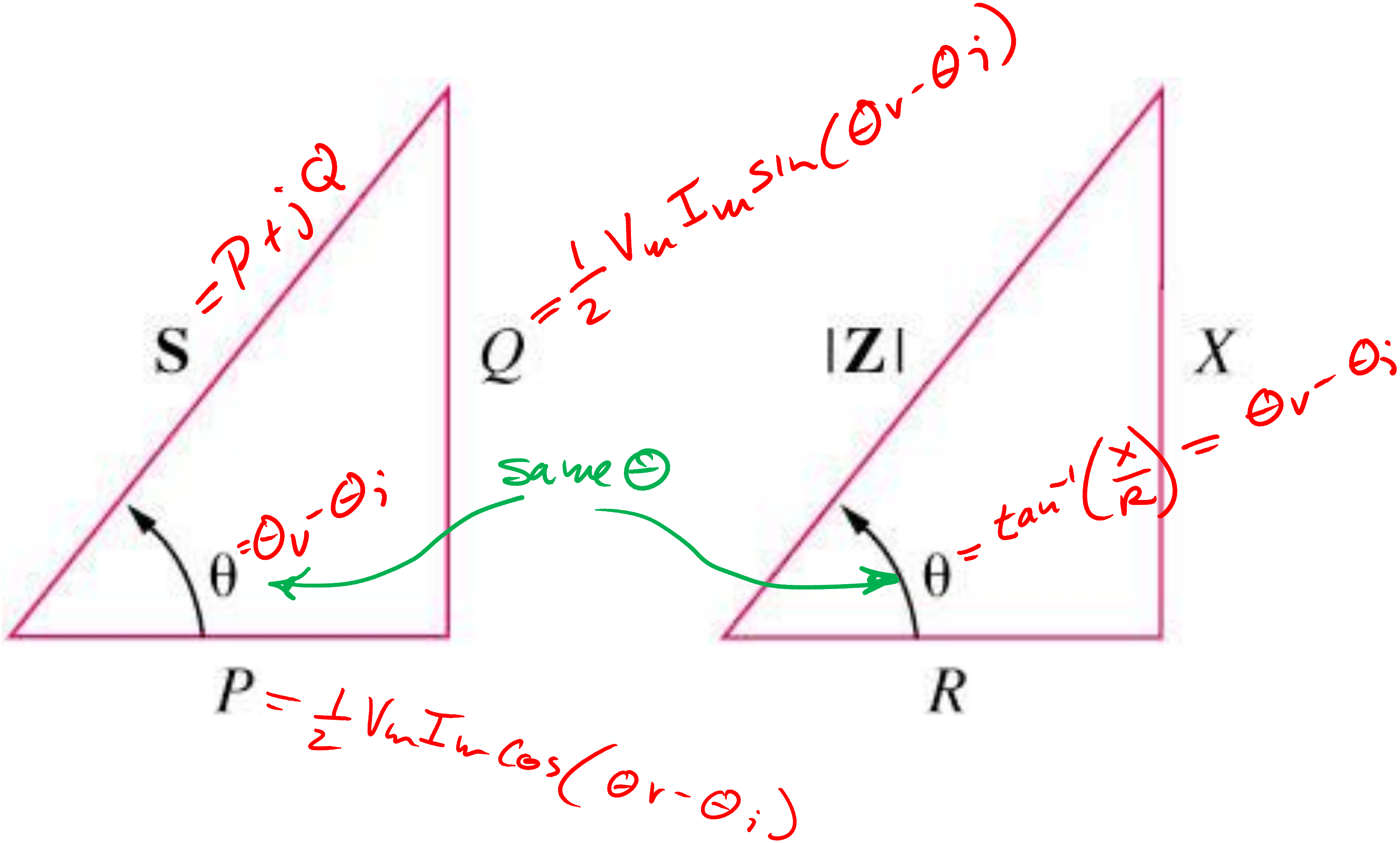






Power Triangle   Impedance Triangle   Power Factor

11.5 Power Triangle



Power Triangle **Impedance Triangle**

Example 6. Find the average power, the reactive power and the complex power delivered by the voltage if v = 6 cos (1000t) V

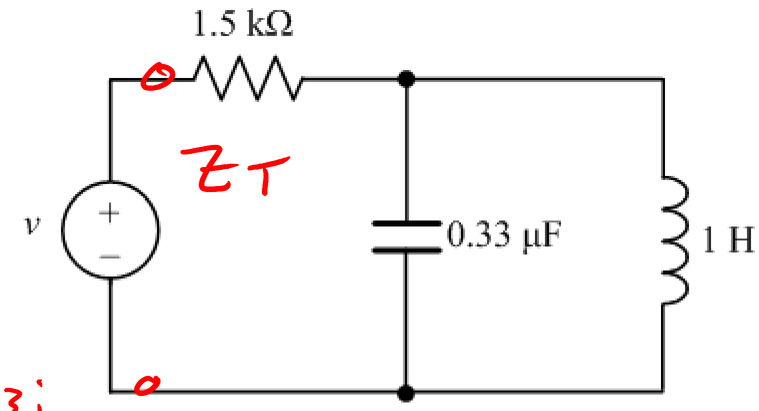
$\omega = 1000$

$Z_C = -j/(1000 \cdot .33\times10^{-6}) = -j3030$

$Z_L = j\omega L = 1000j$

$Z_T = 1.5k + (-j3.03) \| (1j)$

$= 1.5 + (1.493j) = 1.5 + 1.493j$



$Z_T = 2.11 \angle 44.8$

$V = 6\angle 0$, $I = \frac{V}{Z} = \frac{6\angle 0}{2.11\angle 44.8} = 2.844 \angle -44.8$

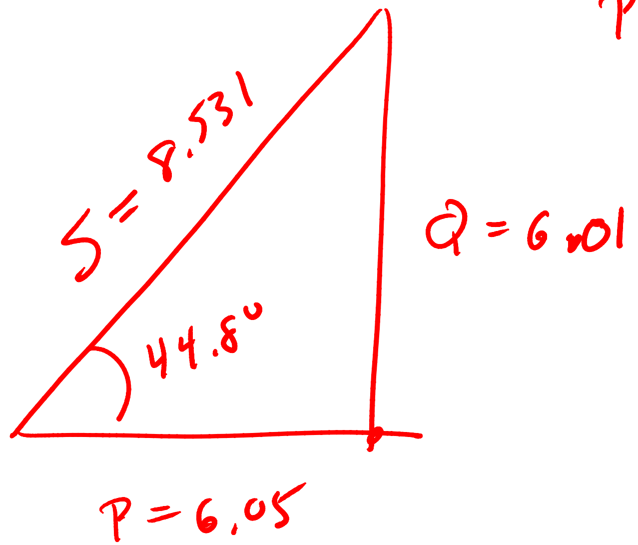
conjugate

$S = \frac{V \cdot I^*}{2} = (6\angle 0)(2.844\angle +44.8) = 6.05 + 6.01j$

$\therefore P = 6.05W$, $Q = +6.01$ VAR

P=6.04 mW, Q=6.01mVAR, S=(6.04+j6.01) mVolt-Amp

Draw the power triangle for example 6. What is power factor? Power factor angle?



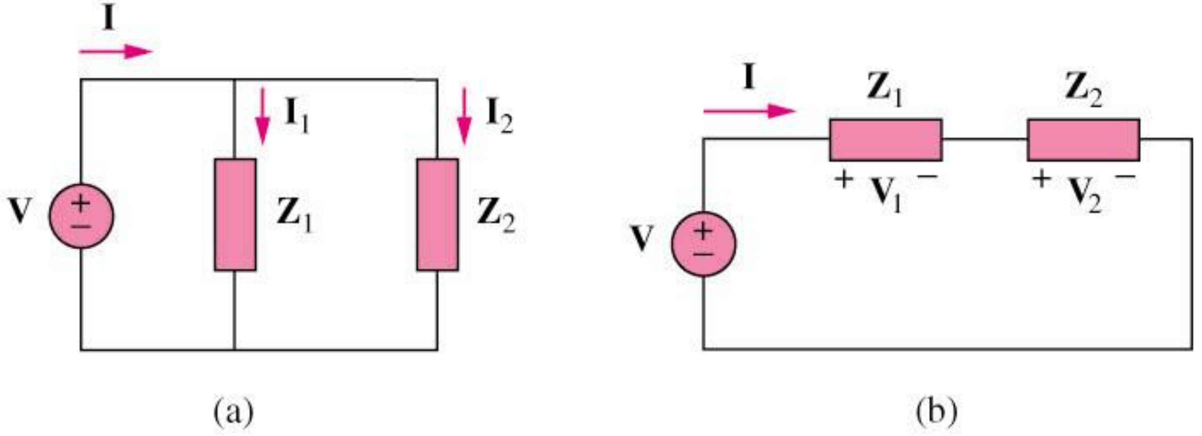
Power Factor Angle 44.8 > 0 → lagging

Power factor = .7095

Pf = .7095, lagging.   PFA = 44.8 deg

11.6 Conservation of AC Power (1)

The complex real, and reactive powers of the sources **equal** the respective **sums of** the complex, real, and reactive powers of the individual loads.



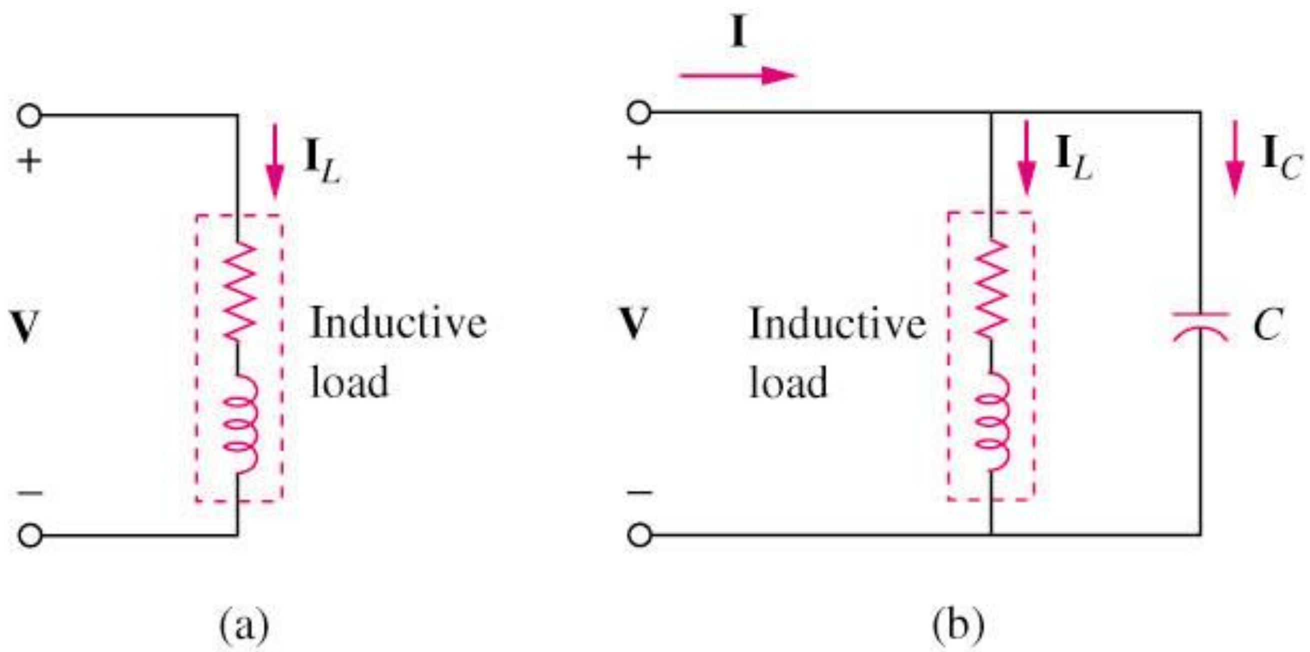
For parallel connection:

$$\bar{S} = \frac{1}{2}\overline{V}\,\overline{I^*} = \frac{1}{2}\overline{V}(\overline{I_1^*} + \overline{I_2^*}) = \frac{1}{2}\overline{V}\,\overline{I_1^*} + \frac{1}{2}\overline{V}\,\underline{\overline{I_2^*}} = \overline{S_1} + \overline{S_2}$$

The same results can be obtained for a series connection.

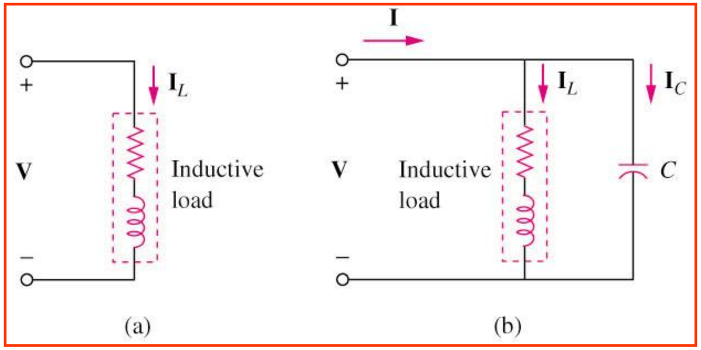
11.7 Power Factor Correction (1)

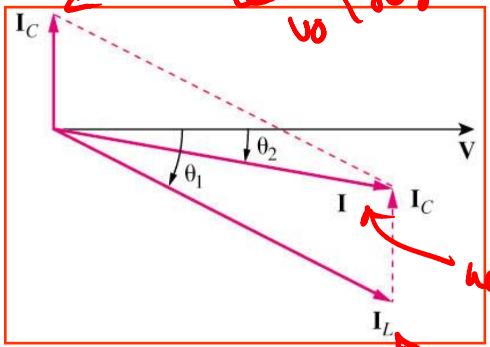
Power factor correction is the process of increasing the power factor without altering the voltage or current to the original load.



Power factor correction is necessary for economic reason.

11.7 Power Factor Correction (2)



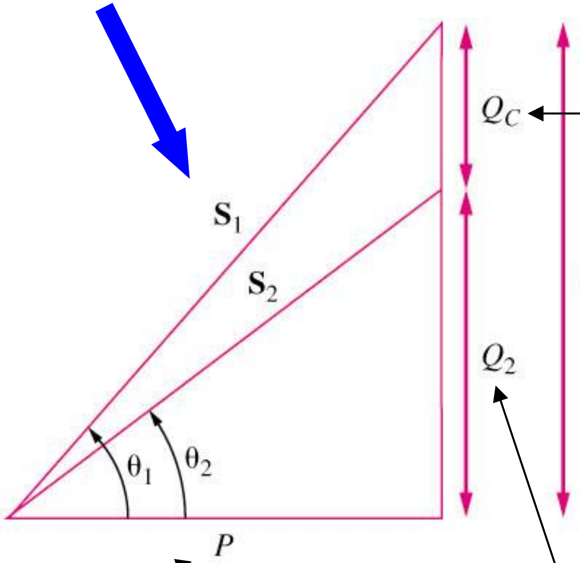


current "leads" voltage

ωt

new I < I_L

current "lags" voltage



$Q_c = Q_1 - Q_2$
$= P(\tan\theta_1 - \tan\theta_2)$
$= \omega C V^2_{rms}$

$I_c = j\omega C V_c$

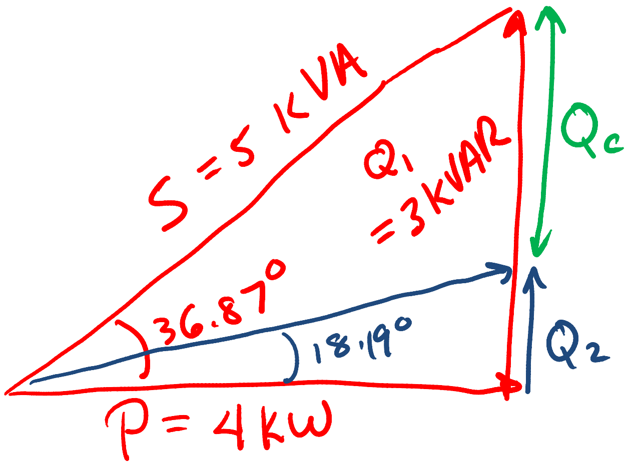
$Q_1 = S_1 \sin\theta_1$
$= P \tan\theta_1$

$Q_2 = P \tan\theta_2$

$P = S_1 \cos\theta_1$

$$C = \frac{Q_c}{\omega V^2_{rms}} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V^2_{rms}}$$

Example 7. When connected to a 120-V (rms), 60-Hz line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the power factor to 0.95



$pf = 0.8 \rightarrow pf_a = \cos^{-1}(0.8) = 36.87^\circ$

$pf = \frac{P}{S}$

$|S| = \frac{P}{pf}$

$\therefore S = \frac{P}{.8} = \frac{4kW}{.8} = 5kW$

$Q_1 = S\sin(36.87^\circ) = 5\sin 36.87 = 3$

$C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2} = \frac{4000(.75 - .3286)}{377(120)^2}$

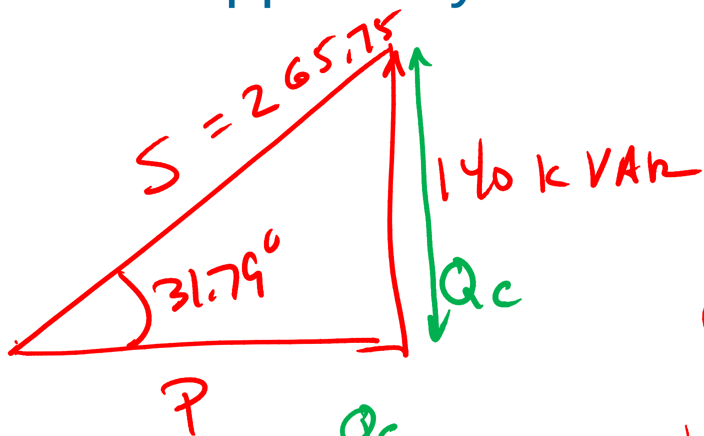
$\theta_1 = 36.87$

$\theta_2 = \cos^{-1}(.95) = 18.19^\circ$

$= 310.5\ \mu F$

C = 310.5 uF

Example 8. Find the value of a parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume the load is supplied by a 110 Vrms, 60 Hz line.`



(1) $pf = .85 \rightarrow pfa = \cos^{-1}(.85)$

$pfa = 31.79°$

(2) $S \sin(31.79) = 140\ kVAR$

$S = \left(\frac{140}{\sin 31.79}\right) kVA$

$= 265.75\ kVA$

(3) $P = S\cos(31.79) = 265.75\cos(31.79)$

$= 225.9\ kW = 225900\ W$

(4) $C = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$ (Q_c)

$\theta_1 = 31.79$

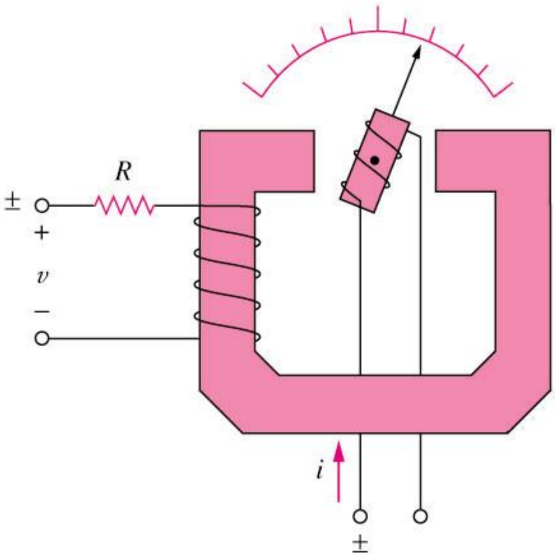
$\theta_2 = 0$

(5) $C = \frac{225900(.6198 - 0)}{(377)(120^2)} = 30.7\ mF$

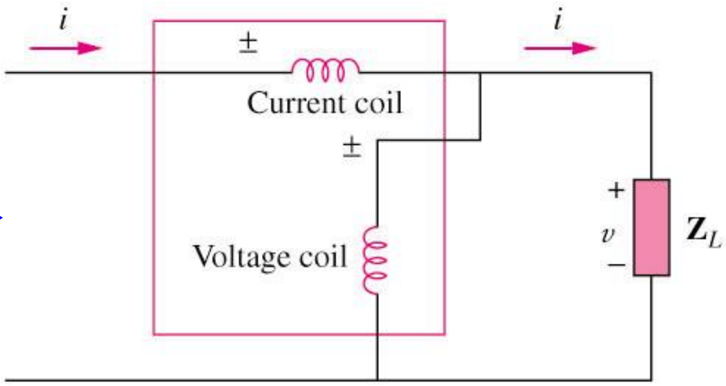
C = 30.7 mF

11.8 Power Measurement (1)

The wattmeter is the instrument for measuring the average power.



The basic structure



Equivalent Circuit with load

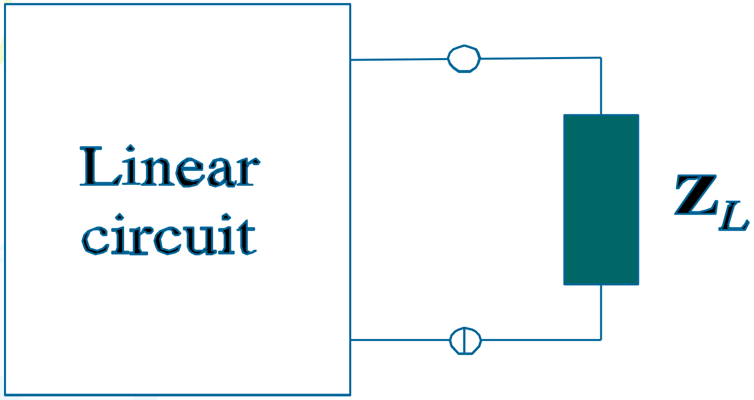
If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

$$P = |V_{rms}||I_{rms}| \cos(\theta_v - \theta_i) = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

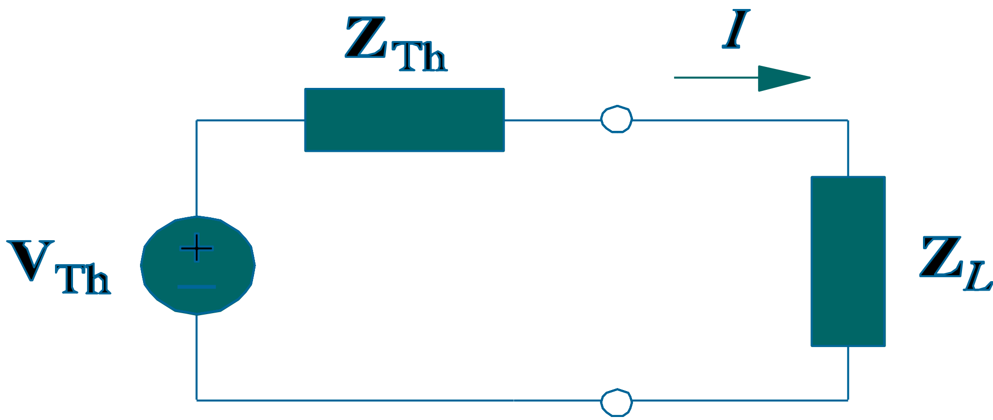
Optional Support Material

Maximum Average Power Transfer

➢ Finding the maximum average power which can be transferred from a linear circuit to a Load connected.



a) Circuit with a load



b) Thevenin Equivalent circuit

- Represent the circuit to the left of the load by its Thevenin equiv.
- Load Z_L represents any element that is absorbing the power generated by the circuit.
- Find the load Z_L that will absorb the **Maximum Average Power** from the circuit to which it is connected.

Maximum Average Power Transfer Condition

- Write the expression for average power associated with Z_L: $P(Z_L)$.

$$Z_{Th} = R_{Th} + jX_{Th} \qquad Z_L = R_L + jX_L$$

$$I = \frac{V_{Th}}{Z_{Th} + Z_L} = \frac{V_{Th}}{(R_{Th} + jX_{Th}) + (R_L + jX_L)} \qquad P = \frac{1}{2}|I|^2 R_L = \frac{|V_{Th}|^2 R_L/2}{(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2}$$

Ajust R_L and X_L to get maximum P

$$\frac{\partial P}{\partial X_L} = \frac{|V_{Th}|^2 R_L (X_{Th} + X_L)}{\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2}$$

$$\frac{\partial P}{\partial R_L} = \frac{|V_{Th}|^2 \left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2 - 2R_L(R_{Th} + R_L)\right]}{2\left[(R_{Th} + R_L)^2 + (X_{Th} + X_L)^2\right]^2}$$

$$\frac{\partial P}{\partial X_L} = 0 \Rightarrow X_L = -X_{Th} \qquad \frac{\partial P}{\partial R_L} = 0 \quad \Rightarrow \quad R_L = \sqrt{R_{Th}^2 + (X_{Th} + X_L)^2} = R_{Th}$$

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z_{Th}^*$$

Maximum Average Power Transfer Condition

- Therefore: $Z_L = R_{Th} - X_{Th} = Z_{Th}$ will generate the maximum power transfer.
- Maximum power $P_{\max}$ $$P_{\max} = \frac{I_L^2 R_L}{2} = \frac{|V_{Th}|^2}{8R_{Th}}$$

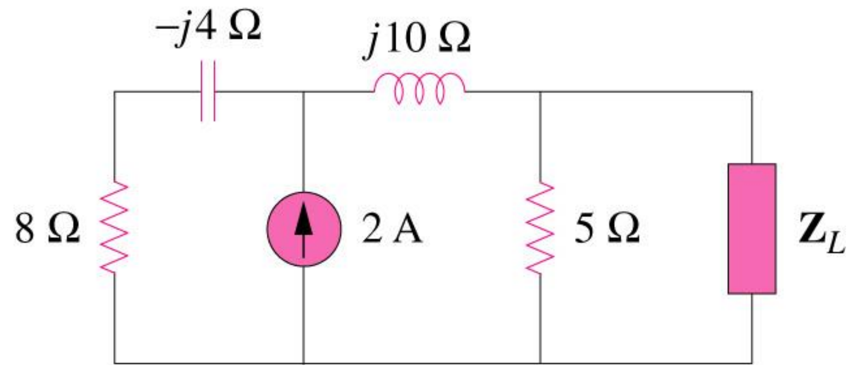
➢ For Maximum average power transfer to a load impedance Z_L we must choose Z_L as the complex conjugate of the Thevenin impedance Z_{Th}.

$$Z_L = R_L + jX_L = R_{Th} - jX_{Th} = Z^*_{Th}$$

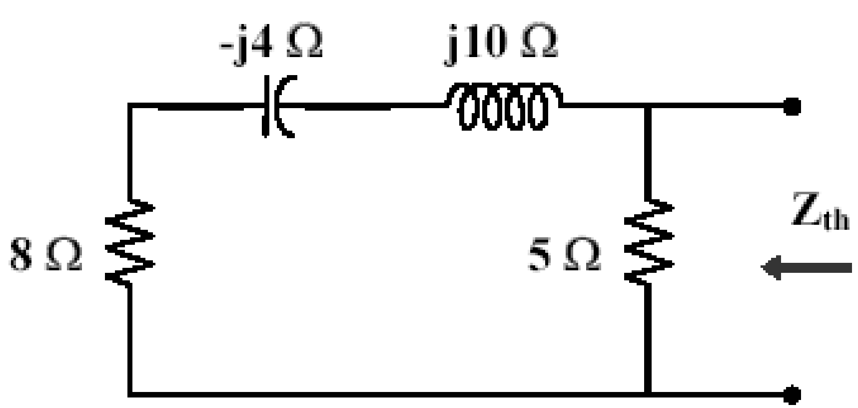
$$P_{\max} = \frac{|V_{Th}|^2}{8R_{Th}}$$

Maximum Average Power Transfer

➢ Practice Problem 11.5: Calculate the load impedance for maximum power transfer and the maximum average power.



P.P.11.5 We first obtain the Thevenin equivalent circuit across $\mathbf{Z}_L$. $\mathbf{Z}_{Th}$ is obtained from the circuit in Fig. (a).



(a)

$$\mathbf{Z}_{Th} = 5 \,||\, (8 - j4 + j10) = \frac{(5)(8 + j6)}{13 + j6} = 3.415 + j0.7317$$

Handouts

Example 1

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 80\cos(10t + 20^\circ)$$

$$i(t) = 15\sin(10t + 60^\circ)$$

Answer: $385.7 + 600\cos(20t - 10^\circ)\text{W}, 387.5\text{W}$

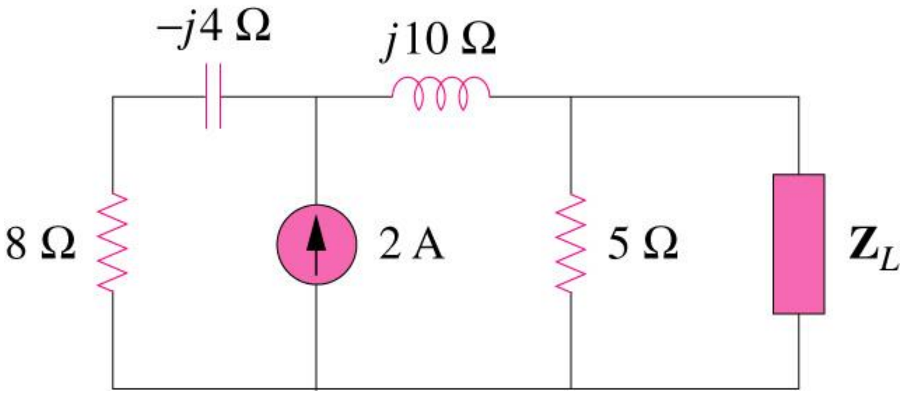
Example 2

A current $\mathbf{I} = 10\angle 30^\circ$ flows through an impedance $\mathbf{Z} = 20\angle -22^\circ\Omega$. Find the average power delivered to the impedance.

Answer: 927.2W

11.2 Maximum Average Power Transfer

Example 3 For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate that maximum average power.



Answer: 3.415 – j0.7317Ω, 1.429W

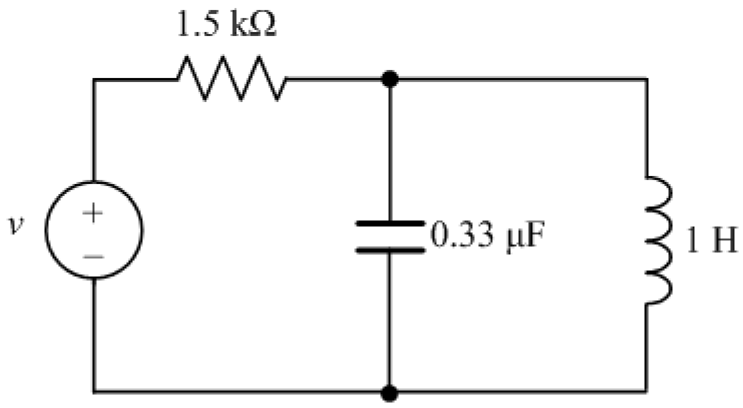
Example 4

A sinusoidal voltage having a max amplitude of 625 V is applied to the terminals of a 50Ω resistor. Find a) RMS voltage. b) RMS current. c) Average power

a) Vrms = 441.9V, b) Irms = 8.839A, c)P =3906W

Example 5 Obtain the power factor and the apparent power of a load whose impedance is Z = 60 + j40 when the applied voltage is $160\cos(377t+10^{o})$

Example 6. Find the average power, the reactive power and the complex power delivered by the voltage if v = 6 cos (1000t) V



P=6.04 mW, Q=6.01mVAR, S=(6.04+j6.01) mVolt-Amp

Draw the power triangle for example 6. What is power factor? Power factor angle?

Pf = .7095, lagging.   PFA = 44.8 deg

Example 7. When connected to a 120-V (rms), 60-Hz line, a load absorbs 4 kW at a lagging power factor of 0.8. Find the value of capacitance necessary to raise the power factor to 0.95

C = 310.5 uF

Example 8. Find the value of a parallel capacitance needed to correct a load of 140 kVAR at 0.85 lagging pf to unity pf. Assume the load is supplied by a 110 Vrms, 60 Hz line.`

C = 30.7 mF