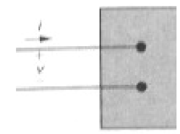
From the beginning, we have used p = iv for power, where positive means absorbing and negative means delivering. With sinusoidal signals, this formula still applies, but there are many variations/ways of applying it to AC circuits.



1) **<u>Instantaneous Power</u>** $p(t) = v(t)\,i(t)$ (Watts) Defined at every instant, t, $p(t)$ fluctuates in sinusoidal circuits, so is not very useful for analysis. But it underlies all other approaches to power.

   For example, if $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

   Various trig identities can be used to show that,

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$
$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i) \qquad (1)$$

2) **<u>Average Power</u>** Although $p(t)$ is a time varying sinusoid, the first term in the expression above is constant. It represents the average of the sinusoidal power waveform.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$ **AVERAGE Power (Unit: Watts)**

   P, the Average or Real power, is the power that is transformed from electric to non-electric energy. P is the power a device is actually consuming.

3) **<u>Maximum Average Power Tansfer</u>** If we connect a load Z_L to a circuit, and can adjust Z_L, then the value of Z_L that will allow the load to receive the most average power is $Z_L = Z_{TH}^*$. In other words,

   a) Find V_{TH}, and Z_{TH} . Note that $Z_{TH} = R_{TH} + jX_{TH}$ (it has a real and imaginary part)

   b) For max power transfer, Z_L= complex conjugate of $Z_{TH} = R_{TH} - jX_{TH}$

   c) The maximum power receive by Z_L, $P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$

   d) If Z_L can only be resistive (R_L), then choose $R_L = |Z_{TH}| = \text{sqrt}(R_{TH}^2 + X_{TH}^2)$

4) **<u>Effective or RMS Value</u>** When measuring sinusoidal signals, it can be more convenient to measure the RMS value (Digital Multimeters to this automatically). RMS is the root means square of a value over one period, for example, $Y_{eff} = \sqrt{\frac{1}{T}\int_0^T y^2(t)\,dt} = Y_{rms}$ For sinewaves, $Y_{rms} = \frac{Y_m}{\sqrt{2}}$ and we can write

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos(\theta_v - \theta_i) = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)$$

   The effective (rms) value of a periodic current is the <u>dc current</u> that delivers the <u>same average power</u> to a resistor as <u>the periodic current</u>. The rms value of a periodic voltage has similar meaning.

5) **Apparent Power** (referred to as S), is the product of the rms values of voltage and current

$$S = V_{rms}\, I_{rms} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = \frac{V_m I_m}{2}$$ **APPARENT Power (Unit: VA)**

Therefore, $P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = S\cos(\theta_v - \theta_i)$

6) **Power Factor** pf = $\cos(\theta v - \theta i)$ and is considered a **lagging power factor** (inductive load) if current lags voltage, that is, $(\theta v > \theta i)$ and a **leading power factor** (capacitive load) if $(\theta v < \theta i)$

This quantity determines how resistive the load is and what type the reactive portion is. The power factor is also the cosine of the load angle, since $\boldsymbol{Z} = \frac{V_m < \theta_v}{I_m < \theta_i} = \frac{V_m}{I_m} < (\theta_v - \theta_i)$

7) **Complex Power** **S** = P + j Q (unit: Volt-Amps or VA) is the product of the Voltage phasor with the complex conjugate of the Current phasor. If phasor (boldface) $\mathbf{V} = V_m \cos(\theta_v)$, $\mathbf{I} = I_m \cos(\theta_i)$, then

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\,\mathbf{I}^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i = \mathbf{V_{rms}\, I^*_{rms}}$$

$$\mathbf{S} = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) + j\, V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

$$\mathbf{S} = \quad P \quad + \quad j\,Q$$

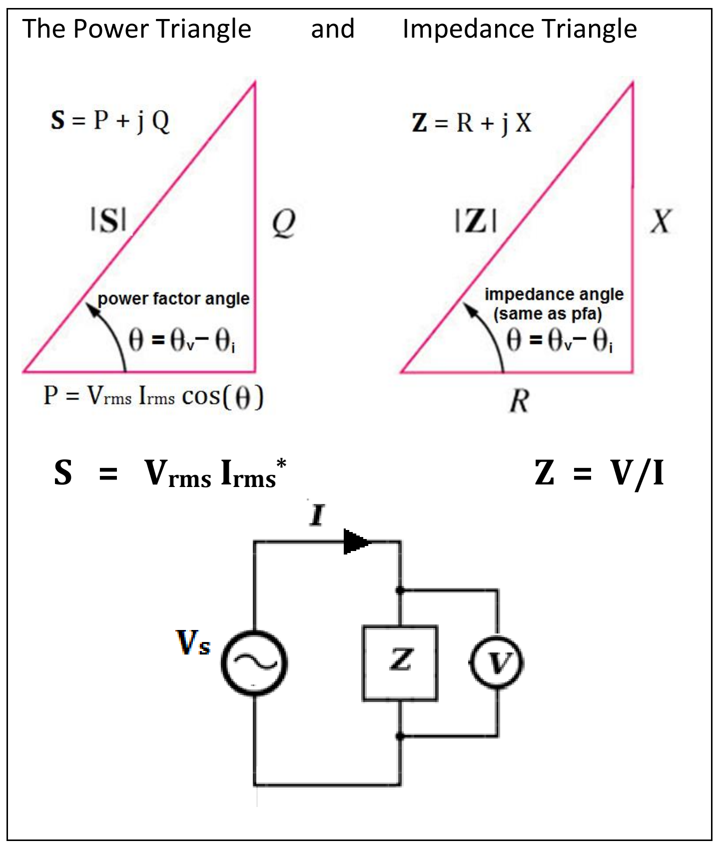
$$P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)$$

= the <u>average power (in Watts)</u> delivered to a load, the only useful power.

$$Q = V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)$$

= the <u>reactive power exchange</u> between the source and the reactive part of the load. It is measured in VAR.

Note that Apparent power S is the magnitude of complex power **S**, both in units of VA



8) **Power Factor Correction** energy savings encourage us to balance inductive loads with capacitive loads. If a load is inductive, with a phase angle of θ_1, the power factor angle can be brought back to a smaller angle, θ_2, with a capacitor attached in parallel to the load. The value of the capacitor is

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega\, V_{rms}^2}$$