Circuit Theory

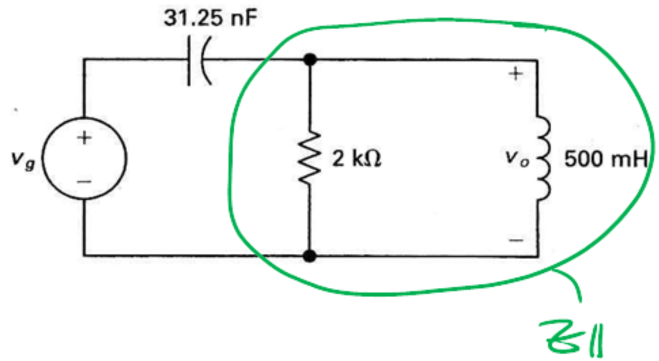
Chapter 9B

Sinusoidal Steady-State : Nodal Analysis, Mesh Analysis, Thevenin, etc

As the problems get more complex..

Ex 10

The circuit is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g = 64\cos 8000t$ V.



1) Find $Z_C = \frac{-j}{8000(31.25\times10^{-9})}$

$Z_C = -4000j$

2) Find $Z_L = j(8000)(.5)$

$Z_L = +4000j$

3) $V_g = 64\angle 0$

4) Voltage Divider $V_o = \left(\frac{Z_{||}}{Z_{||}+Z_C}\right) V_g$

$Z_{||} = \frac{(2000)(4000j)}{(2000+4000j)} = 1600 + 800j$

$V_o = \frac{(1600+800j)}{(1600+800j-4000j)} V_g = \left(\frac{1}{2}j\right)(64\angle 0) = 32\angle 90$

$\therefore v_o(t) = 32\cos(8000t + 90)$

2

vo(t) = -32 sin(8000t) (or 32cos(8000t + 90))

…we need more computational help. Using MATLAB as calculator (ver 1)

```
--> Vg = 64*(cosd(0) + j*sind(0))   % convert polar to rect
Vg =
   64.0000 +   0.0000i
--> ZC = -j/(8000*31.25e-9)      % impedance of capacitor
ZC =
   -0.0000 -4000.0000i
--> ZL = j*8000*0.5              % impedance of inductor
ZL =
    0.0000 +4000.0000i
--> ZP = 2000*4000j/(2000+4000j) % parallel inductor/resist
ZP =
  1600.0000 +800.0000i
--> Vo = (1600+800j)*(64)/(1600+800j - 4000j) % V-divider
Vo =
   -0.0000 + 32.0000i
--> abs(Vo)                         % magnitude of Vo
ans =
  32
--> angle(Vo)*180/pi                % phase angle of Vo
ans =
  90
```

Therefore, Vo = 32cos(8000t + 90)

Re-using variables for clarity (ver 2)

```
--> Vg = 64*ang(0)                % convert polar to rect
Vg =
   64.0000 +   0.0000i
--> ZC = -j/(8000*31.25e-9)       % impedance of capacitor
ZC =
  -0.0000 -4000.0000i
--> ZL = j*8000*0.5               % impedance of inductor
ZL =
    0.0000 +4000.0000i
--> ZP = 2000*ZL/(2000 + ZL)      % parallel resistor/inductor
ZP =
 1600.0000 +800.0000i
--> Vo = ZP*Vg/(ZP + ZC)          % voltage divider
Vo =
  -0.0000 + 32.0000i
--> magPhs(Vo)                    % print in polar form
32.000 < 90.00
```

Writing a script file for added clarity (ver 3)

```
% ENGR12 prob7_13.m  solution to phasor response problem

% circuit parameters
w = 8000;
L = 0.5;
C = 31.25E-9;
R = 2000;

% phasor domain
Vg = 64*ang(0);
ZL = j*w*L;
ZC = -j/(w*C);

% circuit calculations
ZP = (R*ZL)/(R+ZL);    % parallel combination of R and ZL


Vo = Vg * ZP / (ZP + ZC)   %  voltage divider formula

disp('Vo in magnitude and phase: ');
magPhs(Vo)
```

Program Output

```
Vo =
  -0.0000 + 32.0000i

Vo in magnitude and phase:
32.000 < 90.00
```

Two helper commands you can download into your work folder: 1) ang.m

- ang.m -- converts polar angle to rect

  Instead of:

  Vs = 10*(cosd(45) + j*sind(45))

  You can type: **Vs = 10*ang(45)**

Available on ENGR12 and 12L websites

Or click here: ang.m magPhs.m

Two helper commands you can download into your work folder: 2) magPhs.m

magPhs.m -- prints in polar form

Instead of:

```
--> abs(Vo)
ans =
 32
--> angle(Vo)*180/pi
ans =
 90
```

You can just type:

```
-->magPhs(Vo)
32.000<90.00
```

For those who know Matlab Functions

```
function complexValue = ang( thetaDeg )
   % function complexValue = ang( thetaDeg )
   %   returns complexValue associated with
   %      e^(i*thetaDeg)

   complexValue = cosd(thetaDeg) + j * sind(thetaDeg);
```

```
function  magPhs( complexValue )
   % function magPhs( complexValue )
   %  prints magnitude and phase in degrees of a complex value
   %

   fprintf('%1.3f < %1.2f\n',abs(complexValue),angle(complexValue)*180/pi);
```

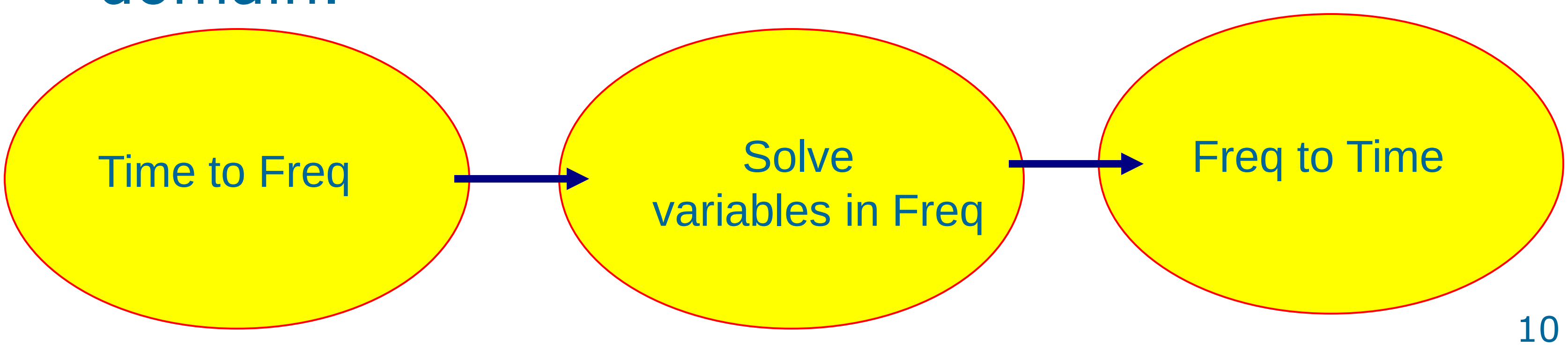
Sinusoidal Steady-State Analysis
Chapter 10

10.1 Basic Approach
10.2 Nodal Analysis
10.3 Mesh Analysis
10.4 Superposition Theorem
10.5 Source Transformation
10.6 Thevenin and Norton Equivalent Circuits

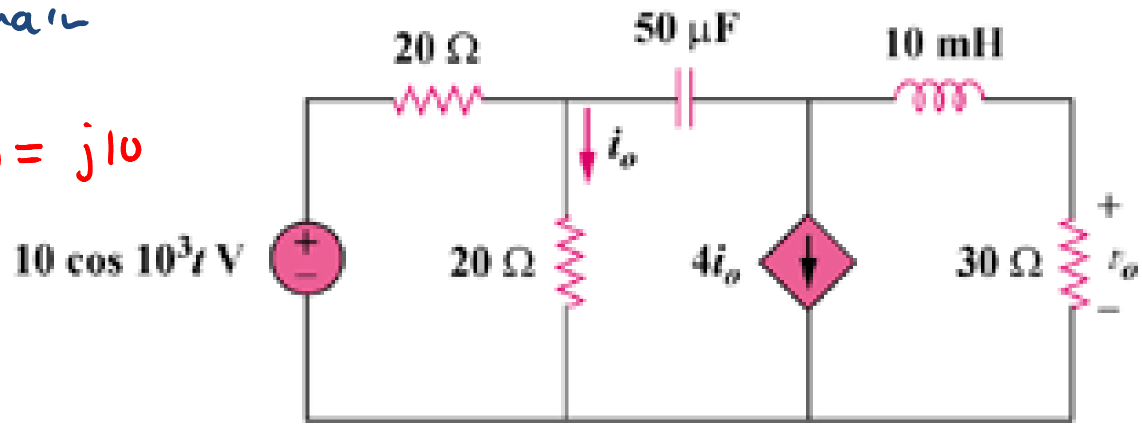
10.1 Basic Approach (1)

Steps to Analyze AC Circuits:

1. Transform the circuit to the phasor or frequency domain.
2. Solve the problem using circuit techniques (nodal analysis, mesh analysis, superposition, etc.).
3. Transform the resulting phasor to the time domain.



Example 1 Find Vo(t) using NA



1) Convert to Phasor Domain

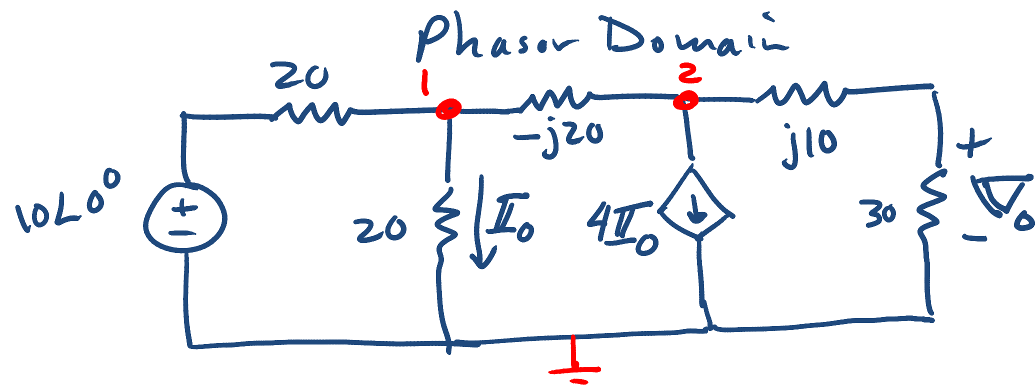
$\omega = 1000$

$Z_L = j\omega L = j1000(.01) = j10$

$Z_C = -j/\omega C = \frac{-j}{(1000)(50\times10^{-6})}$

$= -j20$

2) Set up Nodal Eqs



1) $\frac{V_1-10}{20} + \frac{V_1}{20} + \frac{V_1-V_2}{-j20} = 0$

2) $\frac{V_2-V_1}{-j20} + 4I_o + \frac{V_2}{30+j10} = 0$

$I_o = \frac{V_1}{20}$

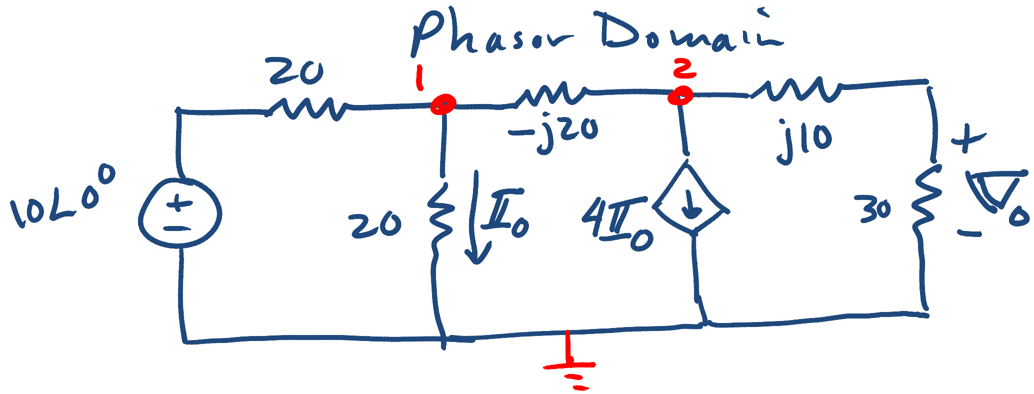
1) $V_1\left(\frac{1}{20} + \frac{1}{20} - \frac{1}{j20}\right) + V_2\left(\frac{1}{j20}\right) = \frac{10}{20}$

2) $V_1\left(\frac{1}{j20} + \frac{4}{20}\right) + V_2\left(\frac{-1}{j20} + \frac{1}{30+j10}\right) = 0$

→ FREEMAT

```
--> A = [(2/20 + j/20), (-j/20); (-j/20+.2),(j/20 + 1/(30+10j))]
A =
   0.1000 +  0.0500i  -0.0000 -  0.0500i
   0.2000 -  0.0500i   0.0300 +  0.0400i
--> B = [0.5;0]
B =
    0.5000
         0
--> V = A\B
V =
   1.4356 -  0.6436i
   0.1485 +  6.4851i
--> Vo = V(2)*30/(30+10j)
Vo =
   2.0792 +  5.7921i
--> magPhs(Vo)
6.154 < 70.25
```

$\therefore V_o(t) = 6.15 \cos(1000t + 70.3°)$



Check, KCL node 1:

$$\frac{(V_1 - 10)}{20} + \frac{V_1}{20} + \frac{V_1 - V_2}{-j20} \stackrel{?}{=} 0$$

```
--> (V(1)-10)/20 + V(1)/20 + (V(1)-V(2))/(-20j)
ans =
 0.0000e+000 -1.3878e-017i
```

Check!

Example 2

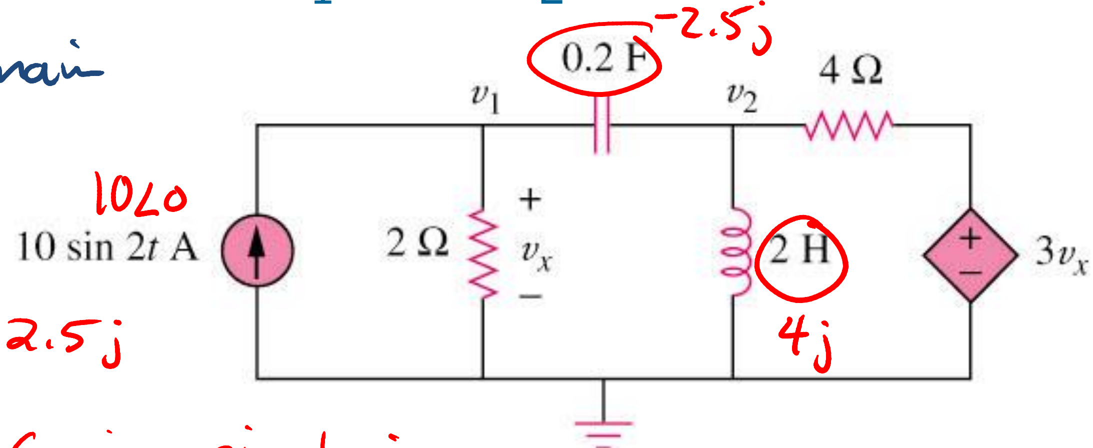
Using nodal analysis, find v_1 and v_2 in the circuit

1) Convert to Phasor Domain

$\omega = 2$

$Z_L = j\omega L = 4j$

$Z_C = -j/\omega C = \frac{-j}{(2)(.2)} = -2.5j$



$10 \sin 2t \Longrightarrow 10\angle 0$ (using sin basis, or $10\angle -90$ using cos)

2) Nodal Eqs

$-10 + \frac{V_1}{2} + \frac{V_1 - V_2}{-2.5j} = 0 \rightarrow V_1\left(\frac{1}{2} + \frac{j}{2.5}\right) + V_2\left(\frac{-j}{2.5}\right) = 10$

$\frac{V_2 - V_1}{-2.5j} + \frac{V_2}{4j} + \frac{V_2 - 3V_x}{4} = 0 \rightarrow V_1\left(\frac{-j}{2.5} - \frac{3}{4}\right) + V_2\left(\frac{j}{2.5} - \frac{j}{4} + \frac{1}{4}\right) = 0$

$V_x = V_1$

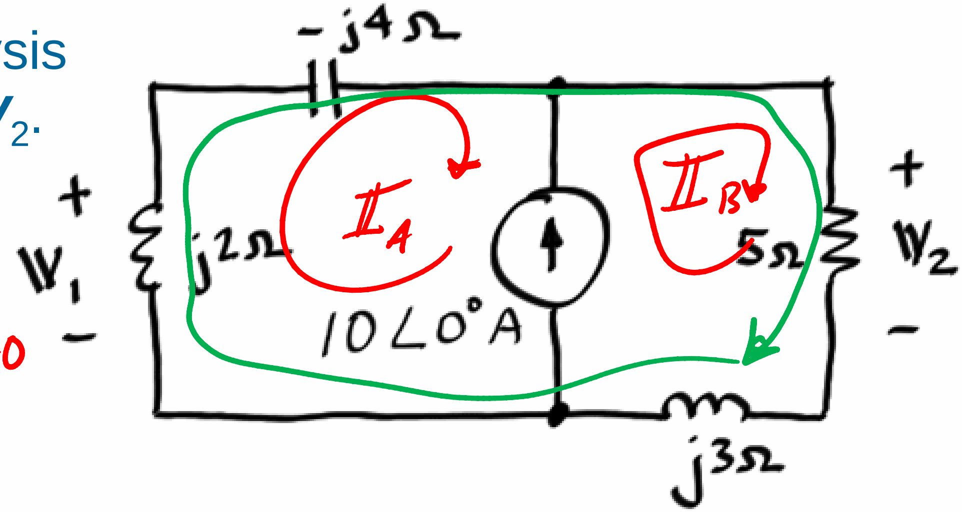
Answer: $v_1(t)$ = **11.32 sin(2t + 60.01°) V** **$v_2(t)$ = 33.02 sin(2t + 57.12°) V**

```
--> A = [(1/2 + j/2.5),-j/2.5; (-j/2.5-3/4),(j/2.5-j/4+1/4)]
A =
   0.5000 +  0.4000i  -0.0000 -  0.4000i
  -0.7500 -  0.4000i   0.2500 +  0.1500i
--> b = [10;0];
--> V= A\b
V =
   5.6604 +  9.8113i
  17.9245 + 27.7358i
--> magPhs(V)
```

11.327 < 60.02 → v1(t)=11.32 sin(2t + 60°)

33.024 < 57.13 → v2(t)=33.02 sin(2t + 57.1°)

Example 3 Use mesh current analysis to find the phasor voltages $\mathbf{V}_1$ and $\mathbf{V}_2$.



Supermesh Eq

$$j2I_A - j4I_A + 5I_B + j3I_B = 0$$

Constraint Eq

$$I_B - I_A = 10\angle 0^\circ$$

Simplify

$$I_A(-2j) + I_B(5+3j) = 0$$

$$I_A(-1) + I_B(1) = 10$$

```
--> A=[-2j, 5+3j; -1, 1];
--> b = [0;10];
--> I = A\b;
--> magPhs(I)
11.43 < -160.4
3.92 < -101.3
```

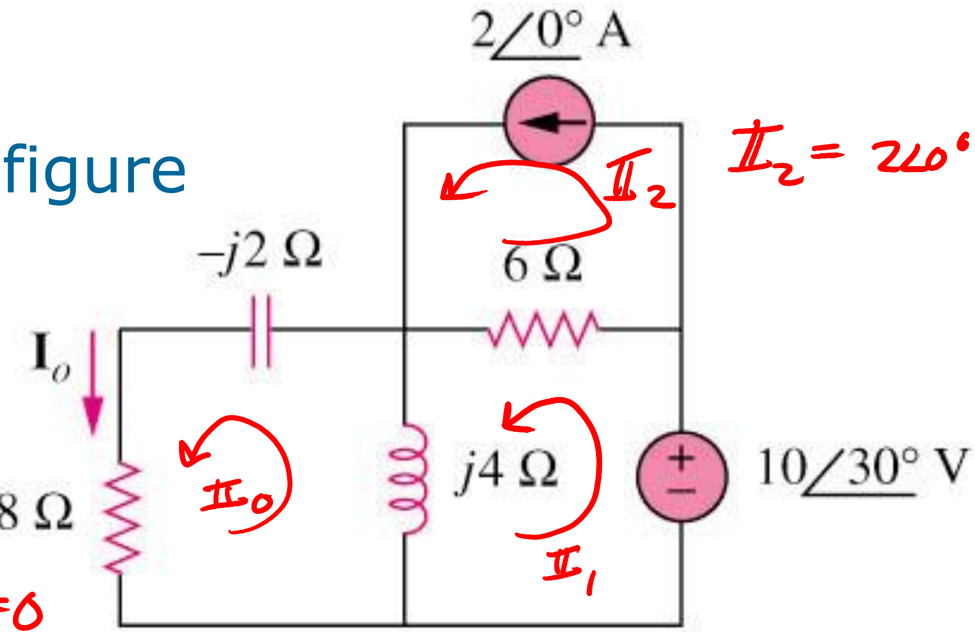
$$V_1 = -j2I_A = 2(-j)(11.43\angle -160.4) = 2(e^{-j90})(11.43)(e^{-j160}) = 22.9\angle 110^\circ$$

$$V_2 = 5I_B = 5(3.92\angle -101) = 19.6\angle -101$$

$\mathbf{V}_1 = 22.9\angle 110^\circ$ V; $\mathbf{V}_2 = 19.6\angle -101^\circ$ V

Example 4

Find I_o in the following figure using mesh analysis.



Mesh Eqs

$$j4(I_o - I_1) - j2 I_o + 8 I_o = 0$$

$$-10\angle 30 + 6(I_1 - 2) + j4(I_1 - I_o) = 0$$

Simplify

$$I_o(8 + j2) - j4 I_1 = 0$$

$$-j4 I_o + (6 + j4) I_1 = 10\angle 30 + 12$$

```
--> A=[8+2j,-4j;-4j,6+4j];
--> b=[0;10*ang(30)+12];
--> magPhs(A\b)
1.194 < 65.45
2.461 < -10.52
```

Answer: $\mathbf{I_o} = \mathbf{1.194\angle 65.44^{0}}$ **A**

10.4 Superposition Theorem

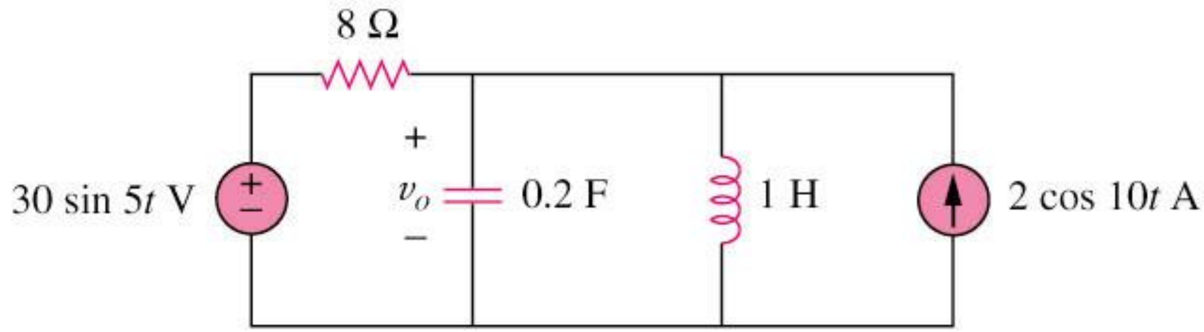
When a circuit has sources operating at different frequencies,

- The separate phasor circuit for each frequency must be solved independently, and
- The total response is the sum of time-domain responses of all the individual phasor circuits.

10.4 Superposition Theorem

Example 5

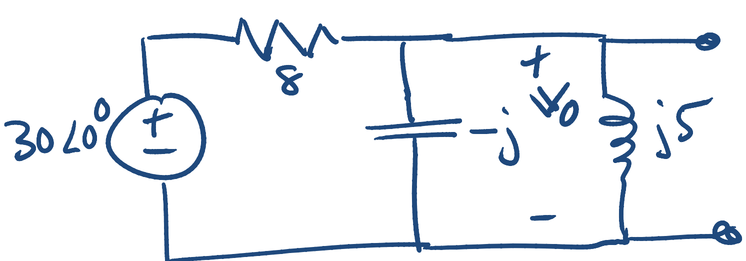
Calculate v_o in the circuit of figure shown below using the superposition theorem.



2 Frequencies!

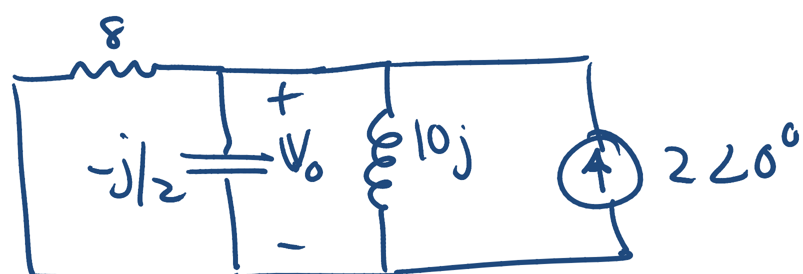
	$\omega=5$	$\omega=10$
Z_L	$j5$	$j10$
Z_C	$-j$	$-j/2$

Turn off I src, $\omega=5$



$$V_o = 30\left(\frac{-j \| j5}{8+(-j\|j5)}\right) = 4.63\angle -81.1^\circ$$

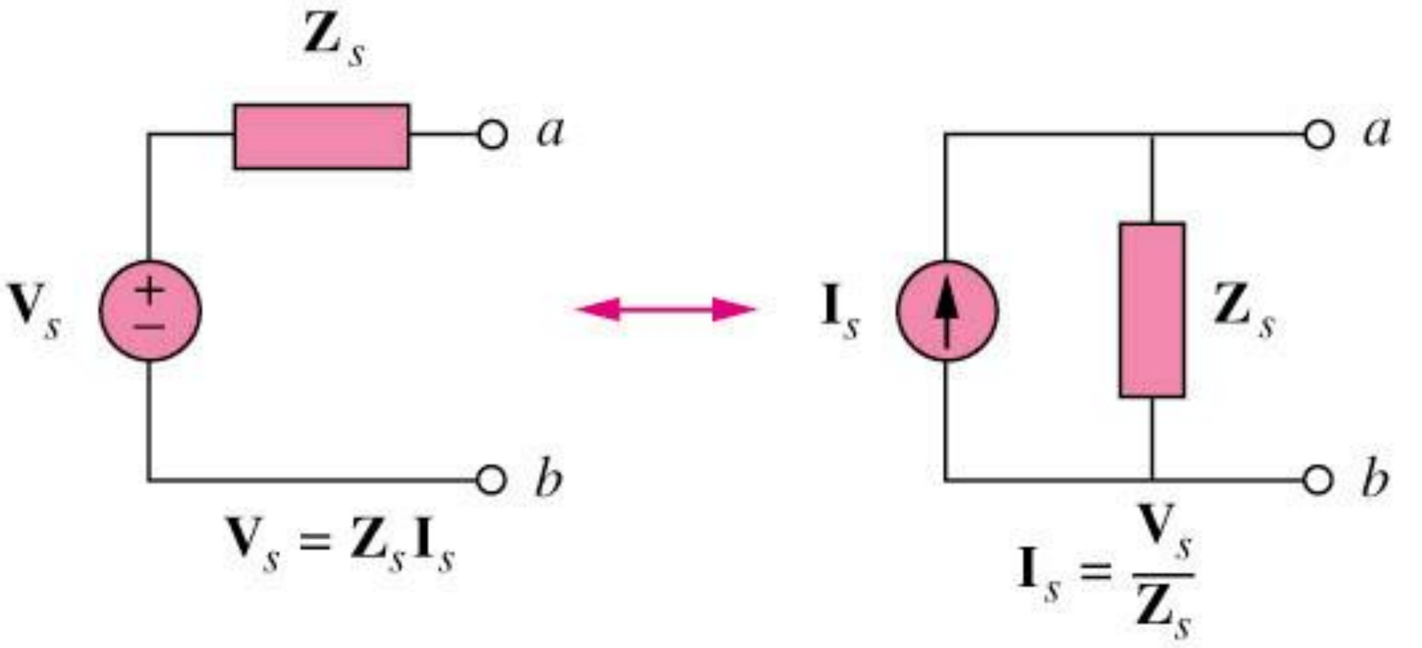
Turn off V src, $\omega=10$



$$V_o = 2\left(\frac{1}{\frac{1}{8}+\frac{1}{-j/2}+\frac{1}{10j}}\right) = 1.05\angle -86.2^\circ$$

V_o = **4.631 sin(5t – 81.12^0) + 1.051 cos(10t – 86.24^0) V**

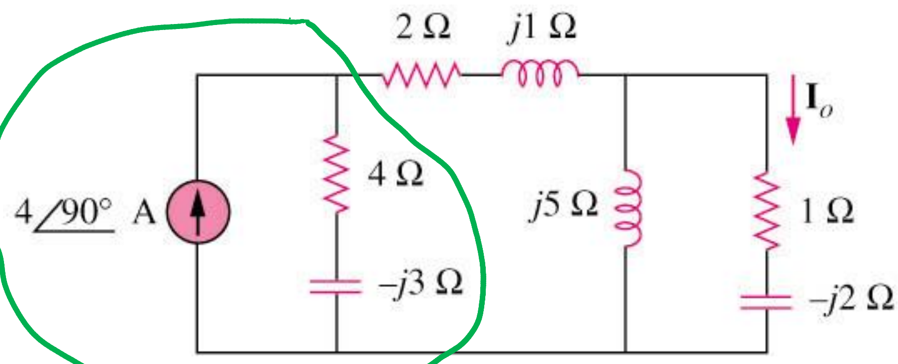
10.5 Source Transformation (1)

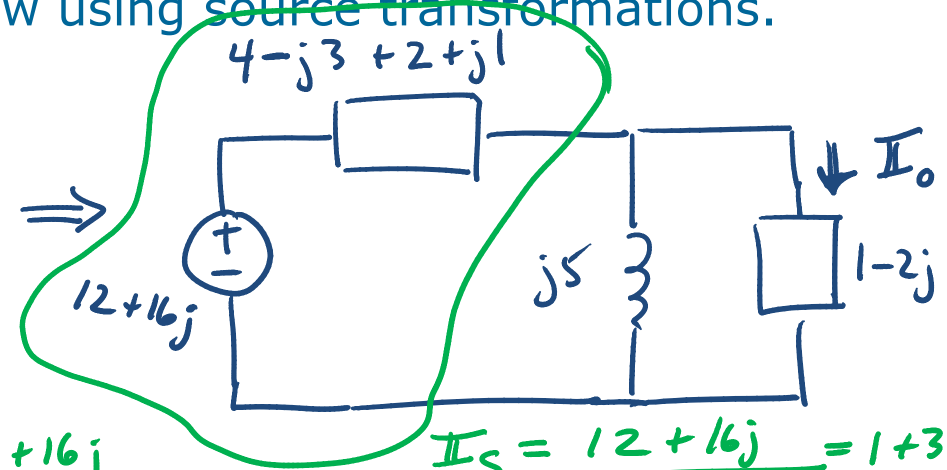


10.5 Source Transformation

Example 6

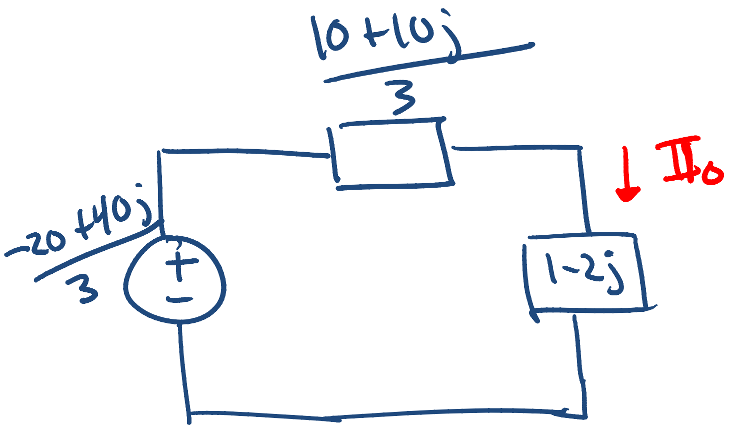
Find Io in the circuit of figure below using source transformations.

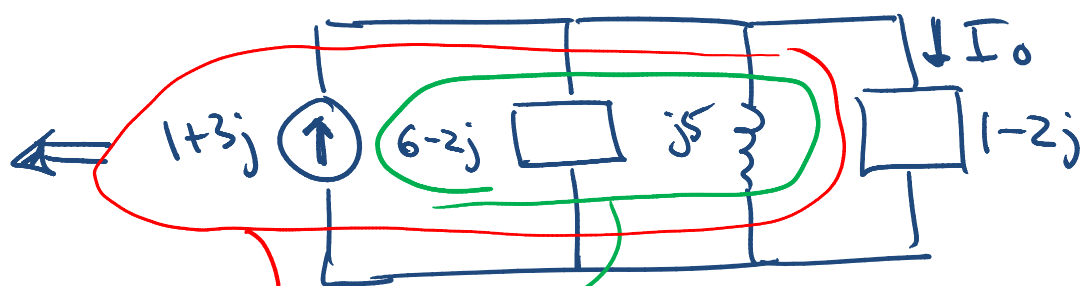




$$V_s = (4\angle 90)(4-j3) = 12+16j$$

$$I_s = \frac{12+16j}{6-2j} = 1+3j$$





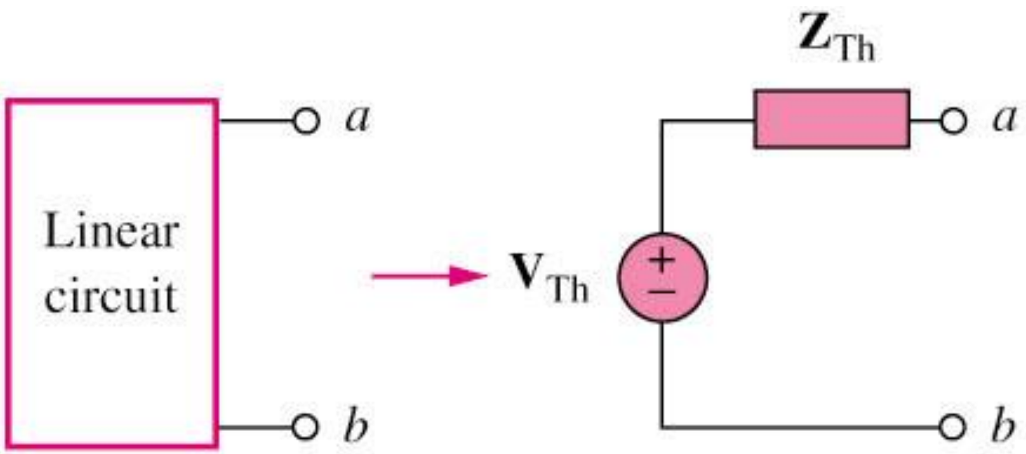
$$(6-2j) \parallel j5 = \frac{10+10j}{3}$$

$$V_s = (1+3j)\left(\frac{10+10j}{3}\right) = \frac{(-20+40j)}{3}$$

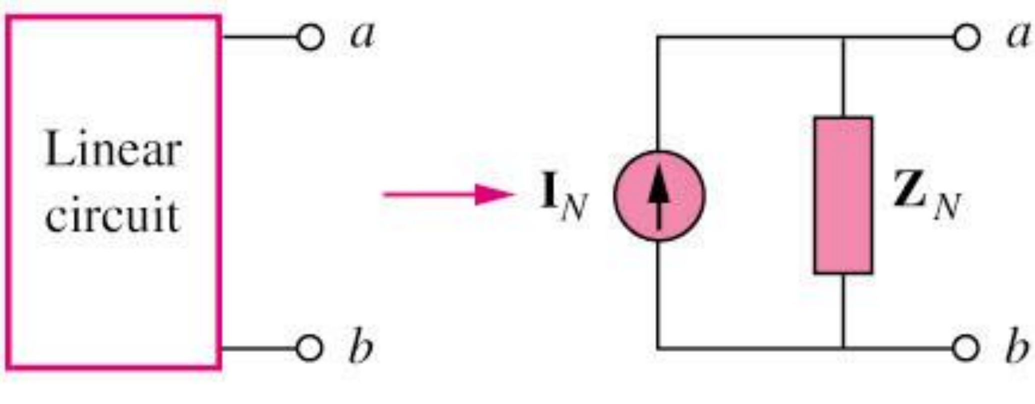
$$I_o = \frac{\left(-\frac{20+40j}{3}\right)}{\left(\frac{10+10j}{3} + 1-2j\right)} = 3.28\angle 99.5^0$$

I_o = **3.288<99.46^0 A**

10.6 Thevenin and Norton Equivalent Circuits (1)



Thevenin equivalent

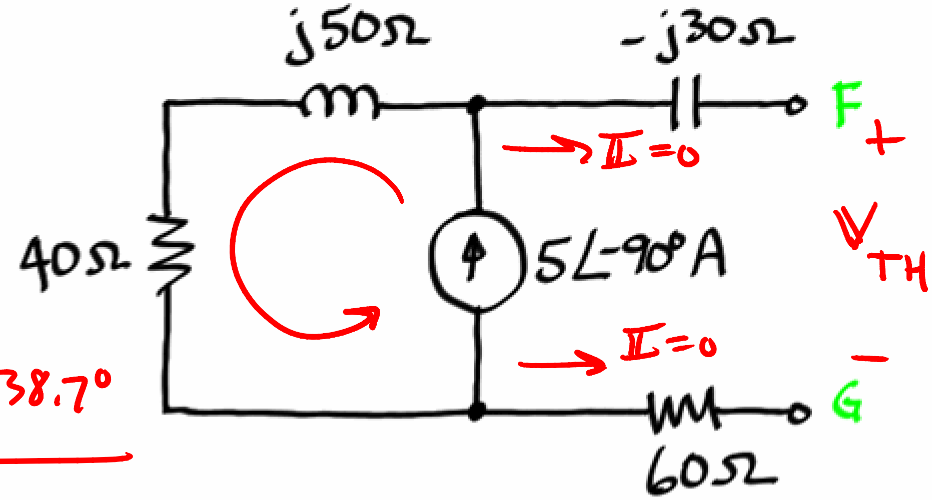


Norton equivalent

Example 7

Find the Thevenin equivalent circuit at the terminals *F-G*. Express all complex values in your solution in both rectangular and polar form.

V_{TH} = OPEN CCT VOLTAGE AT F-G



$$V_{TH} = (5\angle -90)(40 + j50)$$

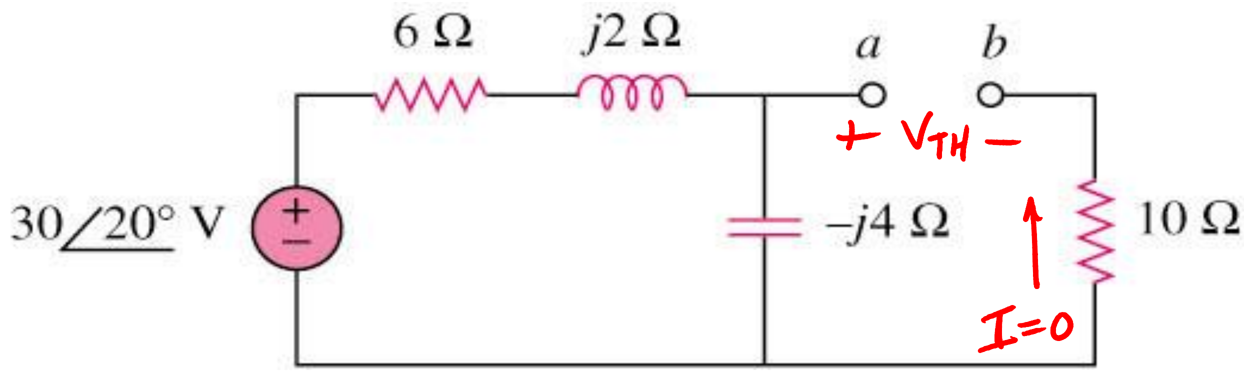
$$= 250 - 200j = 320\angle -38.7^\circ$$

Z_{TH} = "LOOKBACK" IMPEDANCE at F-G (SRC turned off)

$$Z_{TH} = -j30 + j50 + 40 + 60 = 100 + j20 = 102\angle 11.3^\circ\ \Omega$$

$\boldsymbol{V}_{TH}$ = 250 - *j*200 V = 320 ∠ -38.7° V; $\boldsymbol{Z}_{TH}$ = 100 + *j*20 Ω = 102 ∠ 11.3° Ω

Example 8 Find the Thevenin equivalent at terminals a–b of the circuit



V_{TH}

$$V_{TH} = \frac{(30\angle 20)(-j4)}{(6+2j-j4)}$$

$$= 18.97\angle -51.57°$$

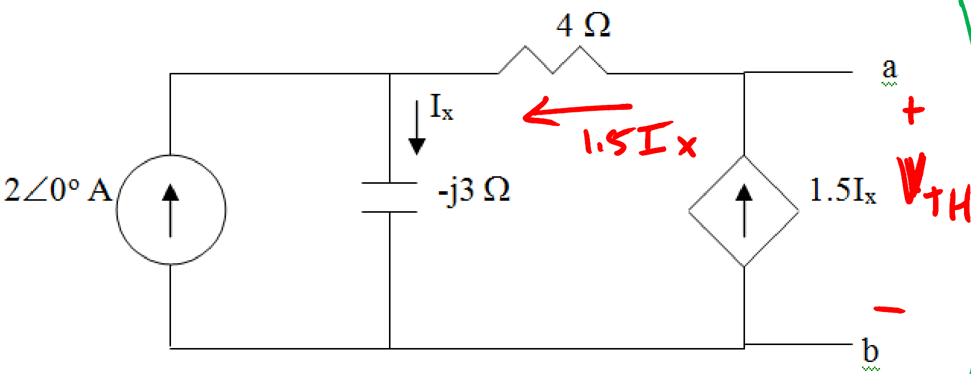
Z_{TH}

$$Z_{TH} = (6+j2)\,||\,(-j4) + 10 = \frac{62-16j}{5} = 12.4-3.2j$$

Z_{th} = **12.4 – j3.2**

V_{TH} = **18.97<-51.57^0 V**

Example 9. Find the Thevenin Equivalent



V_{TH}

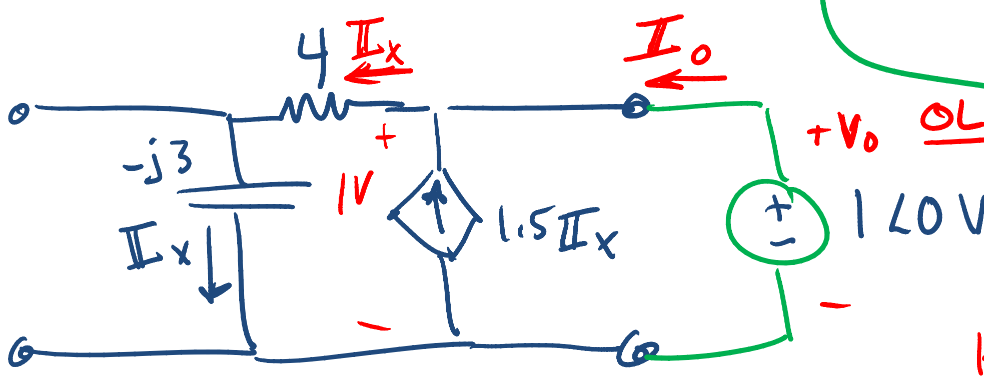
KVL: $V_{TH} = -j3 I_x + 4(1.5)I_x$

KCL: $2 + 1.5 I_x - I_x = 0$

$I_x = \frac{-2}{.5} = -4$

$V_{TH} = I_x(6-3j) = -24 + 12j$

Z_{TH}: Indep Srcs off + Test Src



OL: $I_x = \frac{1V}{4-3j} = \frac{4+3j}{25} = .2\angle 36.8$

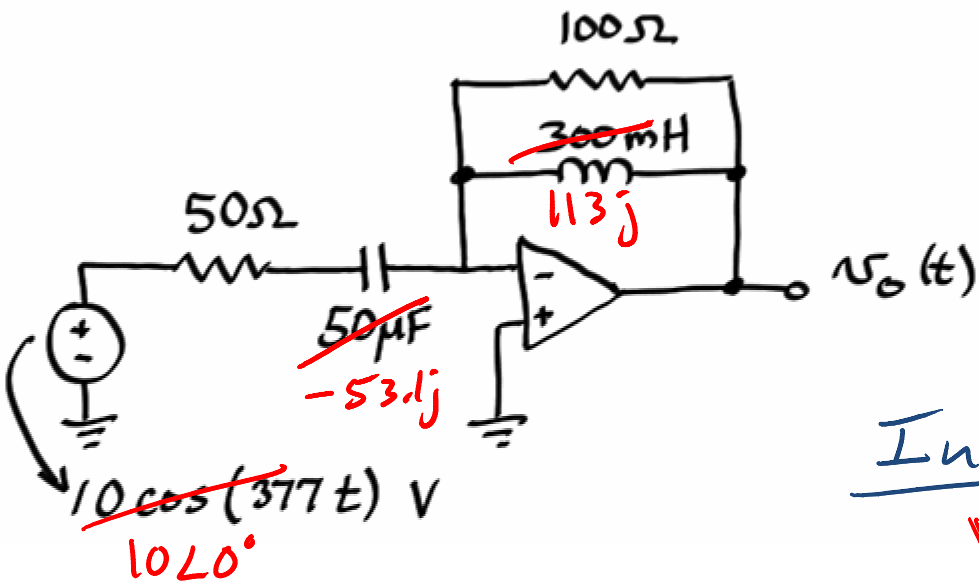
KCL: $I_o + 1.5 I_x - I_x = 0$

$I_o = -0.5 I_x = \frac{-4-3j}{50}$

$Z_{th} = \frac{V_o}{I_o} = \frac{1\angle 0}{(-4-3j)/50} = \frac{50}{-4-3j} = -8+6j\ \Omega$

Vth = -24 + j12 V, Zth = -8 + j6 Ohms

Example 10. Find Vo in the Op-Amp Circuit



Phasor Domain

$$Z_L = j\omega L = j(377)(.3) = 113.1j$$

$$Z_C = -j/\omega C = \frac{-j}{(377)(50\times10^{-6})} = -53.1j$$

Inverting OpAmp

$$V_o = -\frac{Z_f}{Z_s} V_s$$

$$Z_f = 100 \| 113j = 56.1 + 49.6j$$

$$Z_s = 50 - 53.1j$$

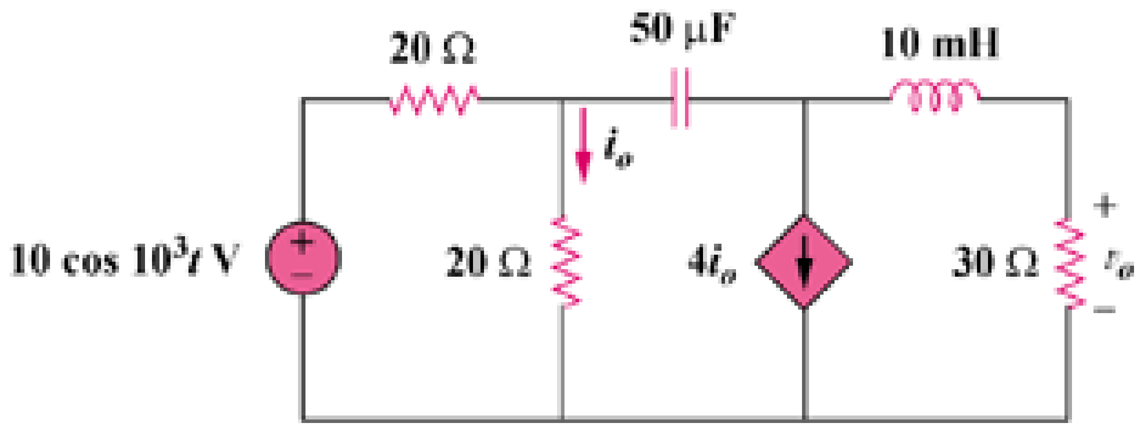
$$\therefore V_o = -\frac{(56.1 + 49.6j)}{(50 - 53.1j)} 10\angle 0^\circ = 10.2\angle -91.7^\circ$$

$$\rightarrow v_o(t) = 10.2\cos(377t - 91.7^\circ)$$

vo(t) = 10.2cos(377t-91.7°)

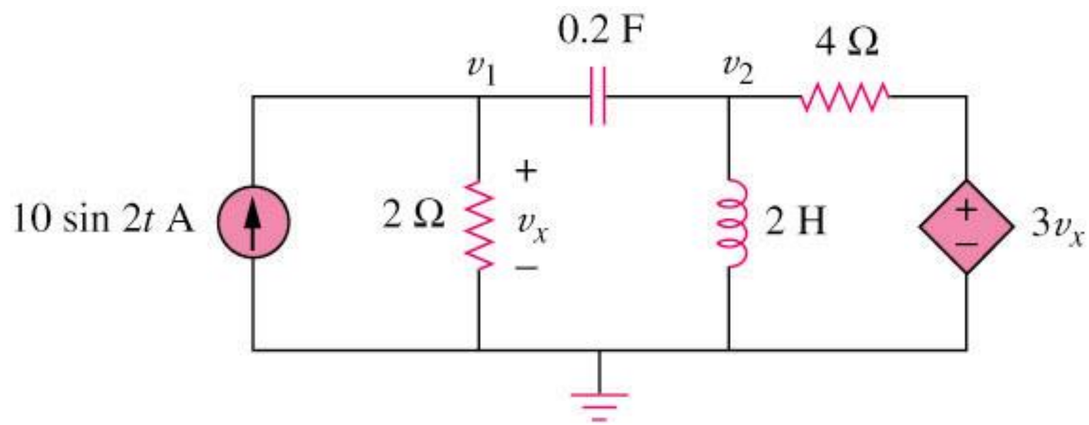
HANDOUTS

Example 1 Find Vo(t) using NA



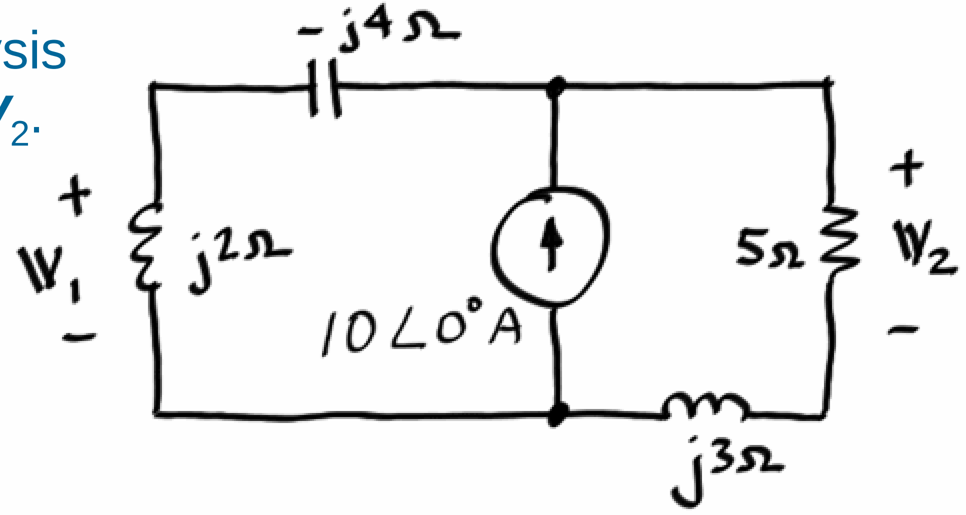
Example 2

Using nodal analysis, find v_1 and v_2 in the circuit



Answer: $v_1(t)$ = **11.32 sin(2t + 60.01°) V** **$v_2(t)$ = 33.02 sin(2t + 57.12°) V**

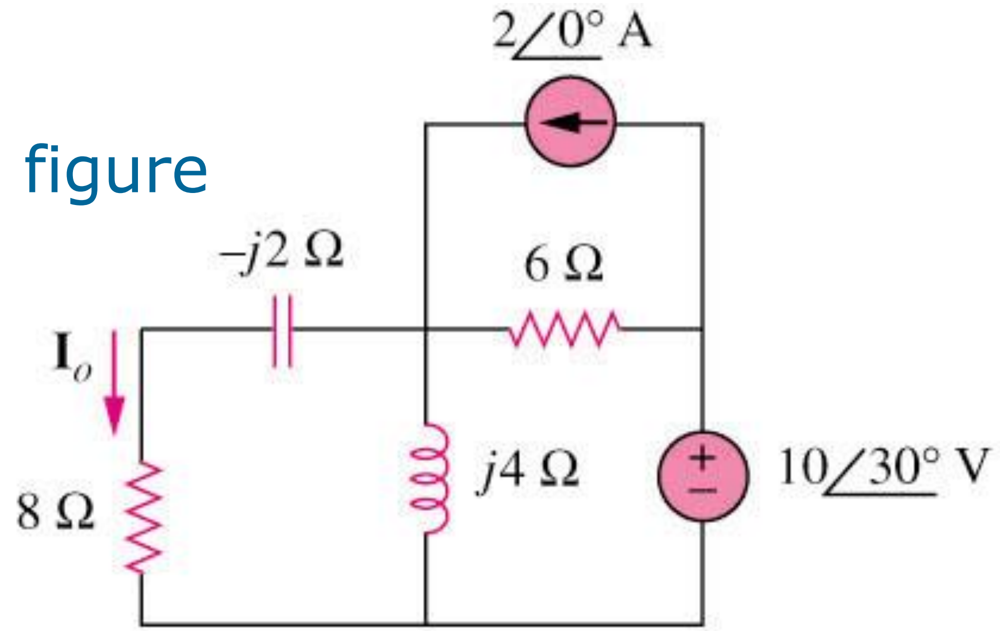
Example 3 Use mesh current analysis to find the phasor voltages $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$.



$\boldsymbol{V}_1 = 22.9\angle 110^\circ$ V; $\boldsymbol{V}_2 = 19.6\angle -101^\circ$ V

Example 4

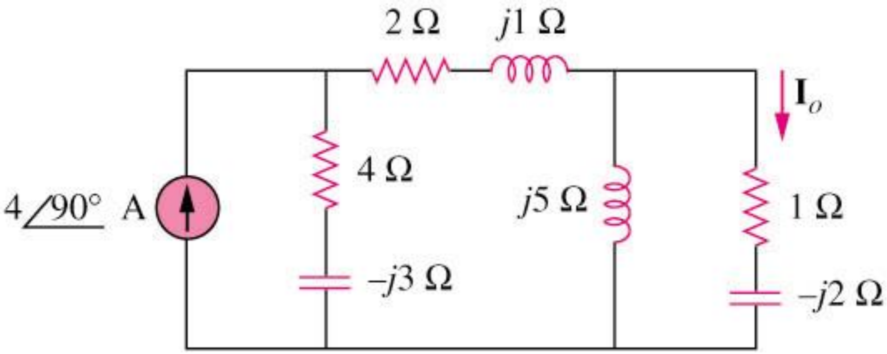
Find I_o in the following figure using mesh analysis.



Answer: **I_o = 1.194**< **65.44 ^{0} A**

Example 6

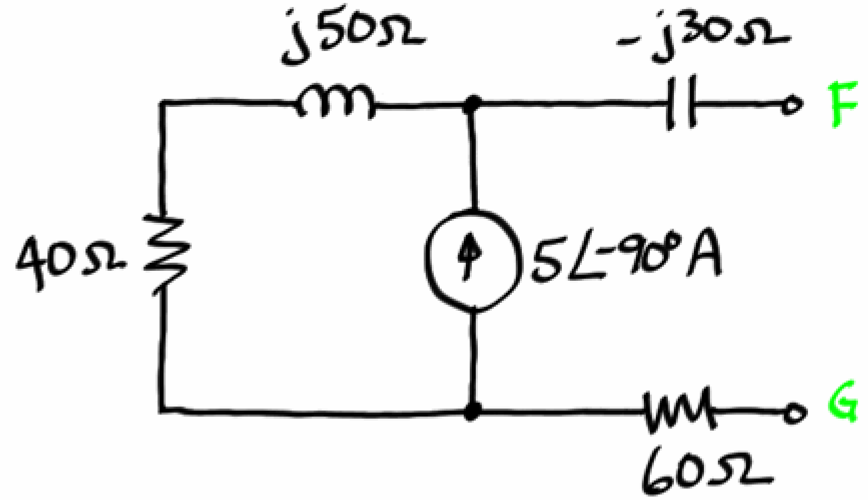
Find Io in the circuit of figure below using source transformations.



I_o = **3.288<99.46^0 A**

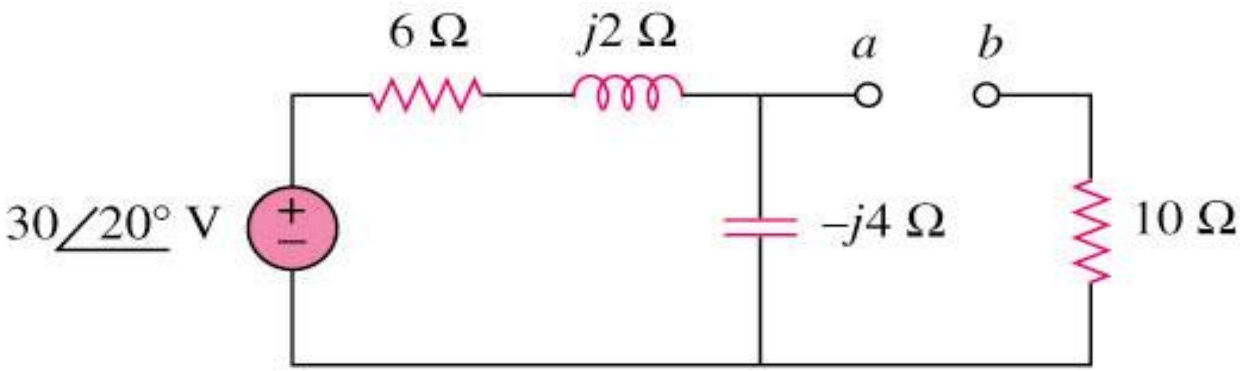
Example 7

Find the Thevenin equivalent circuit at the terminals *F-G*. Express all complex values in your solution in both rectangular and polar form.



$\boldsymbol{V}_{TH} = 250 - j200$ V $= 320 \angle -38.7°$ V; $\boldsymbol{Z}_{TH} = 100 + j20\ \Omega = 102 \angle 11.3°\ \Omega$

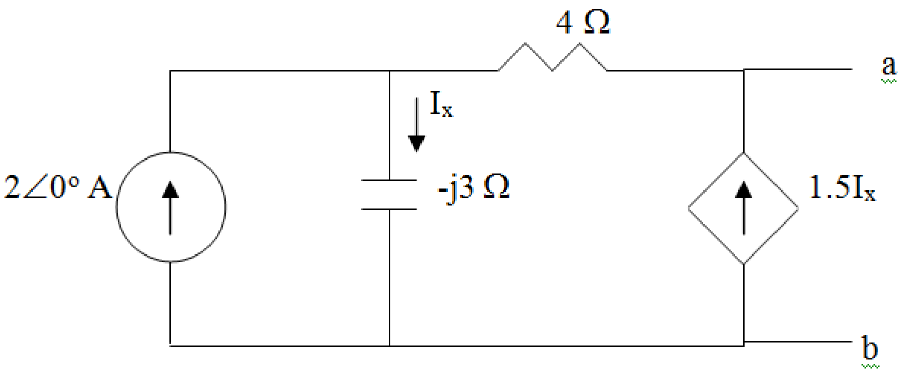
Example 8 Find the Thevenin equivalent at terminals a–b of the circuit



Z_{th} = **12.4 – j3.2**

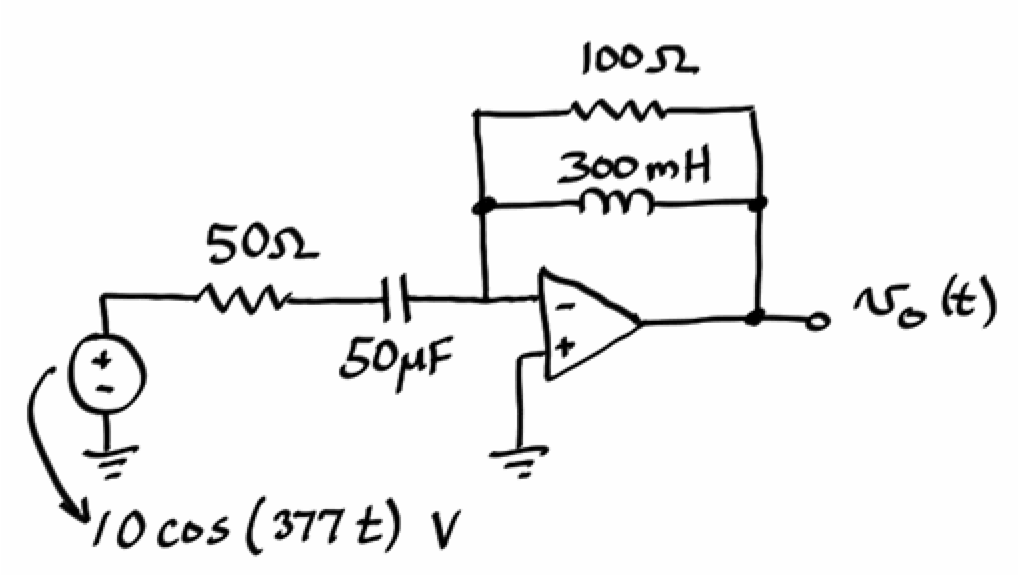
V_{TH} = **18.97<-51.57^{0} V**

Example 9. Find the Thevenin Equivalent



Vth = -24 + j12 V,   Zth = -8 + j6 Ohms

Example 10. Find Vo in the Op-Amp Circuit



vo(t) = 10.2cos(377t-91.7o)