Lecture 10

Chapter 9

Sinusoids and Phasors

Sinusoids and Phasors

Day 1

9.1 Motivation

9.2 Sinusoids, Complex Numbers

Day 2

9.3 Phasors

9.4 Phasor relationships for circuit elements

Day 3

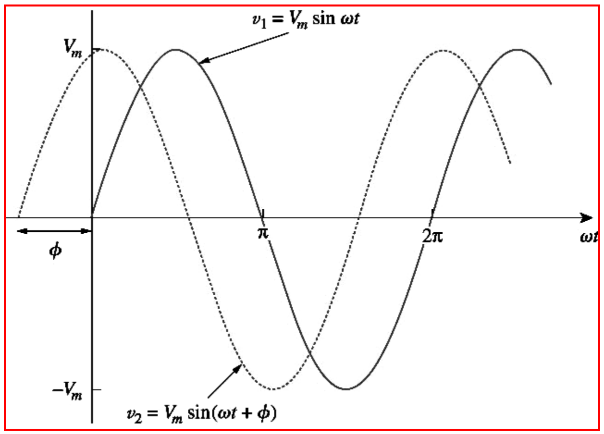
9.5 Impedance and admittance

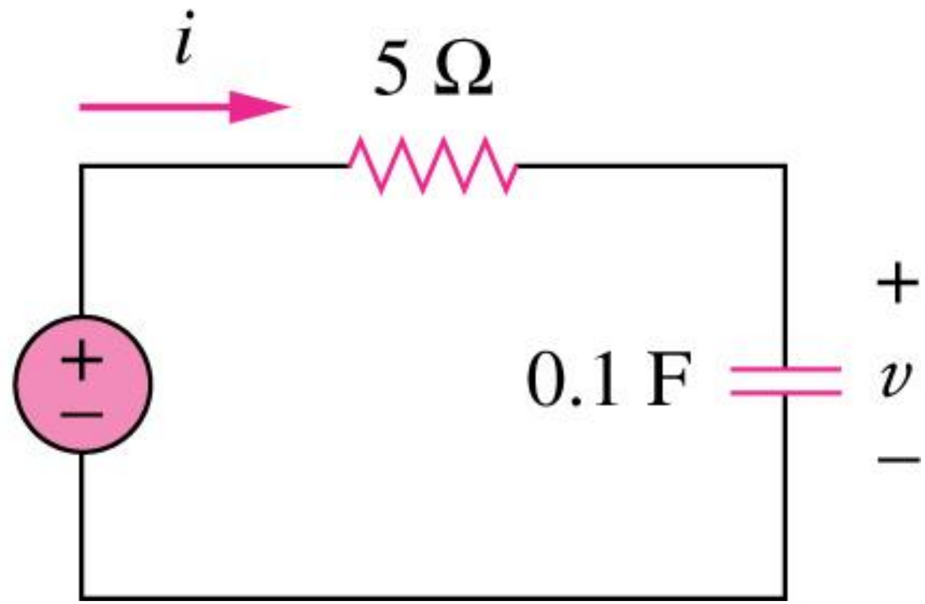
9.6 Kirchhoff’s laws in the frequency domain

9.7 Impedance combinations

9.1 Motivation (1)

How to determine *v(t)* and *i(t)?*





How can we apply what we have learned before to determine *i(t)* and *v(t)*?

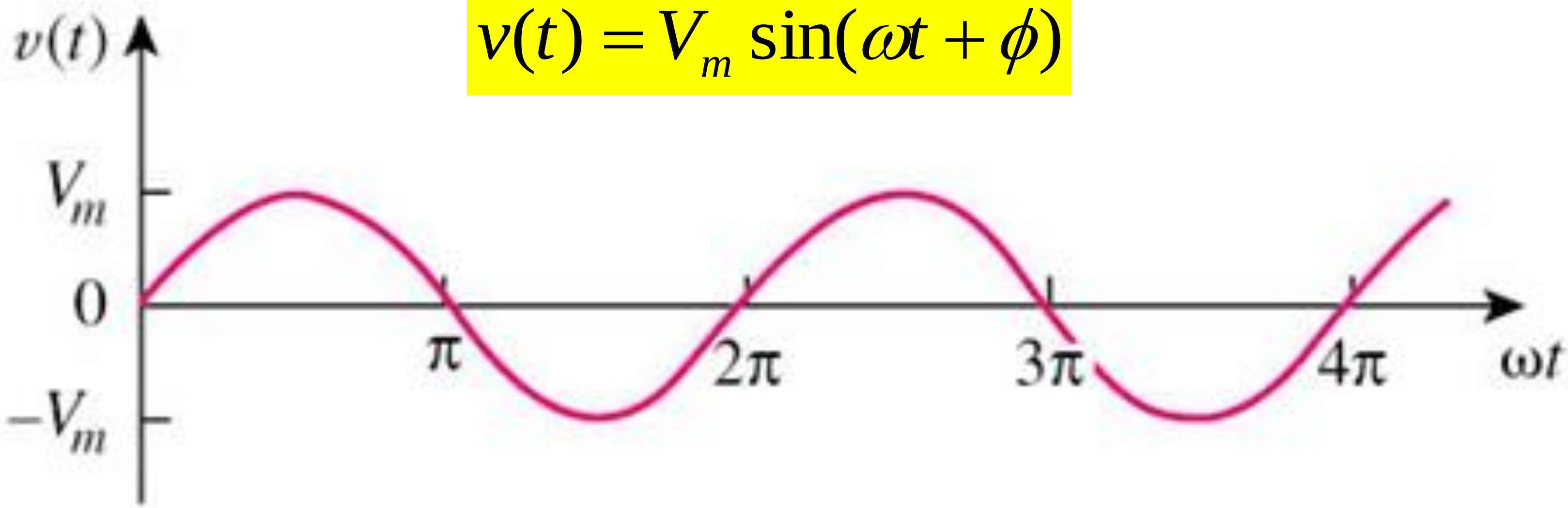
Sinusoidal Signals

- Broad Applications
  - Fourier Series: Fundamental basis for more complicated signals
  - Filters: pass or reject sinewaves of certain frequency
  - Signal processing: how to extract desired information from mix of signal plus noise
  - Radio, Radar, Lasers, TV, Microwave, Cell phones, AC Generators, Piston engines:
    - All express waveforms based on sinewaves
    - Often complex numbers used to represent/analyze

9.2 Sinusoids (1)

- A sinusoid is a signal that has the form of the sine or cosine function.
- A general expression for the sinusoid,

$$v(t) = V_m \sin(\omega t + \phi)$$



(a)

where

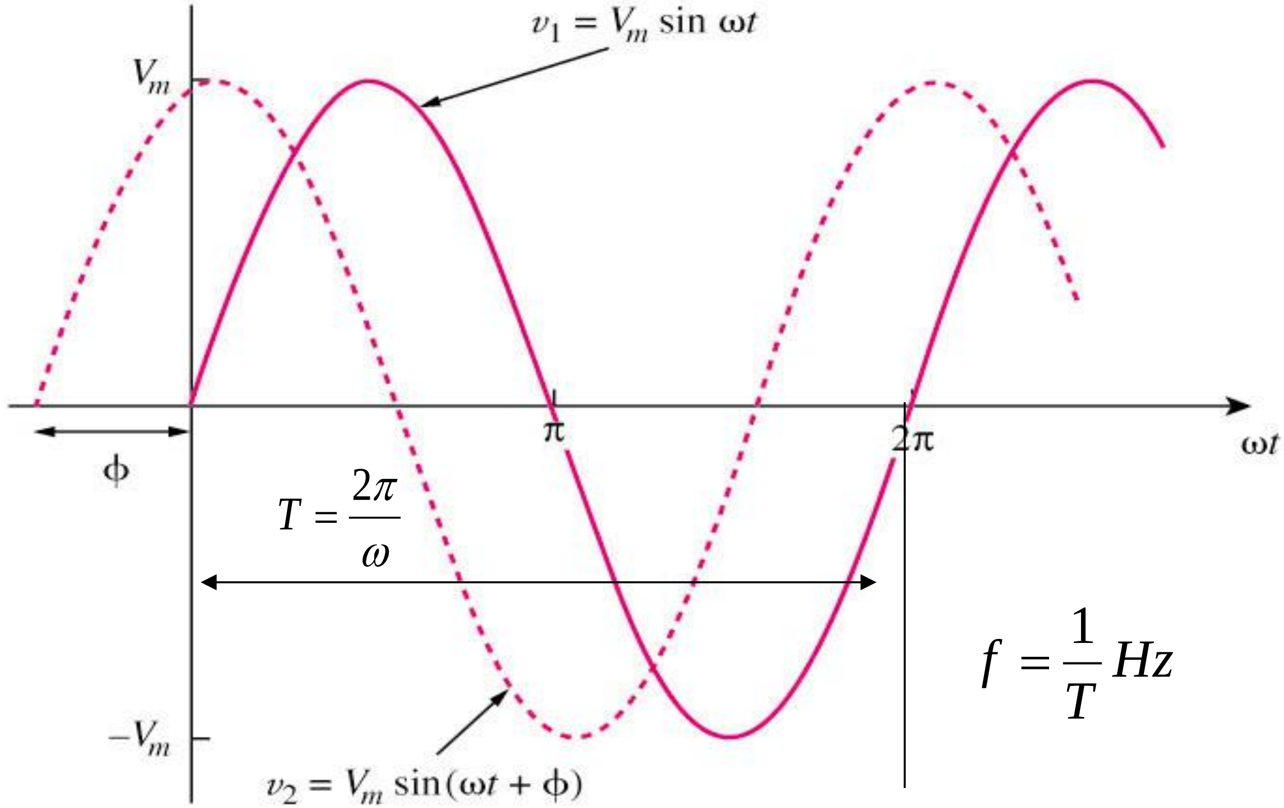
Vm = the **amplitude** of the sinusoid

ω = the angular frequency in radians/s

Φ = the phase

9.2 Sinusoids (2)

A <u>**periodic function**</u> **is one that satisfies *v(t)* = *v(t + nT)*, for all t and for all integers n.**



$$f = \frac{1}{T} Hz \qquad \omega = 2\pi f$$

- Only two sinusoidal values with the <u>same frequency</u> can be compared by their amplitude and phase difference.
- If phase difference is zero, they are in phase; if phase difference is not zero, they are out of phase.

Example 1

Given a sinusoid, $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency (ω, rad/s), period, and cyclical frequency (f, Hz).

$$A\sin(\omega t + \phi) = 5\sin(4\pi t - 60^\circ)$$

$$A = 5$$

$$\omega = 4\pi \text{ rad/s}$$

$$\phi = -60^\circ$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2} \text{ s}$$

$$f = \frac{1}{T} = 2 \text{ Hz}$$

Amplitude = 5, phase = –60°, angular frequency = 4π rad/s, Period = 0.5 s, frequency = 2 Hz.

Example 2

Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$, does i_1 lead or lag i_2?

1) Convert i_1 to cosine

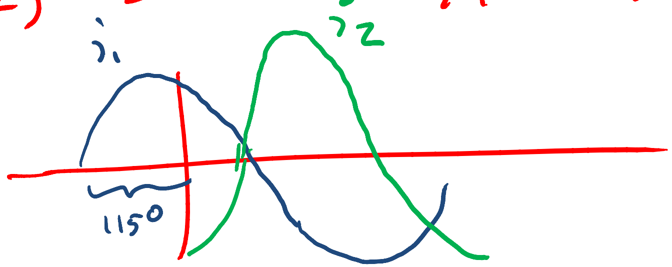
$i_1 = -4 \sin(377t + 25^\circ)$

$= -4 \cos(377t + 25 - 90)$

(180 deg phase shift inverts − sign)

$= 4 \cos(377t + 25 + 90) = 4\cos(377t + 115)$

2) Subtract $\phi_1 - \phi_2 = 115 - (-40) = 155^\circ$



ϕ_1 has larger ϕ angle

$\therefore \phi_1$ leads ϕ_2

i_1 leads i_2 by 155^o.

Complex Numbers

- What if we want to **add** two sinusoids?
  - $V(t) = 5\cos(377t + 45) + 7\sin(377t - 60)$
  - Trig Identity:

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

  - Not very pleasant!
- To deal more effectively with sinusoidal signals we will learn a new tool:
  - Complex Numbers

Complex Numbers,Intro (video)

Origin: roots of polynomials

$$x^2 + 9 = 0$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$x = \pm\sqrt{-1}\sqrt{9}$$

$$x = \pm j3$$

Imaginary Unit

$j = \sqrt{-1}$ (for Elect Engrs)

$$j^2 = -1$$

$\cancel{i = \sqrt{-1}}$ (for Math/Physics)

$=$ Current for Elect Engrs

Complex Nums – Rectangular

Rectangular Form

$$a + jb = n$$

$a \downarrow$ Re(n) $jb \downarrow$ Im(n)

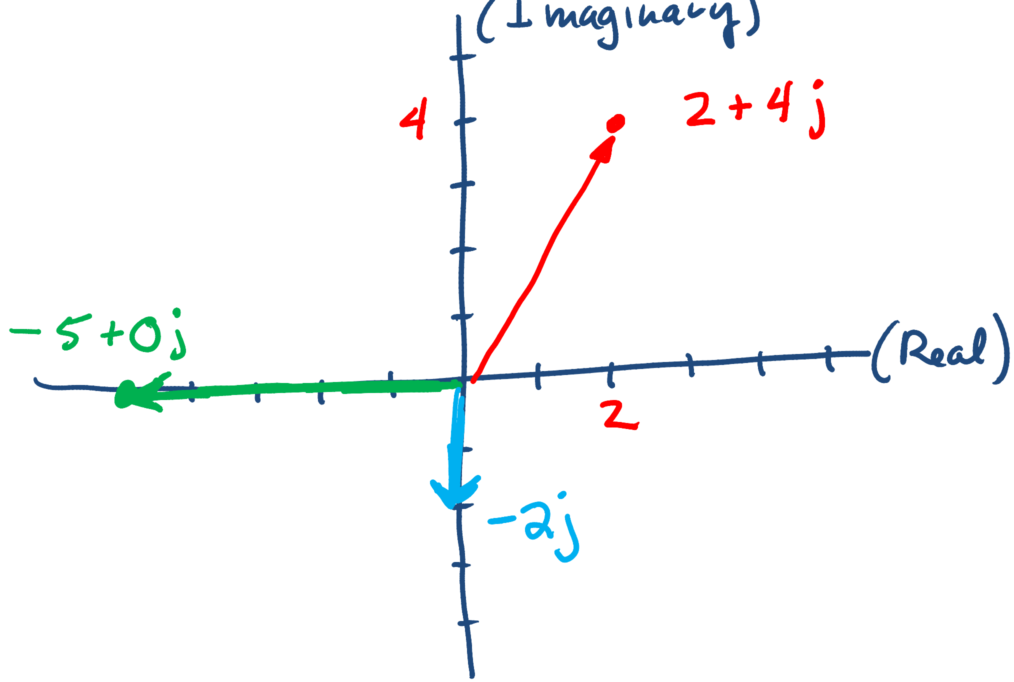
Some Examples

$3 + 2j$

$3 + 0j = 3$

$2j$

The Complex Plane



Rectangular Coordinates

$(x, y) = (\mathrm{Re}(n), \mathrm{Im}(n))$

like a 2D vector

Complex Nums – Rectangular adding and subtracting (video)

Adding & Subtracting

$$(a + jb) + (c + jd) = (a+c) + j(b+d)$$

$$(3 - 2j) - (4 + 8j) = \boxed{-1 - 10j}$$

$$3 + (3 + 8j) = (3 + 0j) + (3 + 8j) = \boxed{6 + 8j}$$

Complex Nums – Rectangular Multiplying (video)

Multiplying

use binomial "FOIL" method

$$(3+2j)(1-3j) = 3 - 9j + 2j - 6j^2$$
$$= 3 - 7j + 6$$
$$= \boxed{9-7j}$$

Complex Conjugate

$n = 2 + 3j \quad , \quad n^* = 2 - 3j$

product of n and n^*:

$$(2+3j)(2-3j) = 4 + \underbrace{6j - 6j}_{\text{always cancels}} - 9j^2$$
$$= 4 + 0j + 9$$
$$= 13 \quad \text{(always Real)}$$
$$= a^2 + b^2$$

Complex Nums – Rectangular Dividing (video)

Dividing – multiply num + den by complex conj of den

$$\frac{(1+2j)}{(3-j)} = \frac{(1+2j)}{(3-j)}\frac{(3+j)}{(3+j)}$$

$$= \frac{(3+j+6j+2j^2)}{9+3j-3j-j^2}$$

$$= \frac{(3-2)+(1+6)j}{(9+1)+(3-3)j}$$

$$= \frac{1+7j}{10} = \frac{1}{10} + \frac{7}{10}j$$

Complex Arithmetic (Rectangular)

Given:

a = 3 + 4j

b= -5 + 6j

Find

a + b $= (3+4j)+(-5+6j) = -2+10j$

a – b $= (3+4j)-(-5+6j) = 8-2j$

a * b $= (3+4j)(-5+6j) = -15+18j-20j+24j^2$

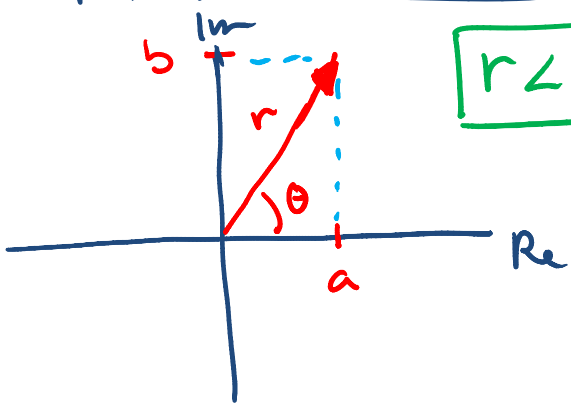
$= -39-2j$

a / b $= \frac{(3+4j)}{(-5+6j)}\frac{(-5-6j)}{(-5-6j)} = \frac{-15-18j-20j-24j^2}{25+30j-30j-36j^2} = \frac{9-38j}{61}$

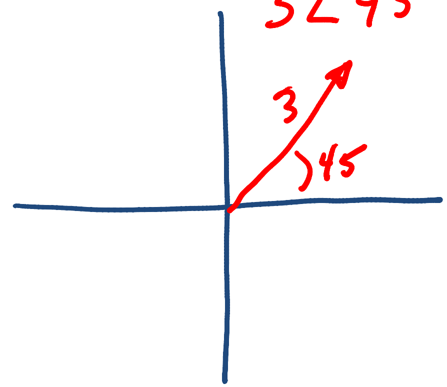
Complex Numbers – Polar (video)

Complex number $a + jb$ can be represented using <u>polar</u> <u>coordinates</u>

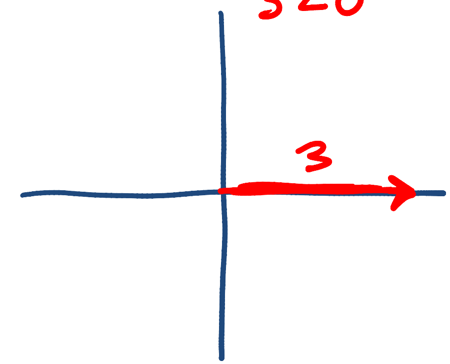
$\boxed{r\angle\theta}$



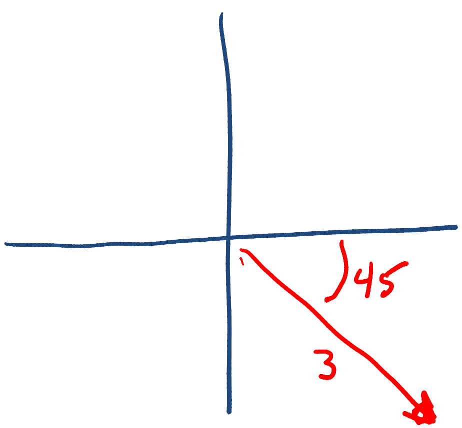
$3\angle 45$



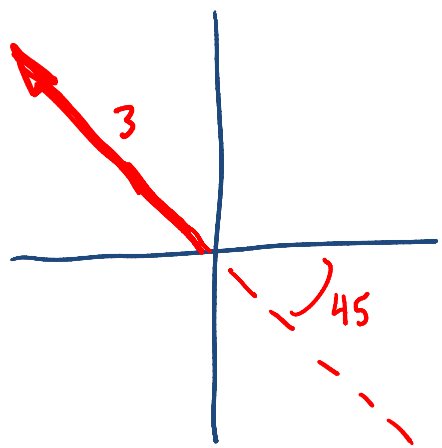
$3\angle 0$



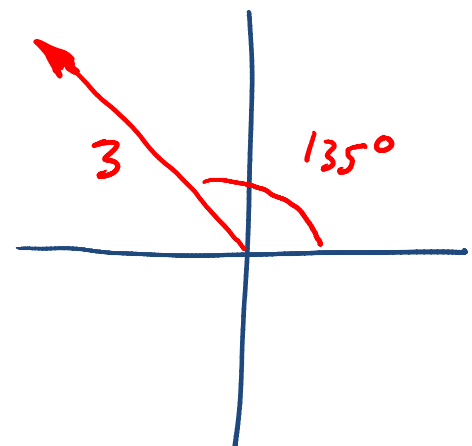
$3\angle -45$



$-3\angle -45$

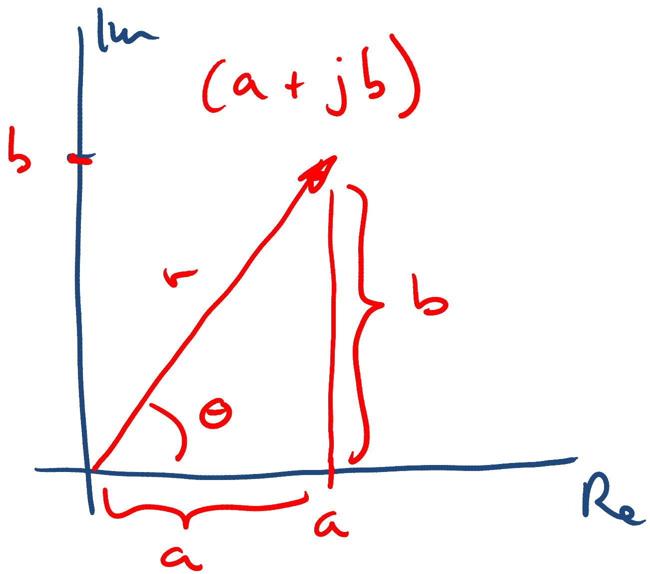


$3\angle 135$



Conversion: Rect to Polar (video)

Convert Rect to Polar Form



$$a^2 + b^2 = r^2$$

$$\boxed{r = \sqrt{a^2+b^2}}$$

$$\tan\theta = \frac{b}{a}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{b}{a}\right)}$$

Careful !

$$-\frac{\pi}{2} < \tan^{-1}\left(\frac{b}{a}\right) < \frac{\pi}{2}$$

Conversion: Rect to Polar (video)

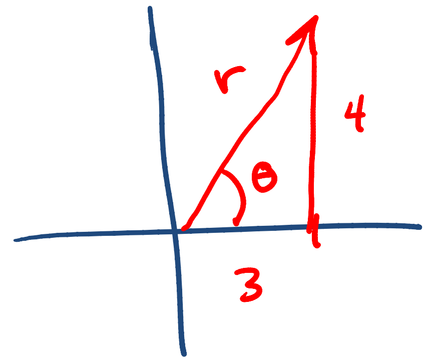
Example

$3+4j$

$r=\sqrt{3^2+4^2}=5$

$\theta=\tan^{-1}\left(\frac{4}{3}\right)=53.1^\circ$

$\therefore\ 3+4j \longleftrightarrow 5\angle 53.1^\circ$



In General

$\theta=\tan^{-1}\left(\frac{b}{a}\right)$

or

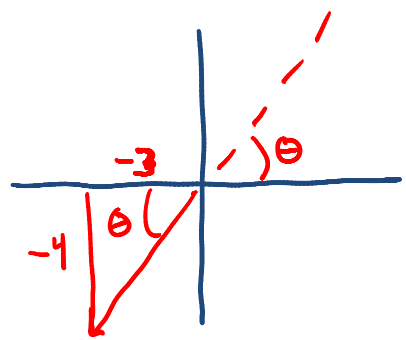
$\theta=\tan^{-1}\left(\frac{b}{a}\right)\pm 180^\circ$

Example

$-3-4j$

$r=\sqrt{(-3)^2+(-4)^2}=5$

$\theta=\tan^{-1}\left(\frac{-4}{-3}\right)=\cancel{53.1^\circ}$



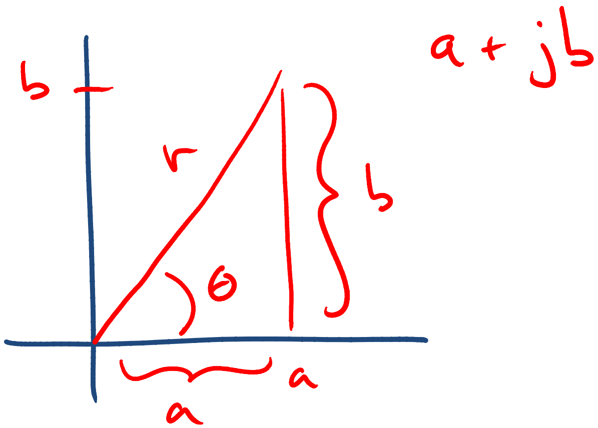
Found wrong θ!

$\theta=53.1^\circ+180^\circ=\underline{233.1^\circ}$

$\therefore\ -3-4j \longleftrightarrow 5\angle 233.1^\circ$

Conversion: Polar to Rect ([video]())

Convert Polar to Rect Form



$$\sin\theta = \frac{b}{r}$$

$$\boxed{b = r\sin\theta}$$

$$\cos\theta = \frac{a}{r}$$

$$\boxed{a = r\cos\theta}$$

$$\therefore\ a + jb = r\cos\theta + jr\sin\theta$$

$$= r(\cos\theta + j\sin\theta)$$

Euler's Formula (video)

Euler's Formula

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

3 ways to express complex number $\underline{n}$:

$$n = a + jb = r\cos\theta + jr\sin\theta \quad \text{(Rectangular Form)}$$

$$= r(\cos\theta + j\sin\theta)$$

$$= re^{j\theta} \quad \text{(Exponential Form)}$$

$$= r\angle\theta \quad \text{(Polar Form)}$$

Polar to Rect (and back)

- Convert a = 2 – 3j and b = -4 + 5j
  - to polar form:

$a = 2 - 3j \qquad r = \sqrt{2^2 + (-3)^2} = \sqrt{13} = 3.61, \quad \theta = \tan^{-1}\left(\frac{-3}{2}\right) = -56.3$

$= 3.61 \angle -56.3^\circ$

$b = -4 + 5j \qquad r = \sqrt{(-4)^2 + (5)^2} = \sqrt{41} = 6.40, \quad \theta = \tan^{-1}\left(\frac{5}{-4}\right) = -51.3 + 180$

$= 128.6$

$= 6.40 \angle 129^\circ$

- Convert c = 2<30° and d = 4<200°
  - To rectangular form

$c = 2\angle 30^\circ = 2\cos 30 + j\,2\sin 30 = \underline{1.73 + j}$

$d = 4\angle 200^\circ = 4\cos 200 + j\,4\sin 200 = \underline{-3.75 - 1.37j}$

Complex: Add/Multiply in Polar Form (video)

3 Forms

$$\boxed{\begin{aligned} a + jb &= r\angle\theta \\ &= re^{j\theta} \end{aligned}}$$

Adding: $r\angle\theta + s\angle\phi = re^{j\theta} + se^{j\phi}$

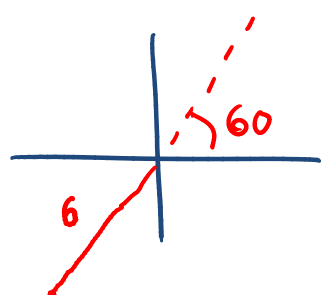
$\boxed{\text{convert to Rectangular then add}}$

Multiplying: we can use law of exponents: $a^n \cdot a^m = a^{n+m}$

$$(r\angle\theta)(s\angle\phi) = re^{j\theta} se^{j\phi} = r \cdot s e^{j(\theta+\phi)}$$

$$= rs\angle(\theta+\phi)$$

$$(3\angle 45)(-2\angle 15) = -6\angle 60 = 6\angle 240$$



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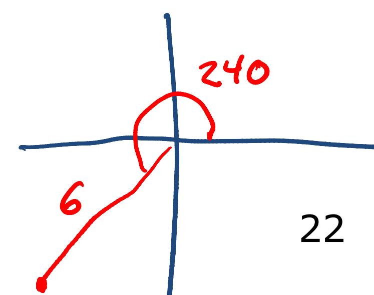


Complex: Divide in Polar (video)

Dividing use law of exponents again: $\frac{a^n}{a^m} = a^{n-m}$

$$\frac{r\angle\theta}{s\angle\phi} = \frac{re^{j\theta}}{se^{j\phi}} = \frac{r}{s}e^{j(\theta-\phi)} = \underline{\frac{r}{s}\angle(\theta-\phi)}$$

$$\frac{3\angle 45}{2\angle 15} = \frac{3}{2}\angle(45-15) = \underline{1.5\angle 30^\circ}$$

$$\frac{1}{3\angle 10^\circ} = \frac{1\angle 0^\circ}{3\angle 10^\circ} = \frac{1}{3}\angle(0-10) = \frac{1}{3}\angle -10^\circ$$

Summary: Mathematic operation of complex numbers

http://www.1728.com/compnumb.htm

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\dfrac{1}{z} = \dfrac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r \angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

Polar Complex Mult/Div

Given:

c = 8<45

d = 12<250

Find

c * d $= (8\angle 45)(12\angle 250) = \underline{96\angle 295^\circ}$

c / d $= \dfrac{8\angle 45}{12\angle 250} = \dfrac{2}{3}\angle(45-250) = \dfrac{2}{3}\angle -205 = \underline{\dfrac{2}{3}\angle 155}$

c* (complex conj of c) $= (8\angle 45)^* = 8\cos 45 \overset{\text{conj}}{-} j8\sin 45$

$= 8\cos(-45) + j8\sin(-45) = \underline{8\angle -45}$

Example 3 Calculator Demo!

- Evaluate the following complex numbers:

a. $[(5+j2)(-1+j4)-5\angle 60^{o}]$

b. $\dfrac{10+j5+3\angle 40^{o}}{-3+j4}+10\angle 30^{o}$

Solution: a. −15.5 + j13.67   b.   8.293 + j2.2

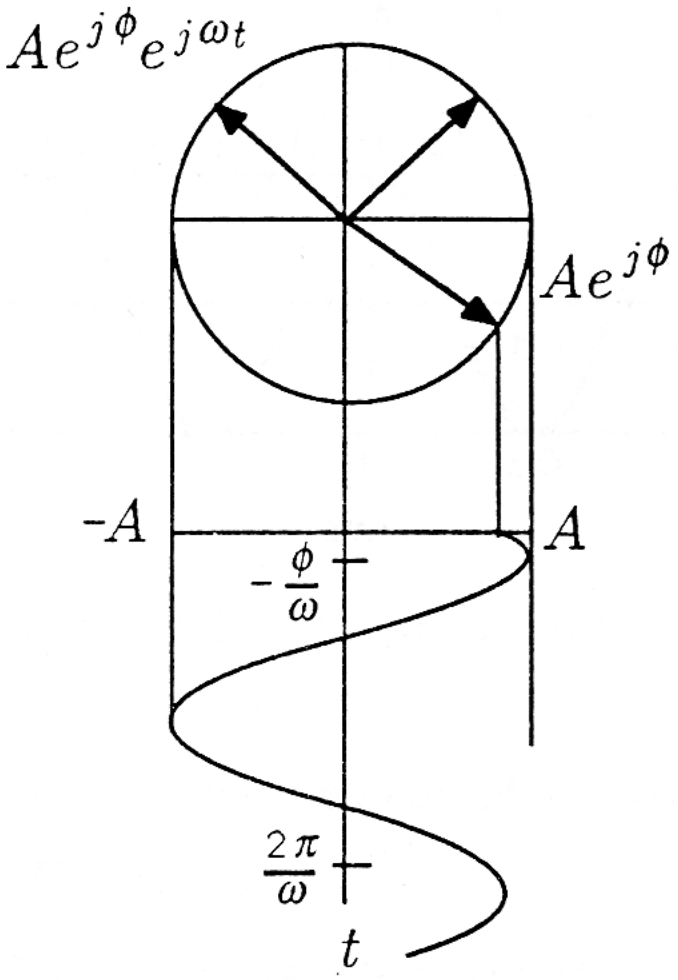
http://ptolemy.eecs.berkeley.edu/eecs20/berkeley/phasors/demo/phasors.html

9.3 Phasor (1)

- A phasor is a complex number that represents the amplitude and initial phase (at t=0) of a sinusoid.

$$A\cos(\omega t + \phi) \leftrightarrow Ae^{j\phi} \leftrightarrow A\angle\phi$$

- It allows us to perform math on sinusoids of the same frequency by using a complex number representation – much easier than trig identities!!



http://ptolemy.eecs.berkeley.edu/eecs20/berkeley/phasors/demo/phasors.html

9.3 Phasor (4)

- Transform a sinusoid to and from the time domain to the phasor domain:

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$$

(time domain) (phasor domain)

- Amplitude and phase difference are two principal concerns in the study of voltage and current sinusoids.
- Phasor will be defined from the cosine function in all our proceeding study. If a voltage or current expression is in the form of a sine, it will be changed to a cosine by subtracting 90 degrees from the phase.
- Note, ϕ is the phase angle of the cosine at t=0

Example 4

9.3 Phasor (5)

Transform the following sinusoids to phasors:

$i = 6\cos(50t - 40^o)$ A

$v = -4\sin(30t + 50^o)$ V

$\mathbf{I} = 6 \angle -40$ A (by inspection)

for v, convert to positive cosine:

$v = -4\sin(30t + 50) = -4\cos(30t + 50 - 90)$

$= 4\cos(30t + 50 + 90)$

$\therefore \mathbf{V} = 4 \angle 140$ V

$I = 6\angle -40^\circ$ A

$V = 4\angle 140^\circ$ V

Example 5: # 9.3 Phasor (6)

Transform to sinusoids the phasors:

a. $\mathbf{V} = -10\angle 30°\ \text{V}$

b. $\mathbf{I} = j(5 - j12)\ \text{A}$

a) $v(t) = -10\cos(\omega t + 30°)$

$v(t) = 10\cos(\omega t + 210°)$

b) $I = j(5 - j12) = 5j - j^2 12 = 12 + 5j$

$= 13 \angle 22.6°$

$\therefore\ i(t) = 13\cos(\omega t + 22.6°)\ A$

v(t) = 10cos(ωt + 210°) V, *i(t) = 13cos(ωt + 22.62°) A*

9.3 Phasor (7)

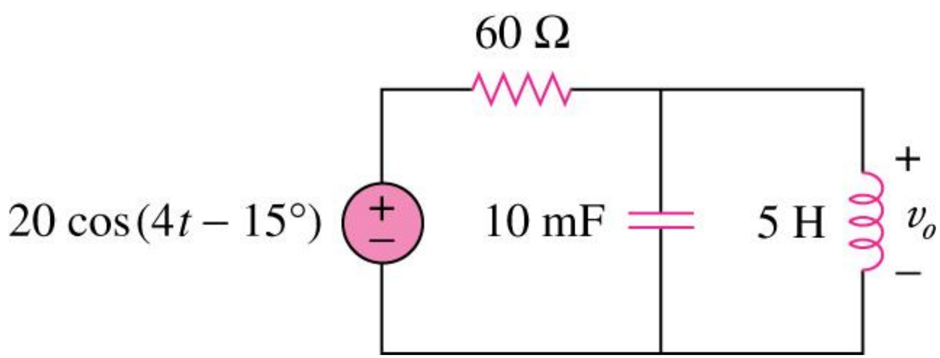
The differences between v(t) and V:

- v(t) is instantaneous or time-domain representation
  V is the frequency or phasor-domain representation.
- v(t) is time dependent, V is not.
- v(t) is always real with no complex term, V is generally complex.

Note: Phasor analysis applies only when frequency is constant; it is applied to two or more sinusoid signals only if they have the same frequency.

9.3 Phasor (10)

- we can derive the differential equations for the following circuit in order to solve for $v_o(t)$ in phase domain V_o.



$$\frac{d^2 v_o}{dt^2} + \frac{5}{3}\frac{dv_0}{dt} + 20v_0 = -\frac{400}{3}\sin(4t - 15^o)$$

- However, the derivation may sometimes be very tedious.

Is there any quicker and more systematic methods to do it?

9.3 Phasor (11)

The answer is YES!

Instead of first deriving the differential equation and then transforming it into phasor domain to solve for V_o, we can transform all the RLC components into the phasor domain first, then apply the KCL laws and other theorems to set up a phasor equation involving V_o directly.

But what does an R, L or C look like in the Phasor Domain??

Resistors in the Frequency (Phasor) Domain

- Ohm's Law, Time Domain

$\boxed{v = iR}$

Suppose $i(t) = I_m \cos(\omega t + \phi)$

then $v(t) = R \cdot I_m \cos(\omega t + \phi)$

- Ohm's Law, Frequency Domain

$\mathbf{I} = I_m \angle \phi$

$\boxed{\mathbf{V} = R\,\mathbf{I}} = R I_m \angle \phi$

Same effect as time domain

Inductors in the Frequency (Phasor) Domain

- Inductor Law, Time Domain

$$\boxed{v = L\frac{di}{dt}}$$

Suppose $i(t) = I_m \cos(\omega t + \phi)$

then $v(t) = L\frac{di}{dt} = -\omega L I_m \sin(\omega t + \phi)$

$= -\omega L I_m \cos(\omega t + \phi - 90)$ add 180^0

$= \omega L I_m \cos(\omega t + \phi + 90)$

- Inductor Law, Frequency Domain

$\mathbf{I} = I_m \angle \phi$

$\mathbf{V} = \omega L I_m \angle(\phi + 90) = \omega L I_m e^{j(\phi + 90)}$

$= \omega L I_m e^{j\phi} e^{j90} = \omega L I_m e^{j\phi} j = j\omega L \mathbf{I}$

$$\boxed{\mathbf{V} = j\omega L \mathbf{I}}$$

same as j

Capacitors in the Frequency (Phasor) Domain

- Capacitor Law, Time Domain

$$\boxed{i = C\frac{dv}{dt}}$$

Suppose $v(t) = V_m \cos(\omega t + \phi)$

then $i(t) = C\frac{dv}{dt} = -\omega C V_m \sin(\omega t + \phi)$

$= \omega C V_m \cos(\omega t + \phi + 90)$

- Capacitor Law, Frequency Domain

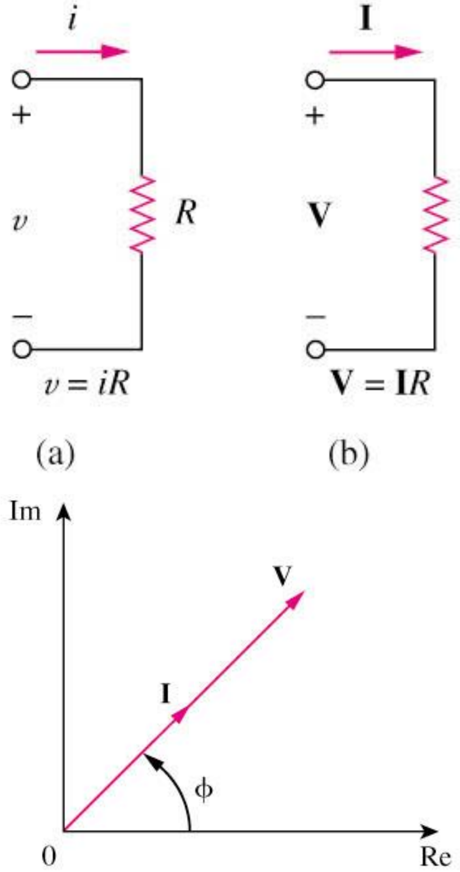
$\mathbf{V} = V_m \angle \phi$

$\mathbf{I} = j\omega C \mathbf{V}$ (same derivation as Inductor)

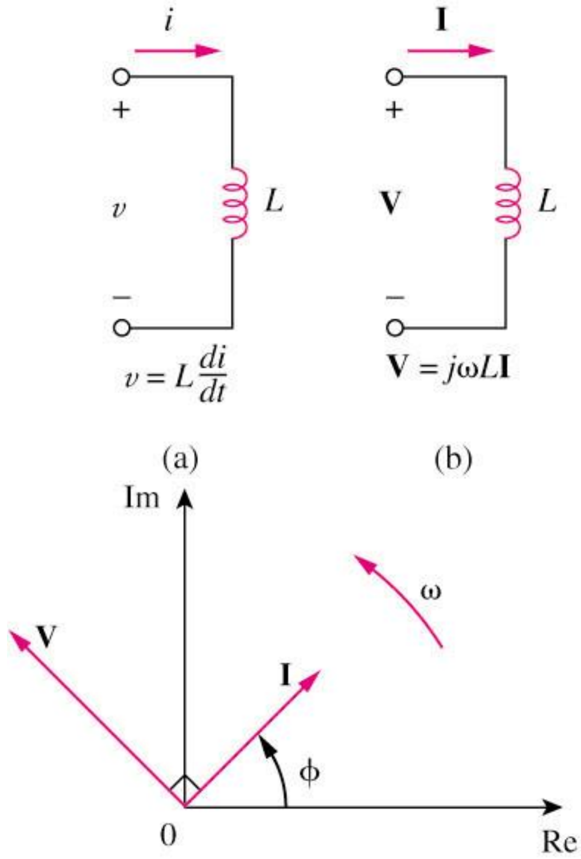
Therefore $\mathbf{V} = \frac{\mathbf{I}}{j\omega C}$, or $\boxed{\mathbf{V} = \frac{-j}{\omega C}\mathbf{I}}$

9.4 Phasor Relationships for Circuit Elements (1)

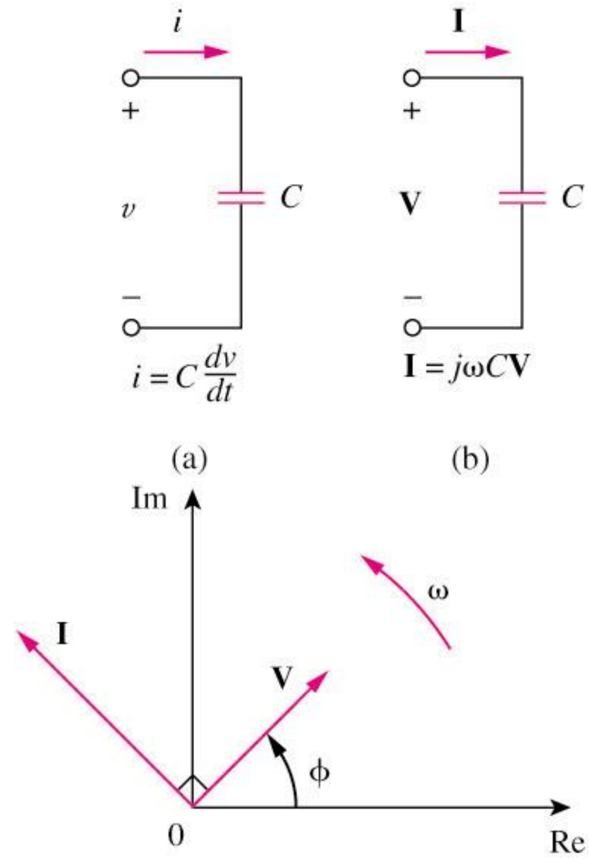
Resistor:



Inductor:



Capacitor:



9.4 Phasor Relationships for Circuit Elements (2)

Summary of voltage-current relationship		
Element	Time domain	Frequency domain
R	$v = Ri$	$V = RI$
L	$v = L\dfrac{di}{dt}$	$V = j\omega L I$
C	$i = C\dfrac{dv}{dt}$	$V = \dfrac{I}{j\omega C}$ $= \dfrac{-j}{\omega C} I$

Example 7A

If current $i(t) = 5\cos(100t + 30^o)\ mA$ is applied to a 2 mH inductor, calculate the voltage, $v(t)$, across the inductor.

$$\mathbf{V} = j\omega L\mathbf{I} = j(100)(.002)(5\angle 30)$$

$$= j(1\angle 30)$$

$$= (1\angle 90)(1\angle 30) = 1\angle 120^0$$

$$\therefore \quad \underline{v(t) = \cos(100t + 120^0)\ V}$$

Answer: v(t) = **cos(100t + 120º) V**

Example 7B

If voltage *v(t) = 6cos(100t – 30^o)* V is applied to a 50 μF capacitor, calculate the current, *i(t)*, through the capacitor.

$$\mathbf{V} = \frac{\mathbf{I}}{j\omega C}, \quad \mathbf{I} = j\omega C \mathbf{V}$$

$$\omega = 100, \quad C = 50 \times 10^{-6}, \quad \mathbf{V} = 6 \angle -30$$

$$\mathbf{I} = j(100)(50\times10^{-6})(6\angle -30)$$

$$= j(.03 \angle -30)$$

$$= (1\angle 90)(.03 \angle -30) = .03 \angle (-30+90)$$

$$= .03 \angle 60 \text{ A}$$

$$\therefore \underline{i(t) = 30\cos(100t + 60^\circ) \text{ mA}}$$

Answer: i(t) = **30 cos(100t + 60^o) mA**

9.5 Impedance and Admittance (1)

- The impedance Z of a circuit is the ratio of the phasor voltage V to the phasor current I, measured in ohms Ω.

$$Z = \frac{V}{I} = R + jX$$

where R = Re, Z is the resistance and X = Im, Z is the reactance. Positive X is for L and negative X is for C.

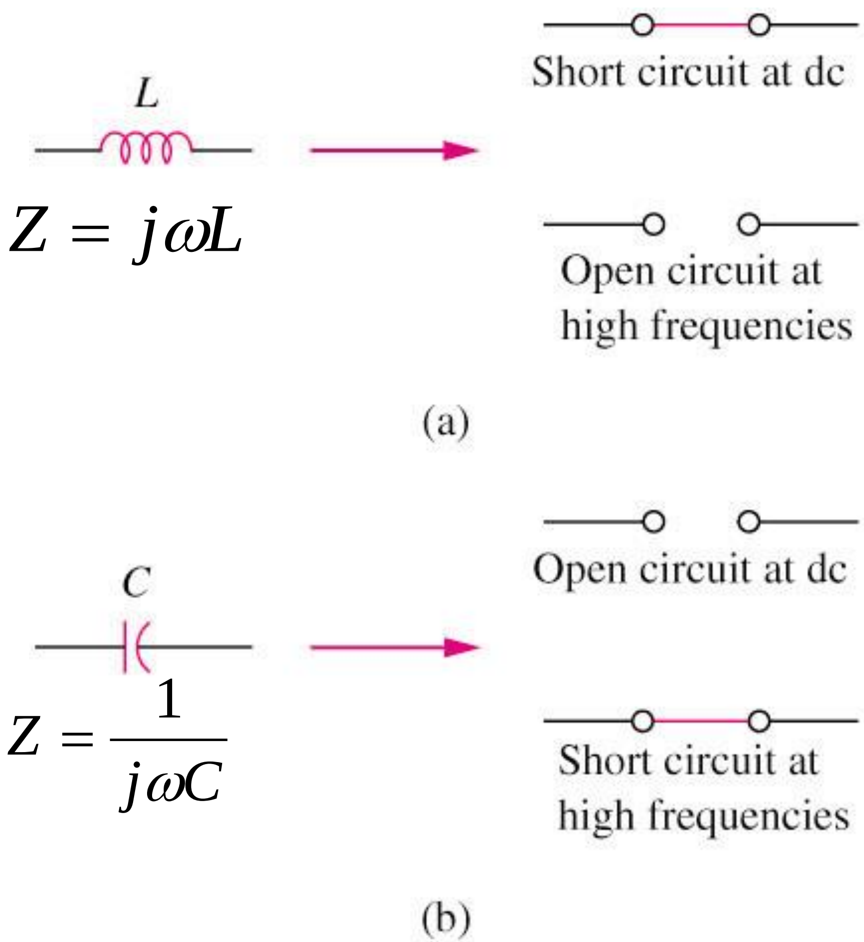
- The admittance Y is the reciprocal of impedance, measured in siemens (S).

$$Y = \frac{1}{Z} = \frac{I}{V}$$

9.5 Impedance and Admittance (2)

Impedances and admittances of passive elements		
Element	**Impedance**	**Admittance**
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C} = \frac{-j}{\omega C}$	$Y = j\omega C$

9.5 Impedance and Admittance (3)



$$\omega = 0; Z = 0$$

$$\omega \to \infty; Z \to \infty$$

$$\omega = 0; Z \to \infty$$

$$\omega \to \infty; Z = 0$$

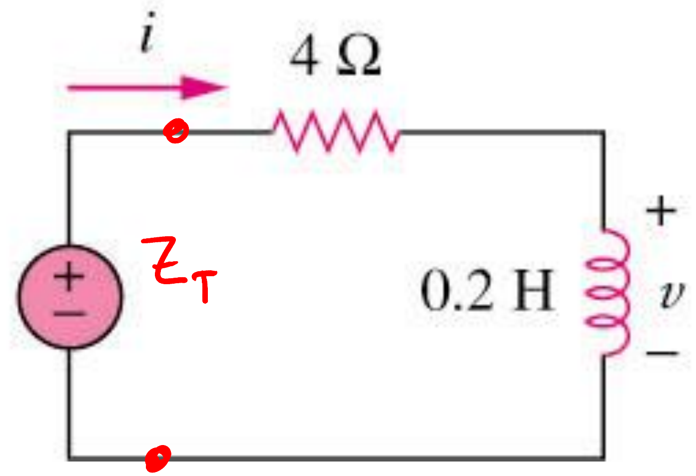
9.5 Impedance and Admittance (4)

After we know how to convert RLC components from time to phasor domain, we can transform a time domain circuit into a phasor/frequency domain circuit.

Hence, we can apply the KCL laws and other theorems to directly set up phasor equations involving our target variable(s) for solving.

Example 8 determine *v(t)* and *i(t)*.

$$v_s = 5\cos(10t)$$



1) Find Z_L

$Z_L = j\omega L$

$= j(10)(.2) = j2$

2) Find Z_T

$Z_T = 4 + Z_L = 4 + j2$

3) Find phasor $\mathbb{V}_s = 5\angle 0^\circ$

4) Use Ohm's Law to find $\mathbb{I}_s = \frac{\mathbb{V}_s}{Z_T} = \frac{5\angle 0^\circ}{(4+j2)}$

$$\mathbb{I}_s = \frac{5\angle 0^\circ}{\sqrt{4^2+2^2}\angle \tan^{-1}\left(\frac{2}{4}\right)} = \frac{5\angle 0^\circ}{4.47\angle 26.57^\circ} = 1.118\angle -26.56^\circ$$

$i_s(t) = 1.118\cos(10t - 26.56^\circ)$

5) Use Ohm's Law to find $\mathbb{V} = \mathbb{I}_s Z_L = (1.118\angle -26.56)(j2)$

$= 2.24\angle(-26.56 + 90) = 2.24\angle 63.43^\circ$

Answers: *i(t) = 1.118cos(10t – 26.56°) A; v(t) = 2.236cos(10t + 63.43°) V*

9.6 Kirchhoff's Laws in the Frequency Domain (1)

- Both KVL and KCL hold in the phasor domain or more commonly called frequency domain.

- Moreover, the variables to be handled are phasors, which are complex numbers.

- All the mathematical operations involved are now in complex domain.

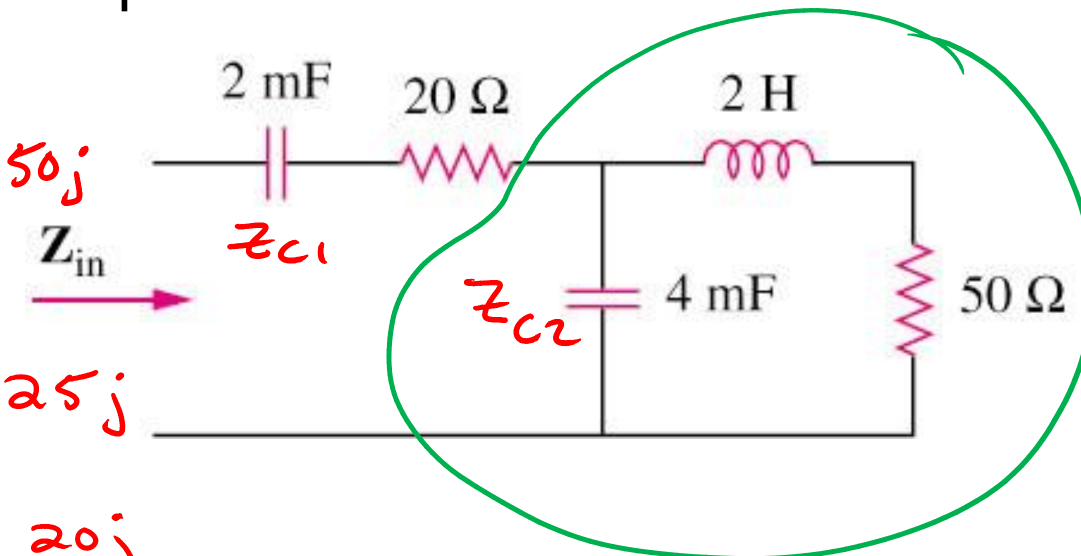
9.7 Impedance Combinations (1)

- The following principles used for DC circuit analysis all apply to AC circuit.

- For example:
  a. voltage division
  b. current division
  c. circuit reduction
  d. impedance equivalence
  e. Y-Δ transformation

9.7 Impedance Combinations (2)

<u>Example 9</u> Determine the input impedance of the circuit in figure below at ω =10 rad/s.



$$Z_{C_1} = \frac{-j}{\omega C_1} = -\frac{j}{10(.002)} = -50j$$

$$Z_{C_2} = \frac{-j}{\omega C_2} = \frac{-j}{10(.004)} = -25j$$

$$Z_L = j\omega L = j(10)(2) = 20j$$

$Z_{\parallel}$

$$Z_{\parallel} = Z_{C2} \parallel (Z_L + 50) = -25j \parallel (50+20j) = \frac{(-25j)(50+20j)}{-25j+50+20j}$$

$$= \frac{500-1250j}{50-5j} \cdot \frac{(50+5j)}{(50+5j)} = \frac{31250-60{,}000j}{2525} = 12.37-23.76j$$

$$Z_{Total} = Z_{\parallel} + Z_{c1} + 20 = 12.37+20-23.76j - 50j = \underline{32.37 - j73.76}$$

<u>Answer</u>: Z_{in} = **32.38 – j73.76**

Ex 10

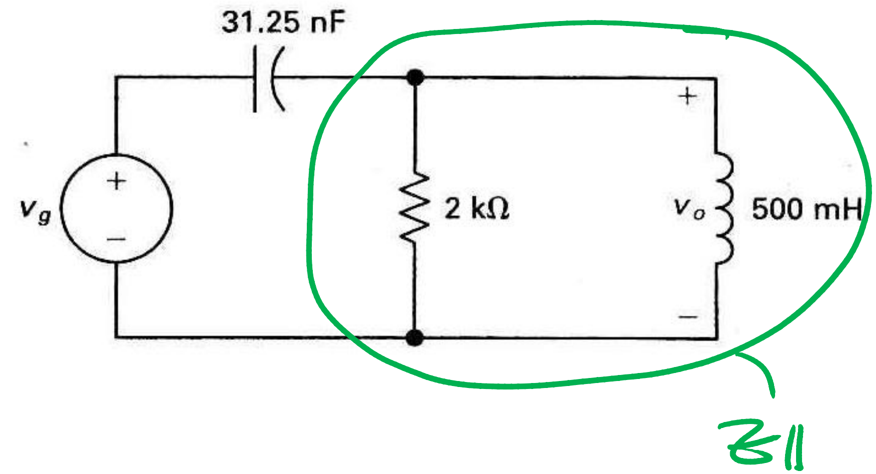
The circuit is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g = 64 \cos 8000t$ V.

1) Find $Z_C = \frac{-j}{8000(31.25 \times 10^{-9})}$

$Z_C = -4000j$

2) Find $Z_L = j(8000)(.5)$

$Z_L = +4000j$



3) $V_g = 64\angle 0$

4) Voltage Divider $V_o = \left(\frac{Z_{\|}}{Z_{\|} + Z_C}\right) V_g$

$Z_{\|} = \frac{(2000)(4000j)}{(2000 + 4000j)} = 1600 + 800j$

$V_o = \frac{(1600 + 800j)}{(1600 + 800j - 4000j)} V_g = \left(\frac{1}{2}j\right)(64\angle 0) = 32\angle 90$

$\therefore v_o(t) = 32\cos(8000t + 90)$

vo(t) = -32 sin(8000t) (or 32cos(8000t + 90))

Handouts

Example 1

Given a sinusoid, $5\sin(4\pi t - 60^o)$, calculate its amplitude, phase, angular frequency (ω, rad/s), period, and cyclical frequency (f, Hz).

Amplitude = 5, phase = –60°, angular frequency = 4π rad/s,
Period = 0.5 s, frequency = 2 Hz.

Example 2

Find the phase angle between $i_1 = -4\sin(377t + 25^o)$ and $i_2 = 5\cos(377t - 40^o)$, does i_1 lead or lag i_2?

i_1 leads i_2 by 155^o.

Complex Arithmetic (Rectangular)

Given:

$a = 3 + 4j$

$b = -5 + 6j$

Find

$a + b$

$a - b$

$a * b$

a / b

Polar to Rect (and back)

- Convert $a = 2 - 3j$ and $b = -4 + 5j$
  - to polar form:

- Convert $c = 2 < 30^o$ and $d = 4 < 200^o$
  - To rectangular form

Polar Complex Mult/Div

Given:

c = 8<45

d = 12<250

Find

c * d

c / d

c* (complex conj of c)

9.3 Phasor (5)

Example 4

Transform the following sinusoids to phasors:

$i = 6\cos(50t - 40^o)$ A

$v = -4\sin(30t + 50^o)$ V

$\mathbf{I} = 6\angle -40^\circ$ A

$\mathbf{V} = 4\angle 140^\circ$ V

9.3 Phasor (6)

Example 5:

Transform to sinusoids the phasors:

a. $\mathbf{V} = -10\angle 30^\circ \ \text{V}$

b. $\mathbf{I} = j(5 - j12) \ \text{A}$

v(t) = 10cos(ωt + 210^o) V, *i(t) = 13cos(ωt + 22.62^o) A*

Example 7A

If current $i(t) = 5\cos(100t + 30^o)\ mA$ is applied to a 2 mH inductor, calculate the voltage, $v(t)$, across the inductor.

Answer: $v(t) = \underline{\mathbf{\cos(100t + 120^o)\ V}}$

Example 7B

If voltage $v(t) = 6\cos(100t - 30^o)$ V is applied to a 50 μF capacitor, calculate the current, $i(t)$, through the capacitor.

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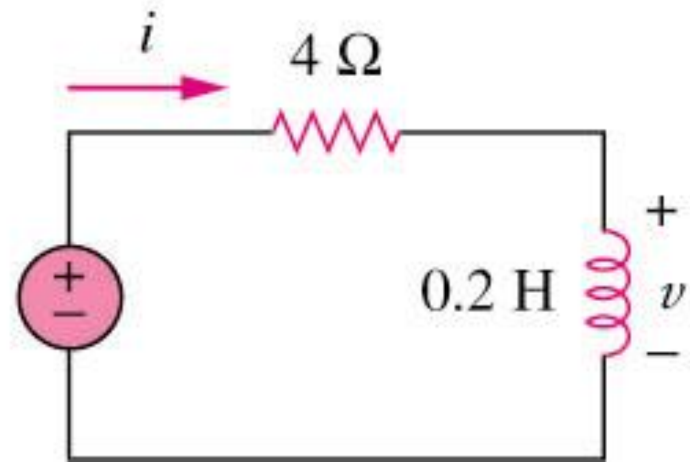
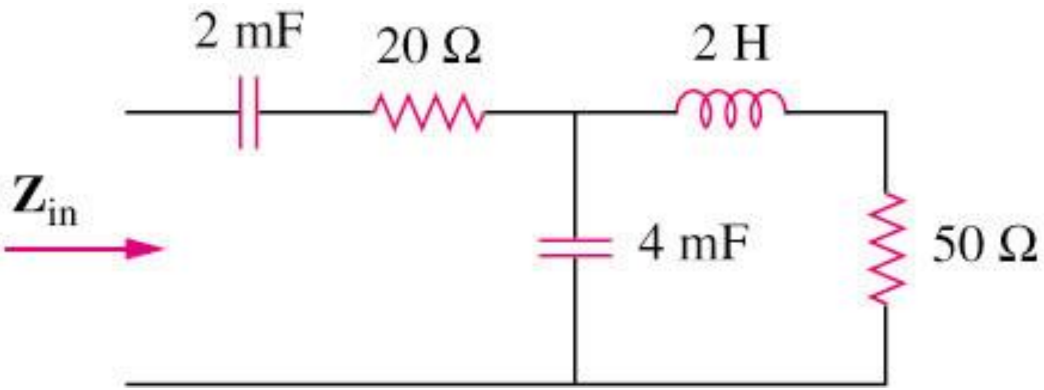
$$v_s = 5\cos(10t)$$



Answers: *i(t) = 1.118cos(10t – 26.56º) A; v(t) = 2.236cos(10t + 63.43º) V*

9.7 Impedance Combinations (2)

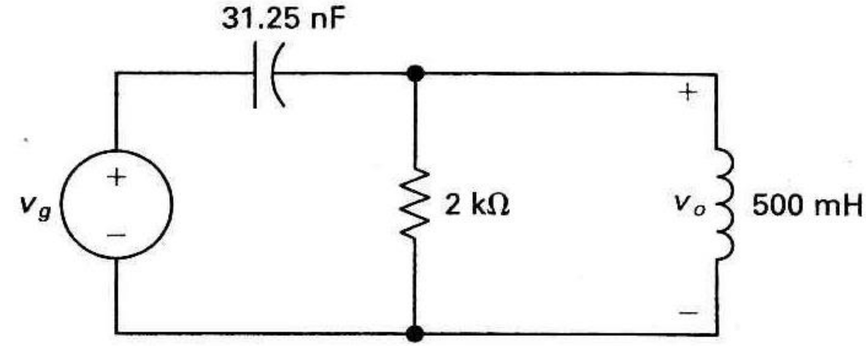
Example 9 Determine the input impedance of the circuit in figure below at ω =10 rad/s.



Answer: Z_{in} = **32.38 – j73.76**

Ex 10

The circuit is operating in the sinusoidal steady state. Find the steady-state expression for $v_o(t)$ if $v_g = 64 \cos 8000t$ V.



vo(t) = -32 sin(8000t) (or 32cos(8000t + 90))