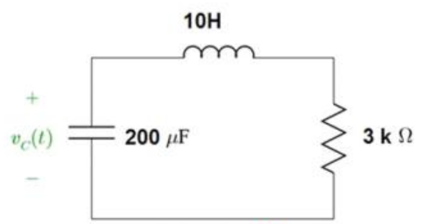
ENGR 12 Assignment 9 SOLUTIONS

Part I. Drills -- 1 point each

Determine the type of response (under, critical, over-damped) for these 2^{nd} order circuits:

1)



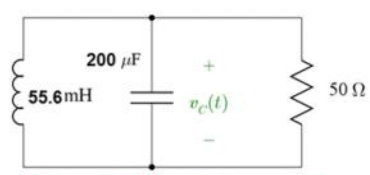
Series, Natural

$\alpha = \frac{3k}{2 \cdot 10} = 150$

$\omega_0 = \frac{1}{\sqrt{LC}} = 22.36$

$\alpha > \omega_0$ OVER

2)



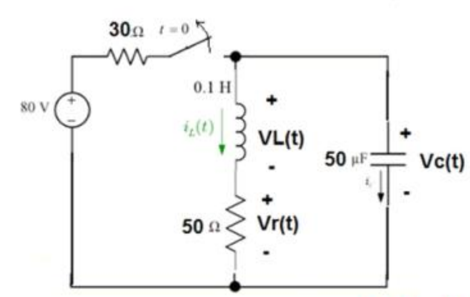
Parallel, Natural

$\alpha = \frac{1}{2RC} = 50$

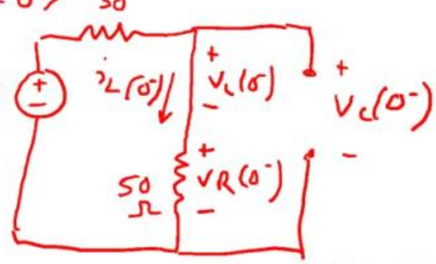
$\omega_0 = \frac{1}{\sqrt{LC}} = 300$

$\omega_0 > \alpha$ UNDER damped

3) For the following circuit, the switch is initially closed and has been for a long time. Find the initial values of Vc(t), iL(t), VL(t) and Vr(t) at t=0-, and diL(t)/dt at t=0+.



$(t < 0)$



$V_L(0^-) = \underline{0}$, $i_L(0^-) = \frac{80V}{80} = \underline{1A}$, $V_C(0^-) = i_L(0^-) \cdot 50 = \underline{50V}$

$V_r(0^-) = V_C(0^-) = 50V$

4) Derive the formula for the inductor current iL(t) in problem 3 for t>0.

Series, Natural: $\alpha = \frac{R}{2L} = 250$, $\omega_0 = \frac{1}{\sqrt{LC}} = 447.2$, Under damped, $\omega_d = 370.8$

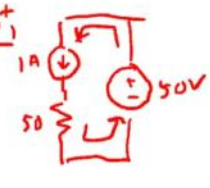
$i(0^+) = 1A = B_1$

$\frac{di}{dt}(0^+) = \frac{V_L(0^+)}{L} = 0 = -\alpha B_1 + \omega_d B_2$

$= -250 B_1 + 370.8 B_2$

$B_2 = 250(1A)/370.8 = 0.674$

at $t = 0^+$,



$\therefore\ i_L(t) = e^{-250t}\left(-\alpha B_1 + \omega_d B_2\right) = e^{-250t}\left(B_1 \cos 370.8t + B_2 \sin 370.8t\right)$

$i_L(t) = e^{-250t}\left(\cos 370.8t + .674 \sin 370.8t\right)$

5) Derive the formula for the capacitor voltage Vc(t) in problem 3 for t>0.

KVL: $-v_R - v_L + v_C = 0$, $v_C = v_L + v_R = L\frac{di_L}{dt} + i_L R$

$v_C = 0.1\left(e^{-250t}(-539.3\sin(370.8t) - 0.0808\cos(370.8t))\right) + \left(e^{-250t}(\cos(370.8t) + 0.674\sin(370.8t))\right)\cdot 50$

$v_C = e^{-250t}\left(50\cos(370.8t) - 20.2\sin(370.8t)\right)$ Volts

6) What value of the inductor in problem 3 would make the circuit critically damped?

$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$, $\frac{50}{2L} = \frac{1}{\sqrt{L\cdot 50\mu F}}$, $\frac{L}{25} = \frac{\sqrt{L\ 50\mu F}}{1}$

$\frac{L^2}{625} = L\cdot 50\mu F$, $L^{\cancel{2}} = \cancel{L}(.03125)$

$L = 31.25\ mH$

Part II. Assisted Problem Solving – 2 pts

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7) The natural voltage response of a parallel RLC circuit is v(t) = $75e^{-8000t}$(cos 6000t – 4sin 6000t) Volts, for t>0

If the inductor is 400 mH, find the values of C, R, Vo, Io, and iL(t).

$\alpha = +8000 = \frac{1}{2RC}$

$\omega_d = 6000 = \sqrt{\omega_o^2 - \alpha^2}$, $36\times10^6 = \omega_o^2 - 8000^2$

$\omega_o = \sqrt{100\times10^6} = 10000 = \frac{1}{\sqrt{LC}}$

$LC = \frac{1}{100\times10^6}$, $C = \frac{1}{L(100\times10^6)} = \frac{1}{40\times10^6} = \boxed{25nF}$

$n: \times10^{-9}$

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PLAN

1) note that this is underdamped
2) compare the equation to the formula for underdamped and notice where different parameters appear and match them up using algebra where needed.

$\alpha = \frac{1}{2RC} = 8000$, $R = \frac{1}{2(8000C)} = 2500\ \Omega$

$V_o = 75\cancel{e^{0}}(\cos 0 - 4\cancel{\sin 0})$

$= 75$

I_o + $I_L(t)$: skip !!

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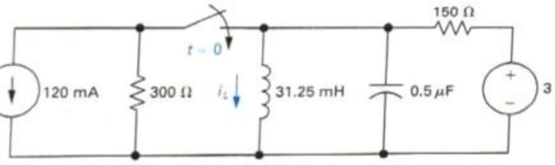
Part III. Unassisted Problem Solving – 3 points

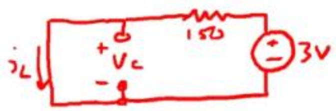
8)

The switch in the circuit has been open a long time before closing at $t = 0$. Find $i_L(t)$ for $t \geq 0$.



Parallel, STEP

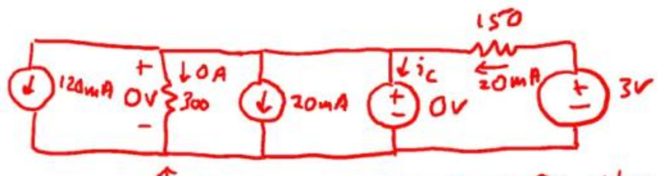
1) Circuit at $t=0^-$ (steady state before switch closes)



$i_L(0^-) = \frac{3}{150} = .02\ A$

$V_c(0^-) = 0\ V$ (V across a short)

2) Circuit at $t=0^+$



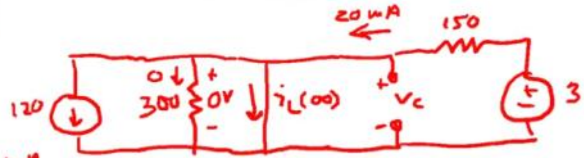
V_c stays at 0V

i_L stays at $\underline{20\ mA}$

0V across 300 Ω → 0A through it

$\frac{di_L}{dt} = \frac{V_L}{L} = V_c = \underline{0\ A/s}$

3) Circuit at $t = \infty$



$V_c = V_L = V_{300} = 0\,V$

$\boxed{KCL}$ $i_L + 120 = 20$, $\underline{i_L(\infty) = -100\ mA}$

3) $R_{eq} = 300 \| 150 = 100\ \Omega$ $\quad \alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 100 \cdot .5 \times 10^{-6}} = 10000$

$\omega_0 = \frac{1}{\sqrt{LC}} = 8000$ $\quad \alpha > \omega_0 \rightarrow$ OVER DAMPED

$s_1 = -10000 + \sqrt{10000^2 - 8000^2} = -10000 + 6000 = -4000$

$s_2 = -10000 - 6000 = -16000$

4) $i_L(0^+) = .02 = I_\infty + A_1 + A_2 \rightarrow A_1 + A_2 = .02 + .1 = .12\ A = 120\,mA$

$\frac{di_L}{dt}(0^+) = 0 = s_1 A_1 + s_2 A_2$, $\quad -4000 A_1 - 16000 A_2 = 0$, $\quad A_1 = -4A_2$

$-4A_2 + A_2 = .12$, $\quad -3A_2 = .12$, $\quad A_2 = -.04\,A = -40\,mA$

$A_1 = .16\,A = 160\,mA$

$i_L(t) = -100 + 160e^{-4000t} - 40e^{-16000t}$ mA