Sunday, April 17, 2011
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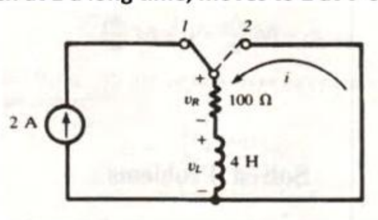
ENGR 12 Assignment 8 SOLNS

Part I. Drills -- 2 point each

1) Switch at 1 a long time, moves to 2 at t=0

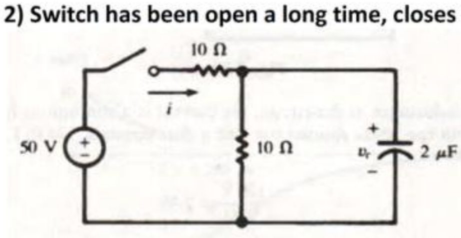


This is a ___NATURAL____ response

For the circuit above, find the:

a) initial current $i_L(0-)$ 2 A
b) final current $i_L(\infty)$ 0
c) Effective R seen by inductor (t>0) 100 ohm
d) time constant for t>0 .04 sec
e) $i_L(t) = 2e^{-25t}\ A$
f) $v_L(t) = \frac{Ldi}{dt} = -200e^{-25t}\ V$

2) Switch has been open a long time, closes at t=0



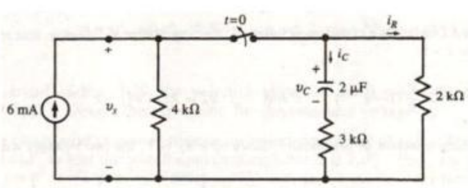
This is a _____STEP______ response

For the circuit above, find the:

a) initial voltage Vc(0-) 0
b) final voltage $Vc(\infty)$ 25V
c) Reff seen by capacitor (t>0) 5 Ohm
d) time constant for t>0 10 usec
e) $v_c(t) = 25 - 25e^{-100000t}\ V$
f) $i_c(t) = \frac{CdVc}{dt} =$
$= 2uF*(-25)(-100000)\ e^{-100000t}$
$= 5e^{-100000t}$ A

Part II. Assisted Problem Solving – 1.5 pts each

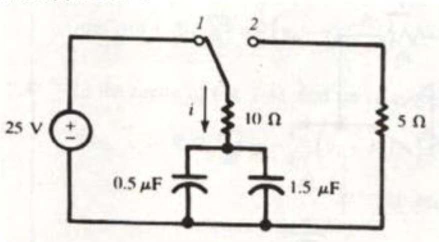
3. Switch has been closed and opens at t=0. Find Vc, Ic, Ir and Vs for t>0



This is a __NATURAL____ response

1) Vc(0-) = Vs(0-) = .006*(4||2) = 8 V
2) Req = 5 k
3) Tau = .01
4) $Vc(t) = 8e^{-100t}\ V$
5) $Ic(t) = \frac{CdVc}{dt} = -1.6e^{-100t}$ mA
6) $Ir(t) = -Ic(t) = +1.6e^{-100t}$ mA
7) Vs(t) = 24V constant for t>0

4. For t<0, swich had been at 2 a long time. At t=0, switch goes to 1. At t = 60 micro-seconds, switch goes back to 2. Find i(t) for t> 0



This is a ___STEP/Natural/STEP__ response

1) Vc(0-) = 0
2) Ceq = .5 + 1.5 = 2 uF
3) Vc(∞) (for t<0<60) = 25V
4) Req = 10 Ohm
5) Tau for t<0<=60 usec = 20 usec
6) Vc (t) for t<0<=60 = 25 – 25$e^{-50000t}$ parallel caps
7) Vc(60 usec) = $25(1 - e^{-50000(60x10^{-6})})$ = 23.75 V
8) For second switch throw (t > 60usec):
9) Vc((60 usec) = 23.75 V
10) Req = 15 Ohm
11) Tau for t>60usec = 15*2x10^{-6} = 30 usec
12) Vc(t>60 usec) = 23.75$e^{-33333(t-60usec)}$ V
13) Finally, use Vc(t) to find i(t):

Region I : 0<t<=60usec:
i(t) = C dVc/dt = 2uF$(1.25x10^{-6})$ $= \mathbf{2.5e^{-50000t}\ A}$

Region II: 60usec < t < ∞
i(t)=2uF(23.75)(-33333) $e^{-33333(t-60usec)}$
$= = \mathbf{-1.583e^{-33333(t-60usec)}\ A}$

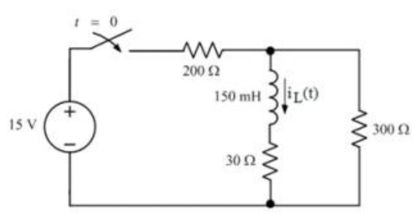
Notice that we must use (t – 60 usec) for region II because we need the exponent to equal zero at t = 60 usec to match the values of Vc at the instant of switching. In other words,

Vc(60usec—) = Vc(60usec+)

and this will only work if we use t-60 usec for the second exponential.

Part III. Unassisted Problem Solving – 2 points each

5) The switch has been open a long time. At t=0 the switch closes. Find $i_L(t)$ for t>=0^+



This is a ____STEP___ response
See SOLN Video: http://www.rose-hulman.edu/CLEO/video/player.php?id=38&embed

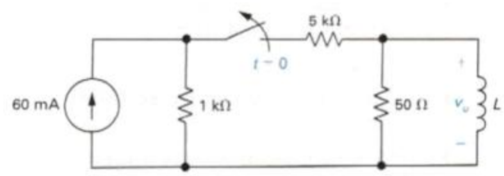
a) iL(0) = 0,
b) In the steady state, inductor is short, and voltage source sees 200 + (30||300) Ohm = 227.27, so isource = 15/227.27 = .066 A, and IL sees only part of that thru the current divider, so iL(∞) = 66*300/330 = 60 mA
c) Req = 200||300 + 30 = 150 Ohm
d) tau = L/R = .150/150 = .001 sec
e) IL(t) = 60 – 60 e^{-1000t} mA

6)

In the circuit the switch has been closed for a long time before opening at $t = 0$.

(a) Find the value of L so that $v_o(t)$ equals $0.25\ v_o(0^+)$ when $t = 5$ ms.

(b) Find the percentage of the stored energy that has been dissipated in the 50 Ω resistor when $t = 5$ ms.



This is a _____NATURAL__ response

a) The solution for IL(t) = Io $e^{-t/\tau}$ (where Io is initial current thru inductor)

and the solution for Vo(t) = $\frac{L di_L}{dt}$ = $\frac{LIo}{\tau} e^{-t/\tau}$, in other words, same time constant as IL(t)

We are given at 5ms Vo(5ms) = 0.25Vo(0+) = Vo(0+) $e^{-0.005/\tau}$.

Therefore, we only need to solve 0.25 = $e^{-\frac{0.005}{\tau}}$, or , taking log of both sides,

$-\frac{0.005}{\tau} = \log(.25) = -1.386$, or, $\tau = 3.607\ ms$ = L/R = L/50, (Req = 50 Ohm for t>0)

so L = R* τ = 180.4 mH

b) Lets solve for IL(t). First, we'll need Io. Before switch opened, Inductor is a short. Due to current divider,

Io = 60 * 1/(1+5) = 10 mA. Therefore:

IL(t) = 10 $e^{-t/\tau} mA = 10\ e^{-277.2t}$

IL(0) = 10 mA and IL(5ms) = 10 $e^{-277.2(0.005)}$ = 2.5 mA (as in V, we are .25 of original value)

Energy in Inductor = $w(t) = \frac{1}{2} L i_L(t)^2$

Ratio of energy in inductor at 5 ms vs 0 ms = $\frac{\frac{1}{2} L i_L(5ms)^2}{\frac{1}{2} L i_L(0)^2} = \frac{2.5^2}{10^2} = .0625\ or\ 6.25\%$

The energy lost in inductor has been dissipated in the 50 Ohm resistor, so the resistor has dissipated 100 – 6.25 = 93.75%