Thursday, June 07, 2012
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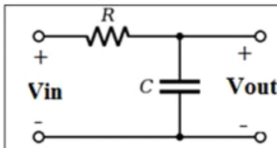
ENGR 12 Assignment 14 SOLUTIONS

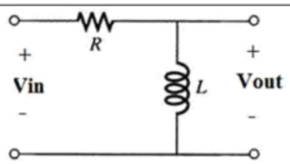
Part I. Drills -- 1 point each

1) A series RC lowpass filter requires a cutoff frequency of 8 kHz. Use R = 10 kΩ and compute the value of C required.



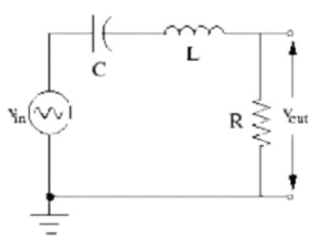
Wo = 1/RC = 8000*2π, RC = 1/(16000 π),
C = 19.894μs /10kΩ , <u>**C = 1.98 nF**</u>

2) A series RL highpass filter with a cutoff frequency of 2 kHz is needed. Using R = 5 kΩ, compute
a) L, b) |H(w)| at 200 Hz, c) Phase(H(w)) at 200 Hz



a) Wo = R/L = 2000*2*π, L = R/(4000π) = 5000/(4000π) , <u>**L= 397 mH**</u>
b) when w = 200*2*π, H(w) = 1/(1+ R/(jwL)) = 0.0099 + 0.0990i
H(200) = 0.100 < 84.29, <u>**|H(w)| = 0.1**</u>
c) <u>**phase(H(w)) = 84.29 degrees**</u>

3) A series RLC has a 0.1 uF capacitor. Find R and L for a bandpass filter with a center frequency of 12 kHz and a Q of 6



$\omega_o = \frac{1}{\sqrt{LC}}$ rad/s = 12000*2*π , LC = 1/(24000π)^2, <u>**L = 1.76 mH**</u>

$$Q = \frac{\text{Peak energy stored in the circuit}}{\text{Energy dissipated by the circuit in one period at resonance}} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

Q = Wo*L/R = 6, R = Wo*L/Q = 24000π*.00176/6, <u>**R = 22.1 Ω**</u>

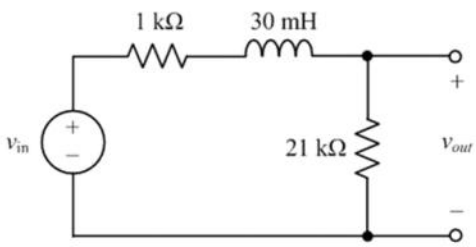
4) Show that $w_0 = \sqrt{w_l w_h}$ for the series RLC circuit

$$\omega_1 = -\frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \qquad \omega_2 = \frac{R}{2L} + \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \qquad \omega_o = \sqrt{\omega_1 \omega_2}$$

w1 = 69376, w2 = 81942, w0 = 75398 and the original w0 = 24000π = 75398

Part II. Assisted Problem Solving – 2 pts

6) Find the transfer function Vout/Vin and the corner frequency w. What kind of filter is it? Plot the magnitude and phase of H(w) using Freemat or WolframAlpha.



Plan

1) Replace the inductor with j w (0.03)
2) Use the voltage divider formula to derive H(w)
3) Then find H(w) by taking Vout/Vin
4) Plot the magnitude and phase of H(w) using Freemat. Something like this:

```
w = [0:100:1e6];
H = 21000 ./ (1000 + j*w*(0.3));
semilogx( w, abs(H));
```

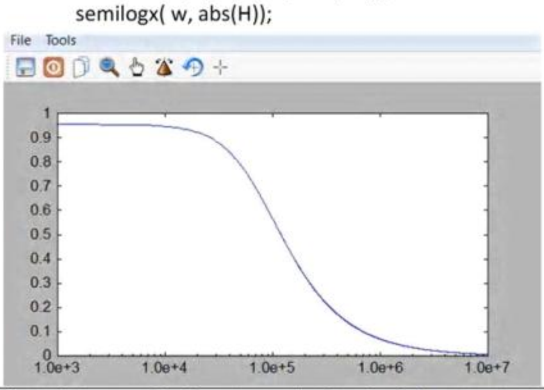
[copy and paste plot into word, then…]

```
semilogx( w, angle(H)*180/pi)
```

make sure to use ./ to divide the two terms of H

Solution to 6)

```
w = [1000:1000:10e6];
H = 21000 ./ (22000 + j*w*(0.3));
semilogx( w, abs(H));
```




```
semilogx( w, angle(H)*180/pi)
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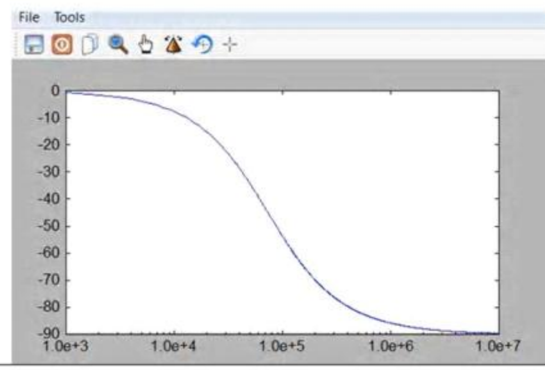
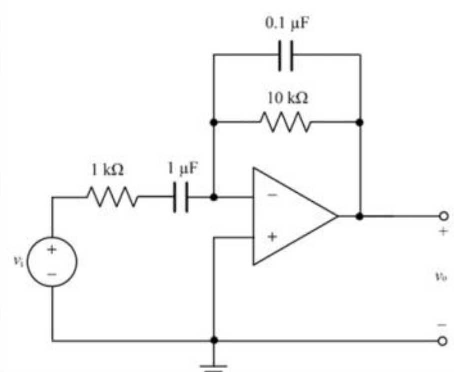


7) Find the transfer function Vo/Vi for the following circuit



PLAN

1) Note that this is an inverting amplifier
2) Determine the values of Rs and Rf and use the formula from Chapter 5 (that's Vo = -(Rf/Rs) Vi)
3) Simplify your formula for H(w) if necessary
4) Plot the magnitude and phase of H(w) using Freemat

Solution to 7)

H(w) = - Zf/Zs =

Zf = 10000 ||(1/ (jw Cf)) (where Cf = 0.1 µF)

$$= \frac{\frac{10000}{jwCf}}{10000+\frac{1}{jwCf}} = \frac{\frac{10000*1*10^7}{jw}}{10000+\frac{1*10^7}{jw}} = \frac{1*10^7}{jw+1000}$$

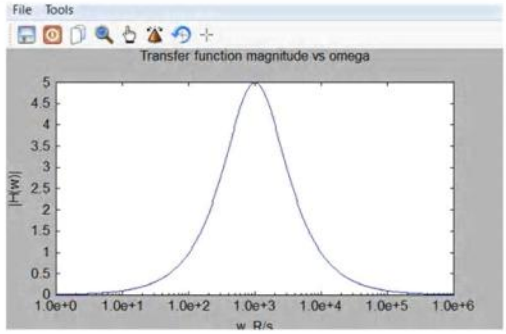
Zs = 1000 + (1/(jw Cs)) (where Cs = 1 µF)

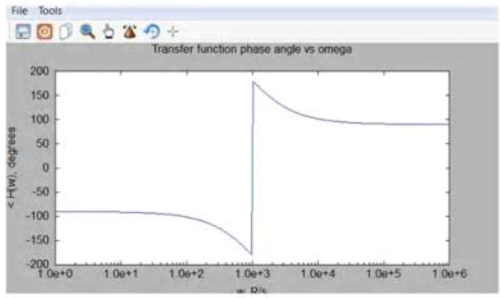
$$= 1000 + \frac{1}{jwCs} = 1000 + \frac{1*10^6}{jw} = \frac{1000jw + 1*10^6}{jw}$$

H(w) = -(Zf)/(Zs)

$$= -\frac{\frac{1*10^7}{jw+1000}}{\frac{1000jw+1*10^6}{jw}} = \frac{-jw\,(1*10^7)}{(1000jw+1*10^6)(jw+1000)}$$

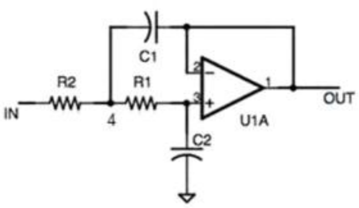
$$\frac{-10000\text{jw}}{(jw+1000)(jw+1000)} = \frac{-10000\text{jw}}{(1*10^6 - w^2 + 2000jw)}$$

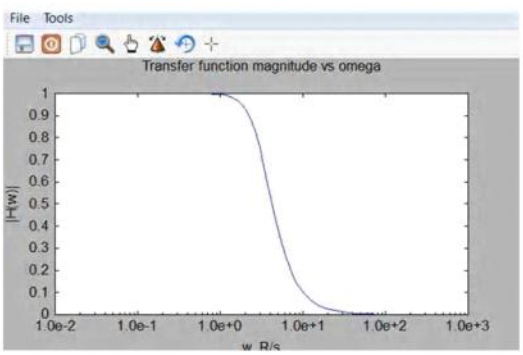




Part III. Unassisted Problem Solving – 3 points

8) Find the transfer function for this circuit given R1=R2=110 k, C1 = 4uF, C2 = 2 uF. Determine the cutoff frequency and plot the magnitude in Freemat. Hint: since there is negative feedback, terminal 3 will have the same voltage as 2.





Solution:

Call terminal 4 (V4) the center node between R2 and R1

Note that V2 = Vout and V3 = Vout

Then,

1) (Vin – V4)/R2 + (V3-V4)/R1 + (V2-V4)/ZC1 = 0

or, (Vin – V4)/R2 + (Vout-V4)/R1 + (Vout-V4)/ZC1 = 0

or, Vin = R2*(V4/R2 + (V4 – Vout)/R1 + (V4 – Vout)/ZC1

and

2) (V3 – V4)/R1 + V3/ZC2 = 0

or, (Vout – V4)/R1 + Vout/ZC2 = 0

solving 2) for V4:

3) V4 = R1*Vout(1/R1 + 1/ZC2) = Vout(1 + R1/ZC2)

Substituting 3) into 1)

Vin = R2*Vout*[(1+R1/ZC2)/R2 + (R1/ZC2)/R1 + (R1/ZC2)/ZC1]

H(w) = Vout/Vin = 1/[(1 + R1/ZC2) + (R2/ZC2) + (R1*R2/(ZC2*ZC1)]

= 1/[1 + jwR1C2 + jwR2C2 – w^2 R1*R2*C1*C2

$$H(w) = \frac{1}{1 - w^2R_1R_2C_1C_2 + jwC_2(R_1+R_2)}$$

when w = 0, H(w) = 1, when w = ∞, H(w) = 0 → low pass filter

note the steeper rolloff of this filter due to the w^2 term.

	To find the cutoff frequency, set magnitude of H to 1/sqrt(2) $\|H(w)\| = .7071 = \frac{1}{1 - w^2R_1R_2C_1C_2 + jwC_2(R_1+R_2)}$ when $\|1 - w^2R_1R_2C_1C_2 + jwC_2(R_1 + R_2)\| = 1.414$ or $(1 - w^2R_1R_2C_1C_2)^2 + \left(wC_2(R_1 + R_2)\right)^2 = 2$ or $1 - 2w^2R_1R_2C_1C_2 + (w^2R_1R_2C_1C_2)^2 + \left(wC_2(R_1 + R_2)\right)^2 = 2$ Substituting in values for R and C gives $w^4 = 106.72$ $w = 3.214$