ENGR 12 Assignment 12 SOLUTIONS

Part I. Drills -- 1 point each

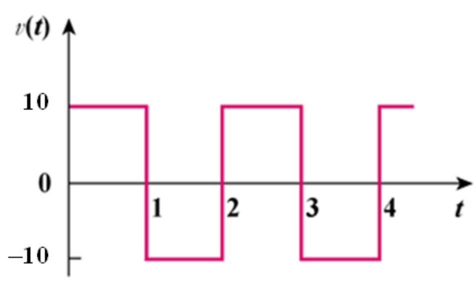
1) If $v(t)$ = 160 cos 50t V and $i(t)$ = - 20 sin (50t - 30°) A, calculate the instantaneous power and the average power.

$$v(t) = 160\cos(50t)$$
$$i(t) = -20\sin(50t - 30°) = 2\cos(50t - 30° + 180° - 90°)$$
$$i(t) = 20\cos(50t + 60°)$$

$$p(t) = v(t)i(t) = (160)(20)\cos(50t)\cos(50t + 60°)$$
$$p(t) = 1600\lfloor \cos(100t + 60°) + \cos(60°) \rfloor \text{ W}$$
$$p(t) = \underline{\mathbf{800 + 1600\cos(100t + 60°)\ W}}$$

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(160)(20)\cos(60°)$$
$$P = \underline{\mathbf{800\ W}}$$

2) Determine the rms value of the waveform



$$T = 2, \qquad \mathbf{v(t)} = \begin{cases} 10, & 0 < \mathbf{t} < 1 \\ -10, & 1 < t < 2 \end{cases}$$

$$\mathbf{V_{rms}^2} = \frac{1}{2}\left[\int_0^1 10^2 \ \mathbf{dt} + \int_1^2 (-10)^2 \ \mathbf{dt} \right] = \frac{100}{2}[1+1] = 100$$
$$V_{rms} = \underline{\mathbf{10\ V}}$$

3) For the following voltage and current phasors, calculate the complex power, apparent power, real power, and reactive power. Specify whether the pf is leading or lagging.

(a) **V** = 220 ∠30° V rms, **I** = 0.5 ∠60° A rms

(b) **V** = 250 ∠-10° V rms, **I** = 6.2 ∠-25°A rms

(c) **V** = 120 ∠0° V rms, **I** = 2.4 ∠-15° A rms

(d) **V** = 160 ∠45° V rms, **I** = 8.5 ∠90° A rms

ANSWERS:

(a) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (220\angle 30°)(0.5\angle -60°) = 110\angle -30°$

$\mathbf{S} = \underline{\mathbf{95.26 - j55\ VA}}$

Apparent power = **110 VA**

Real power = **95.26 W**

Reactive power = **55 VAR**

pf is **leading** because current leads voltage

(b) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (250\angle -10°)(6.2\angle 25°) = 1550\angle 15°$

$\mathbf{S} = \underline{\mathbf{1497.2 + j401.2\ VA}}$

Apparent power = **1550 VA**

Real power = **1497.2 W**

Reactive power = **401.2 VAR**

pf is **lagging** because current lags voltage

(c) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (120\angle 0°)(2.4\angle 15°) = 288\angle 15°$

$\mathbf{S} = \underline{\mathbf{278.2 + j74.54\ VA}}$

Apparent power = **288 VA**

Real power = **278.2 W**

Reactive power = **74.54 VAR**

pf is **lagging** because current lags voltage

(d) $\mathbf{S} = \mathbf{V}\mathbf{I}^* = (160\angle 45°)(8.5\angle -90°) = 1360\angle -45°$

$\mathbf{S} = \underline{\mathbf{961.7 - j961.7\ VA}}$

Apparent power = **1360 VA**

Real power = **961.7 W**

Reactive power = **- 961.7 VAR**

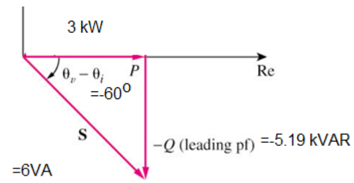
pf is **leading** because current leads voltage

4) An electrical load operates at 120 Vrms. The load absorbs an average power of 3 kW at a leading power factor of 0.5. Use the power triangle to calculate a) the power factor angle, b) the apparent power, |S|, c)

the reactive power |Q| of the load. d) the impedance of the load (use **S** = **Vrms Irms***, or **Irms** = **(S/Vrms)***, then find Z=**Vrms/Irms**)

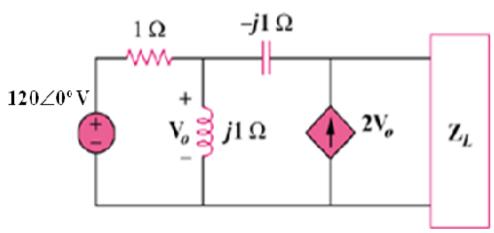
We are given: P = 3kW and a leading power factor = 0.5 = P/|**S**| = cos(pfa).

a) Therefore, the power factor angle = $\cos^{-1}(0.5)$= **<u>-60 degrees</u>** because it's a leading power factor
b) Solving for the apparent power, **<u>|S| = P/0.5 = 6 kVA</u>**
c) Q = |**S**| sin(pfa) = 6 sin(-60), **<u>Q = -5.19 kVAR</u>**
d) We know that **S** = P + jQ = 3 – j5.19 = 6<-60,
   and we know that **S** = **Vrms Irms***, or **Irms** = **(S/Vrms)***
   therefore **Irms** = ((6000<-60)kVA / 120 Vrms)* = 50<60 Amps rms
   Finally Z=**Vrms/Irms** = 120<0/50<60 = 2.4<-60 = 1.2 - 2.08j Ohms



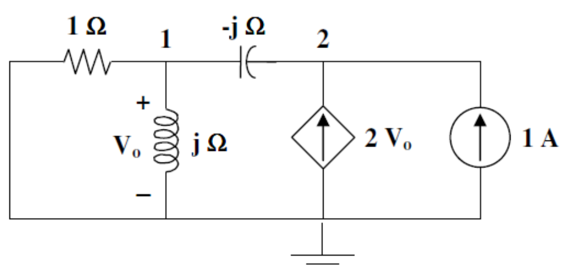
Part II. Assisted Problem Solving – 2 pts

5) find the value of Z_L that will absorb the maximum power and the value of the maximum power.



1) Find the thevenin equivalent
2) Then ZL = Zth*
3) Use the max power transfer formula
   Pmax = Vth^2/Rth

To find $\mathbf{Z}_{Th}$, insert a 1-A current source at the load terminals as shown in Fig. (a)



(a)

At node 1,

$$\frac{\mathbf{V}_o}{1} + \frac{\mathbf{V}_o}{j} = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow \mathbf{V}_o = j\mathbf{V}_2 \qquad (1)$$

At node 2,

$$1 + 2\mathbf{V}_o = \frac{\mathbf{V}_2 - \mathbf{V}_o}{-j} \longrightarrow 1 = j\mathbf{V}_2 - (2+j)\mathbf{V}_o \qquad (2)$$

Substituting (1) into (2),

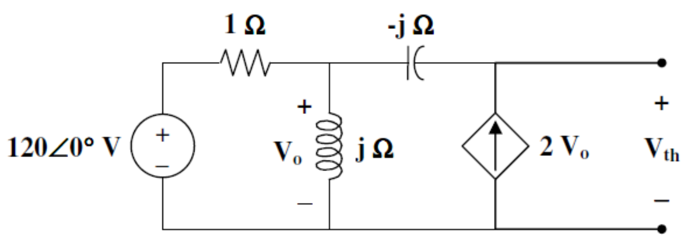
$$1 = j\mathbf{V}_2 - (2+j)(j)\mathbf{V}_2 = (1-j)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{1}{1-j}$$

$$\mathbf{Z}_{Th} = \frac{\mathbf{V}_2}{1} = \frac{1+j}{2} = 0.5 + j0.5$$

$$\mathbf{Z}_L = \mathbf{Z}_{Th}^* = \underline{\mathbf{0.5 - j0.5\ \Omega}}$$

We now obtain $\mathbf{V}_{Th}$ from Fig. (b).



(b)

$$2\mathbf{V}_o + \frac{120 - \mathbf{V}_o}{1} = \frac{\mathbf{V}_o}{j}$$

$$\mathbf{V}_o = \frac{-120}{1+j}$$

$$-\mathbf{V}_o - (-j \times 2\mathbf{V}_o) + \mathbf{V}_{Th} = 0$$

$$\mathbf{V}_{Th} = (1 - j2)\mathbf{V}_o = \frac{(-120)(1-j2)}{1+j}$$

$|\mathbf{V}_{Thev}| = 189.74$ V

I = 189.74/(0.5+j0.5+0.5–j0.5) = 189.74 A

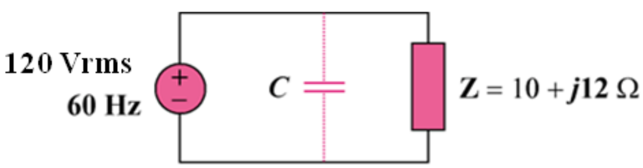
$P_{max} = (189.74)^2(0.5)/2 = \underline{\mathbf{9\ kW}}$

6) Refer to the circuit shown

(a) What is the power factor?

(b) What is the average power dissipated?

(c) What is the value of the capacitance that will give a unity power factor when connected to the load?



1) Ignore capacitor C (it's not attached) until step c
2) Find Irms and use that to find S
3) Draw the power triangle
4) Use the power factor correction formula for the capacitor to balance a load.

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{rms}^2}$$

(a) Given that $\mathbf{Z} = 10 + j12$

$$\tan\theta = \frac{12}{10} \longrightarrow \theta = 50.19°$$

$$pf = \cos\theta = \underline{\mathbf{0.6402}\ \text{lagging}}$$

(b) $$\mathbf{S} = \frac{\mathbf{Vrms}^2}{\mathbf{Z}^*} = \frac{(120)^2}{(10 - j12)} = 590.2 + j708.2$$

The average power absorbed $= \text{P} = \text{Re}(\mathbf{S}) = \underline{590.2\ W}$

(c) For unity power factor, $\theta_1 = 0°$, which implies that the reactive power due to the capacitor needs to be $Q_c = 708.2$

But $$Q_c = \frac{Vrms^2}{X_c} = \omega C\, Vrms^2$$

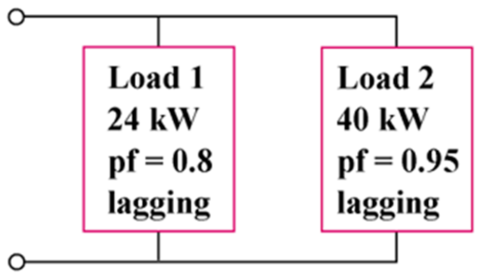
$$C = \frac{Q_c}{\omega Vrms^2} = \frac{(708.2)}{(2\pi)(60)(120)^2} = \underline{\mathbf{130.4\ \mu F}}$$

Part III. Unassisted Problem Solving – 3 points

7) A 120-V rms 60-Hz source supplies two loads connected in parallel, as shown in Fig. 11.89.

(a) Find the power factor of the parallel combination.

(b) Calculate the value of the capacitance connected in parallel that will raise the power factor to unity.



(a)

$$\theta_1 = \cos^{-1}(0.8) = 36.87°$$

$$\text{S}_1 = \frac{\text{P}_1}{\cos\theta_1} = \frac{24}{0.8} = 30\ \text{kVA}$$

$$\text{Q}_1 = \text{S}_1 \sin\theta_1 = (30)(0.6) = 18\ \text{kVAR}$$

$$\mathbf{S}_1 = 24 + j18\ \text{kVA}$$

$$\theta_2 = \cos^{-1}(0.95) = 18.19°$$

$$\text{S}_2 = \frac{\text{P}_2}{\cos\theta_2} = \frac{40}{0.95} = 42.105\ \text{kVA}$$

$$\text{Q}_2 = \text{S}_2 \sin\theta_2 = 13.144\ \text{kVAR}$$

$$\mathbf{S}_2 = 40 + j13.144\ \text{kVA}$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = 64 + j31.144\ \text{kVA}$$

$$\theta = \tan^{-1}\left(\frac{31.144}{64}\right) = 25.95°$$

$$\text{pf} = \cos\theta = \underline{\mathbf{0.8992}}$$

(b) $\theta_2 = 25.95°$, $\theta_1 = 0°$

$$Q_c = P[\tan\theta_2 - \tan\theta_1] = 64[\tan(25.95°) - 0] = 31.144 \text{ kVAR}$$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{31{,}144}{(2\pi)(60)(120)^2} = \underline{\mathbf{5.74\ mF}}$$