

# A Binary Search Tree Implementation

## Chapter 25



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- The Efficiency of Operations
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- An Implementation of the ADT Dictionary

# Objectives

- Decide whether a binary tree is a binary search tree
- Locate a given entry in a binary search tree using fewest comparisons
- Traverse entries in a binary search tree in sorted order
- Add a new entry to a binary search tree

# Objectives

- Remove entry from a binary search tree
- Describe efficiency of operations on a binary search tree
- Use a binary search tree to implement ADT dictionary

# Getting Started

- Characteristics of a binary search tree
  - A binary tree
  - Nodes contain **Comparable** objects
- For each node
  - Data in a node is greater than data in node's left subtree
  - Data in a node is less than data in node's right subtree

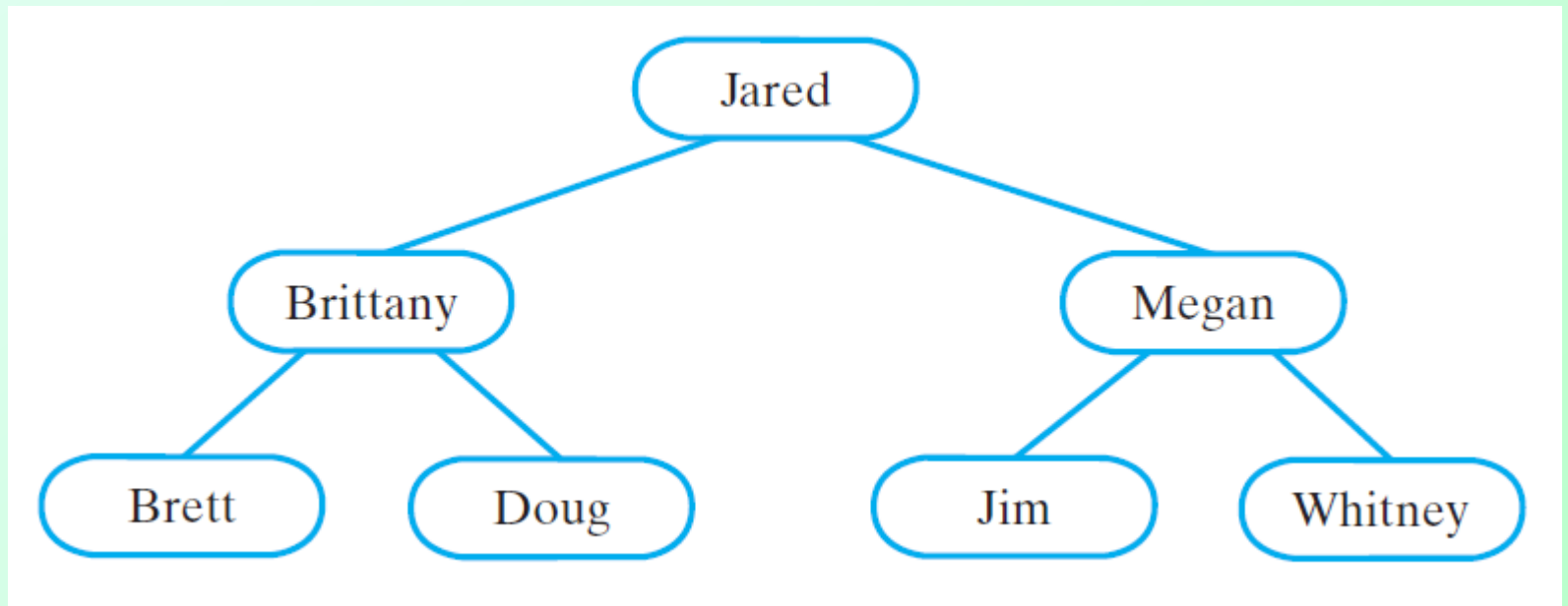


Figure 25-1 A binary search tree of names



# Interface for the Binary Search Tree

- Additional operations needed beyond basic tree operations
- Database operations
  - Search
  - Retrieve
  - Remove
  - Traverse
- Note interface, [Listing 25-1](#)

Note: Code listing files must be in same folder as PowerPoint files for links to work

# Understanding Specifications

- Interface specifications allow use of binary search tree for an ADT dictionary
- Methods use return values instead of exceptions
  - Indicates success or failure of operation

(a) Before `myTree.add(whitney2)` executes

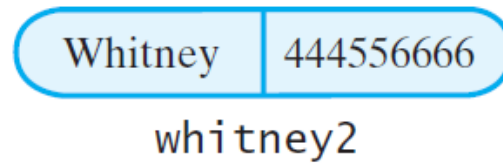


Figure 25-2 Adding an entry that matches an entry already in a binary search tree

(b) After `myTree.add(whitney2)` executes

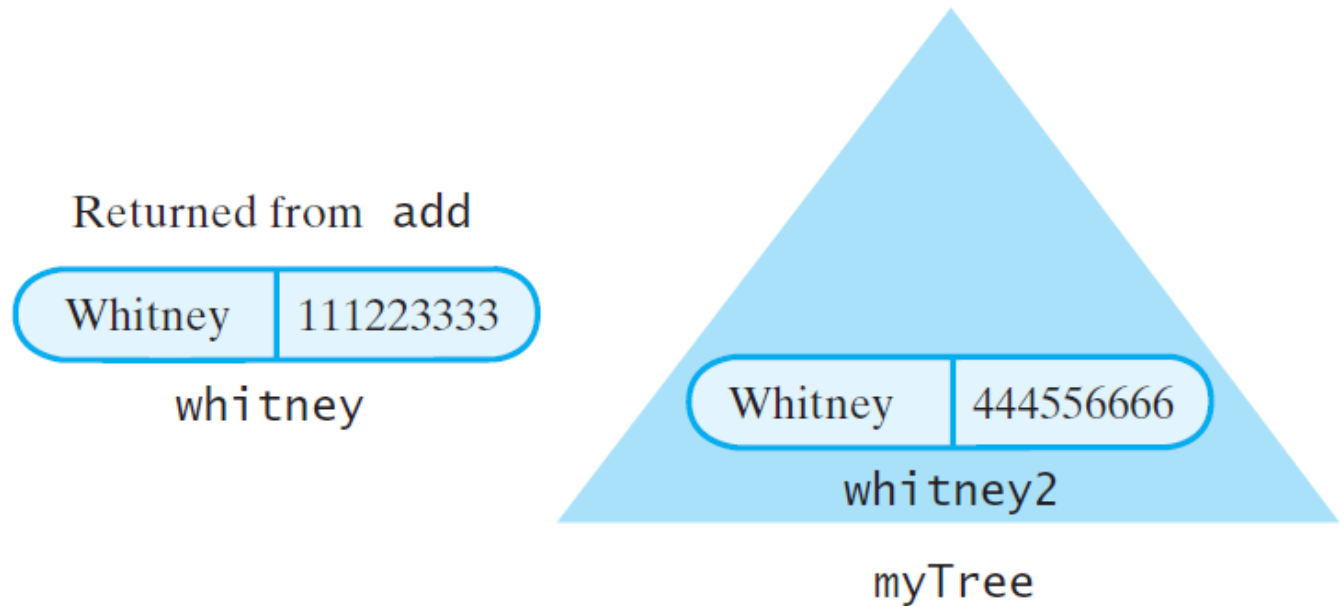


Figure 25-2 Adding an entry that matches an entry already in a binary search tree

# Duplicate Entries

- For simplicity we do not allow duplicate entries
  - **Add** method prevents this
- Thus, modify definition
  - Data in node is greater than data in node's left subtree
  - Data in node is less than *or equal* to data in node's right subtree

Inorder traversal of the tree visits duplicate entry *Jared* immediately after visiting the original *Jared*.

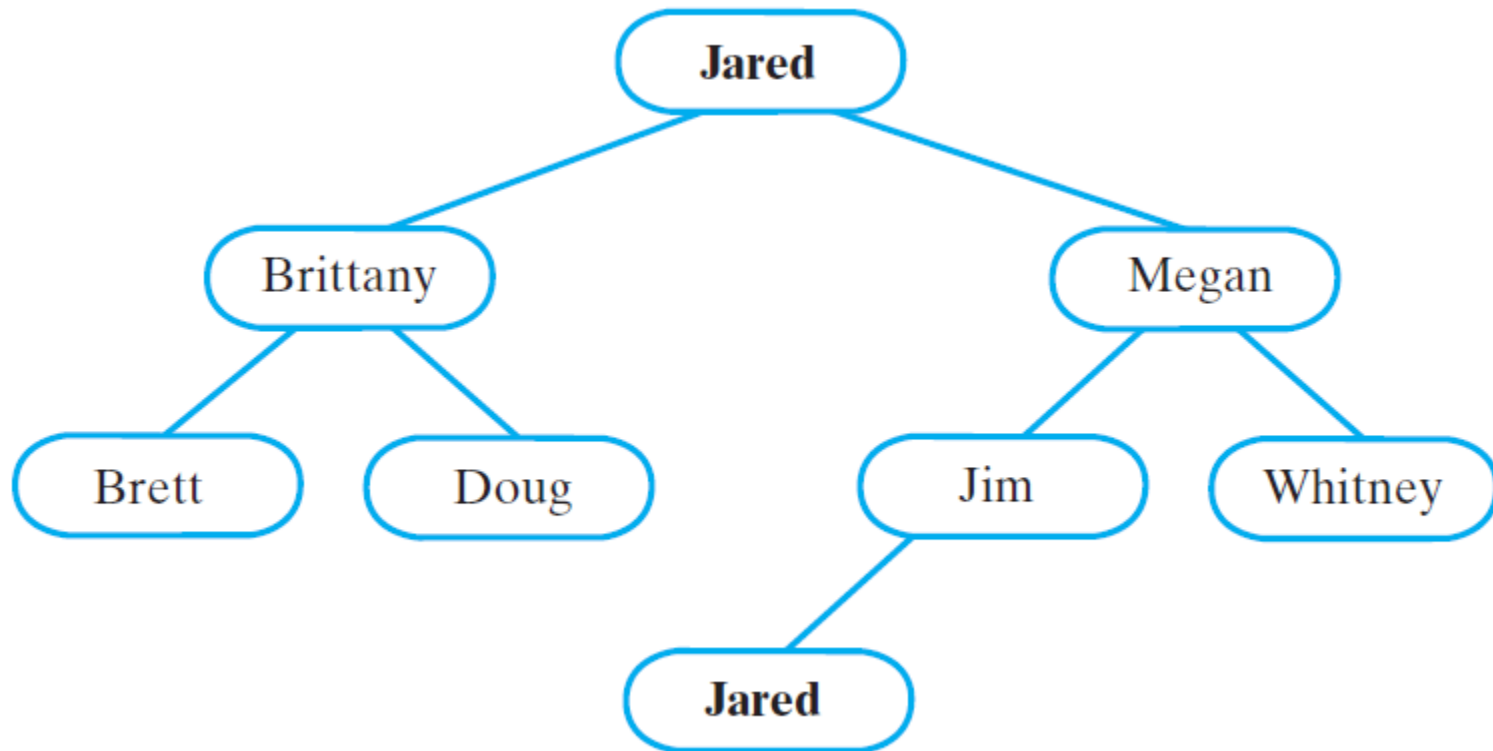
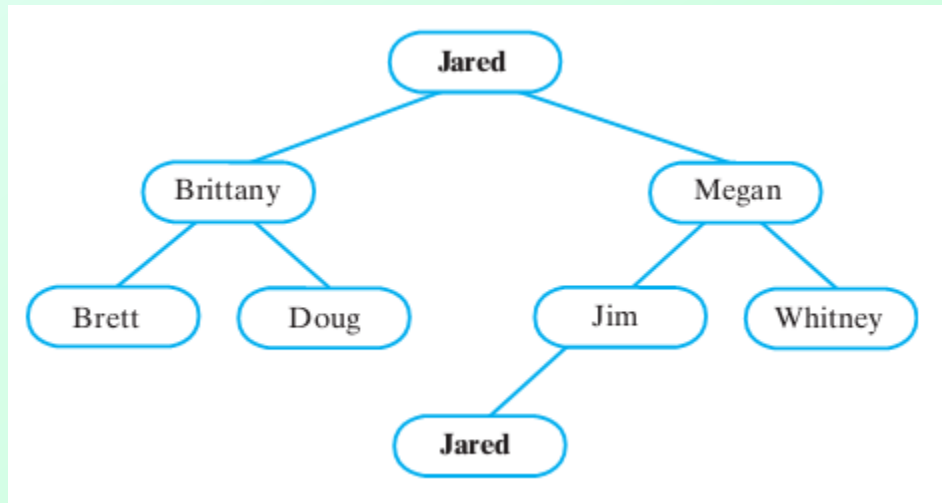
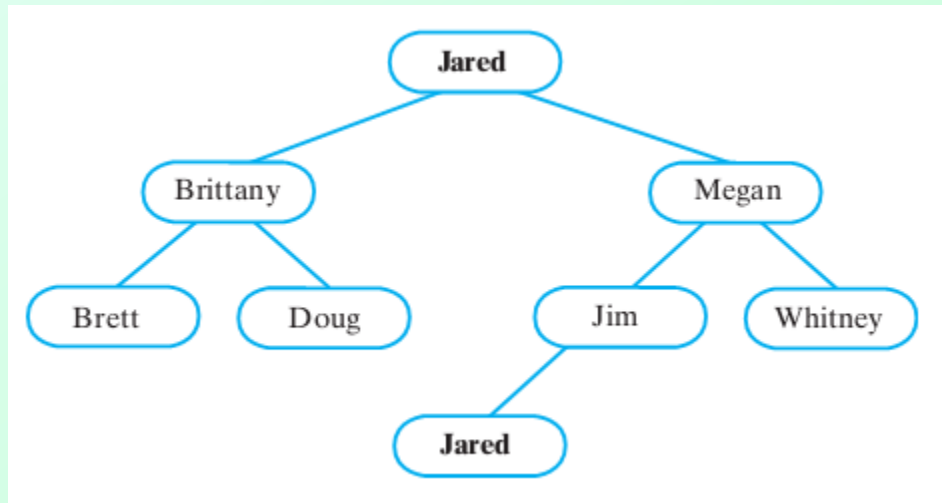


Figure 25-3 A binary search tree with duplicate entries

Question 1 If you add a duplicate entry Megan to the binary search tree in Figure 25-3 as a leaf, where should you place the new node?



Question 1 If you add a duplicate entry Megan to the binary search tree in Figure 25-3 as a leaf, where should you place the new node?



As the left child of the node that contains Whitney.



# Beginning the Class Definition

- Source code of class **BinarySearchTree**, [Listing 25-2](#)
  - Note methods which disable **setTree** from **BinaryTree**
- de

Question 2 The second constructor in the class `BinarySearchTree` calls the method `setRootNode` . Is it possible to replace this call with the call `setRootData(rootEntry)`? Explain.

Question 3 Is it necessary to define the methods `isEmpty` and `clear` within the class `BinarySearchTree` ? Explain.

Question 2 The second constructor in the class `BinarySearchTree` calls the method `setRootNode`. Is it possible to replace this call with the call `setRootData(rootEntry)`? Explain.

No. The constructor first calls the default constructor of `BinaryTree`, which sets `root` to `null`. The method `setRootData` contains the call `root.setData(rootData)`, which would cause an exception.

Question 3 Is it necessary to define the methods `isEmpty` and `clear` within the class `BinarySearchTree`? Explain.

No; `BinarySearchTree` inherits these methods from `BinaryTree`.

Q 4 When `getEntry` calls `findEntry`, it passes `getRootNode()` as the first argument. This argument's data type is `BinaryNodeInterface<T>`, which corresponds to the type of the parameter `rootNode`. If you change `rootNode`'s type to `BinaryNode<T>`, what other changes, if any, must you make?

Question 5 Under what circumstance will a client of `BinarySearchTree` be able to call the other methods in `TreeIteratorInterface`? Under what circumstance will such a client be unable to call these methods?

Q 4 When `getEntry` calls `findEntry`, it passes `getRootNode()` as the first argument. This argument's data type is `BinaryNodeInterface<T>`, which corresponds to the type of the parameter `rootNode`. If you change `rootNode`'s type to `BinaryNode<T>`, what other changes, if any, must you make?

In `getEntry`'s call to `findEntry`, you would cast `getRootNode()` to `BinaryNode<T>`, as follows:

```
findEntry((BinaryNode<T>)getRootNode(), entry)
```

Within `findEntry`, the first recursive call to `findEntry` must be

```
findEntry((BinaryNode<T>)rootNode.getLeftChild(), entry)
```

since the return type of `getLeftChild` is `BinaryNodeInterface<T>`. Analogous comments apply to the second recursive call and `getRightChild`.

Question 5 Under what circumstance will a client of `BinarySearchTree` be able to call the other methods in `TreeIteratorInterface`? Under what circumstance will such a client be unable to call these methods?

The situation is like that described for `setTree` in Segment 25.6. `BinarySearchTree` inherits the methods declared in `TreeIteratorInterface` from `BinaryTree`. An object whose static type is `BinarySearchTree` can invoke these methods, but an object whose static type is `SearchTreeInterface` cannot.

(a)

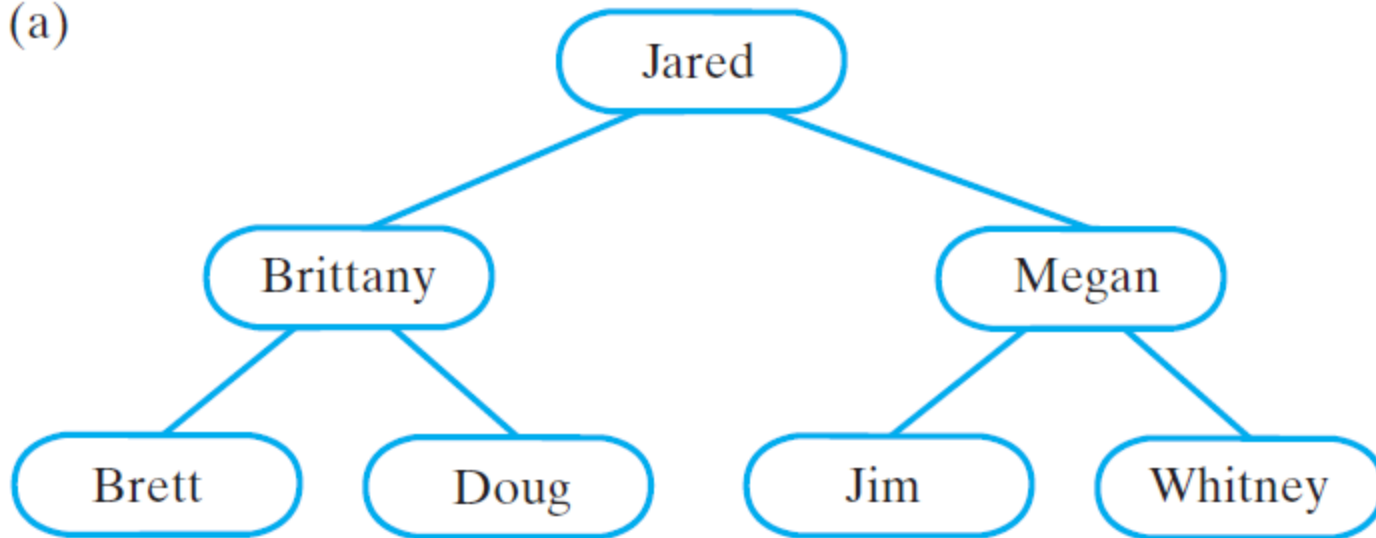


FIGURE 25-4 (a) A binary search tree;

Note recursive implementation

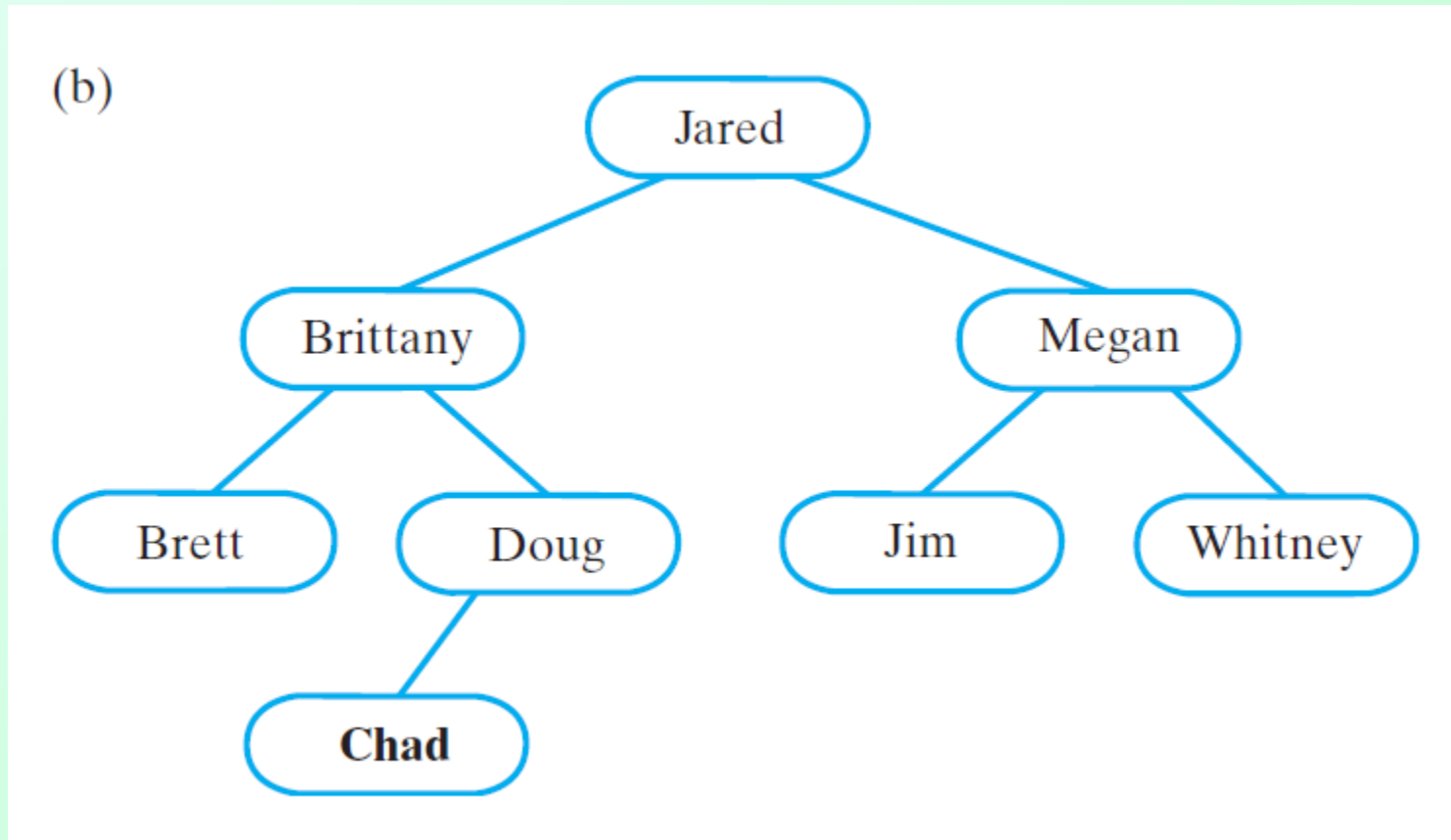
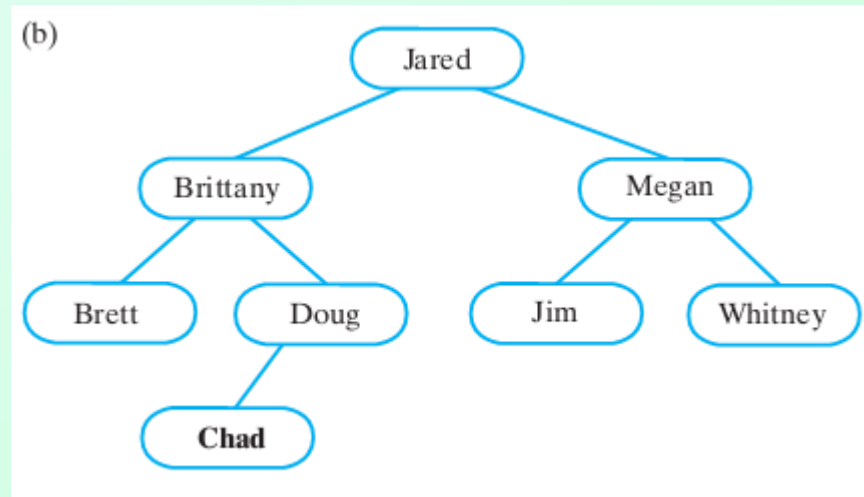


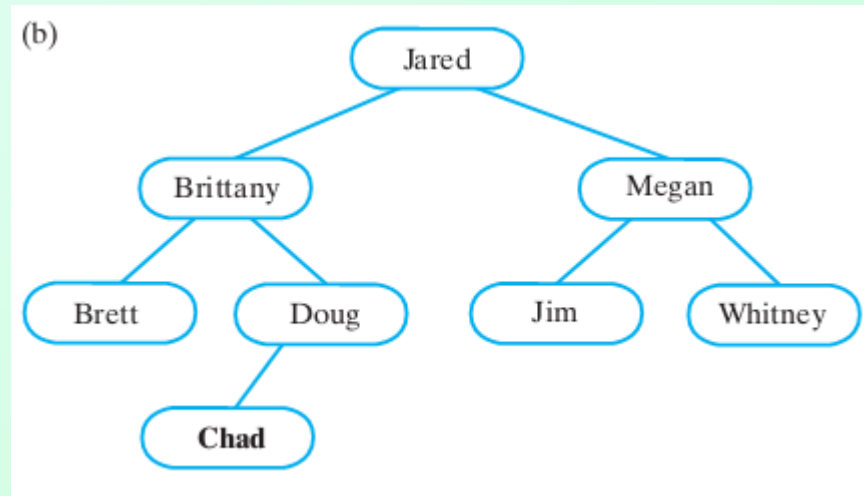
FIGURE 25-4 (b) the same tree after adding *Chad*

Q 6 Add the names Chris, Jason, and Kelley to the binary search tree in Figure 25-4b.



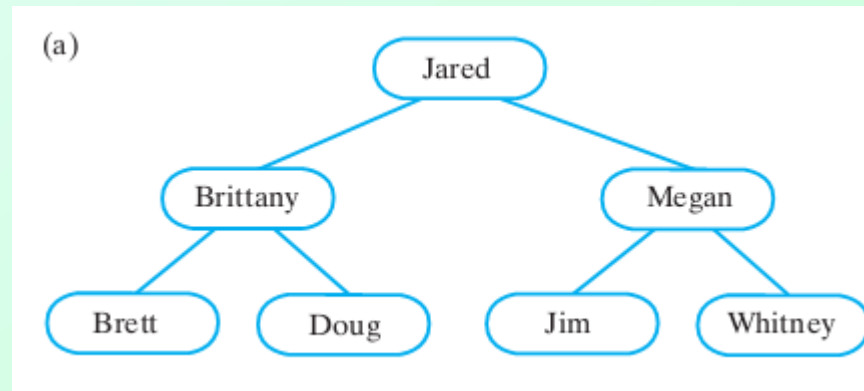


Q 6 Add the names Chris, Jason, and Kelley to the binary search tree in Figure 25-4b.

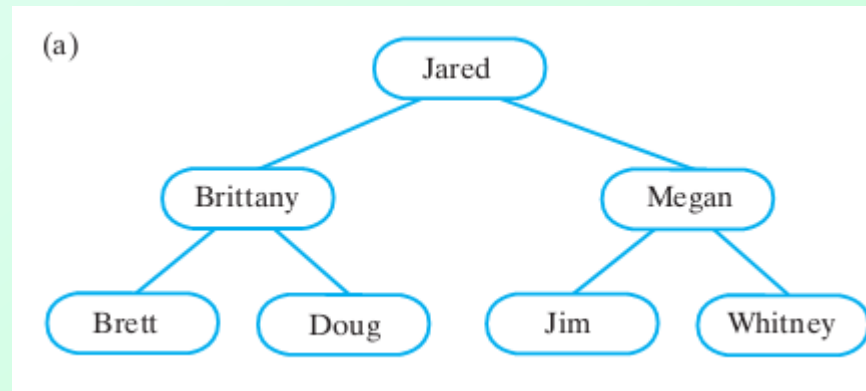


Chris is the right child of Chad. Jason is the left child of Jim . Kelley is the right child of Jim.

Q 7 Add the name Miguel to the binary search tree in Figure 25-4a, and then add Nancy. Now go back to the original tree and add Nancy and then add Miguel. Does the order in which you add the two names affect the tree that results?



Q 7 Add the name Miguel to the binary search tree in Figure 25-4a, and then add Nancy. Now go back to the original tree and add Nancy and then add Miguel. Does the order in which you add the two names affect the tree that results?



When you add Miguel first, Miguel is the left child of Whitney, and Nancy is the right child of Megan. When you add Nancy first, Nancy is the left child of Whitney, and Miguel is the left child of Nancy. Thus, the order of the additions does affect the tree that results.

(a)

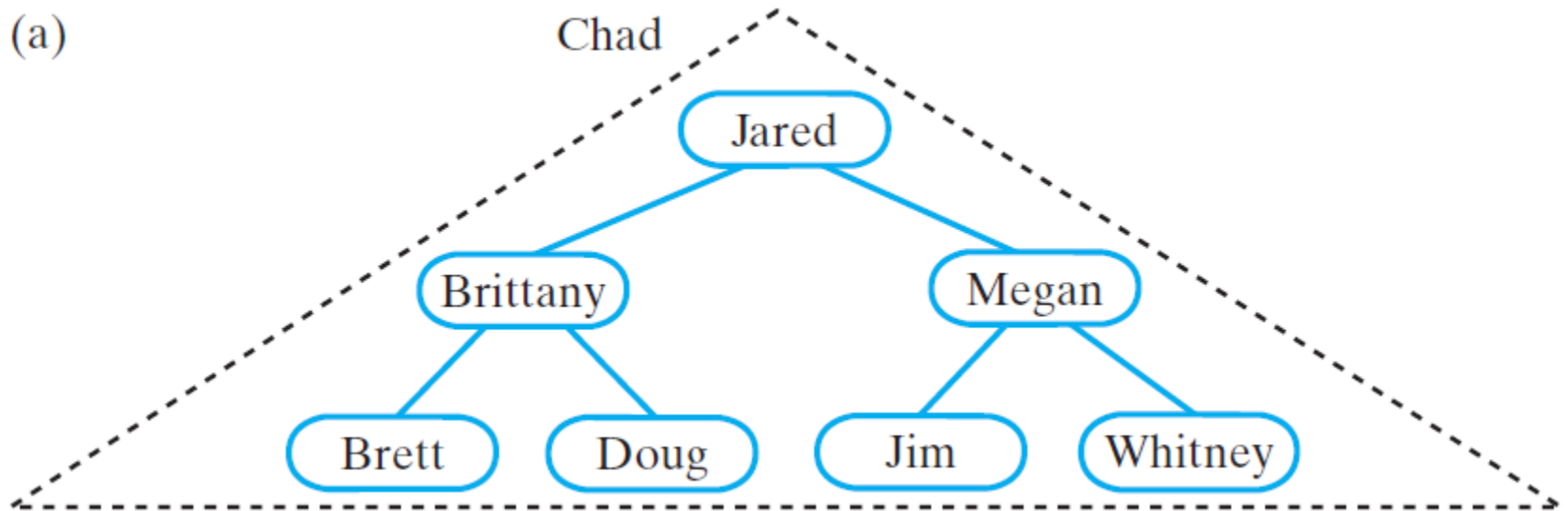


Figure 25-5 Recursively adding Chad to smaller subtrees of a binary search tree

(b)

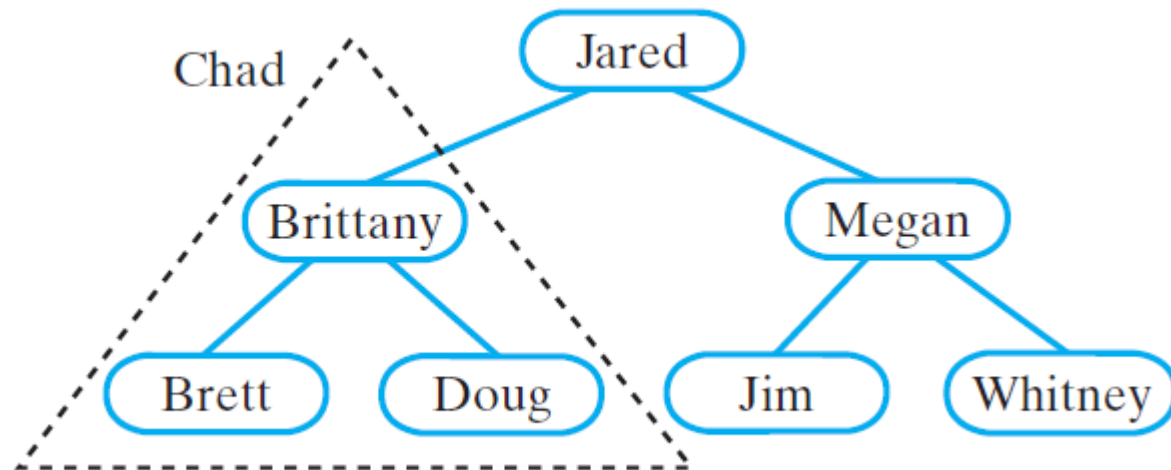


Figure 25-5 Recursively adding Chad to smaller subtrees of a binary search tree

(c)

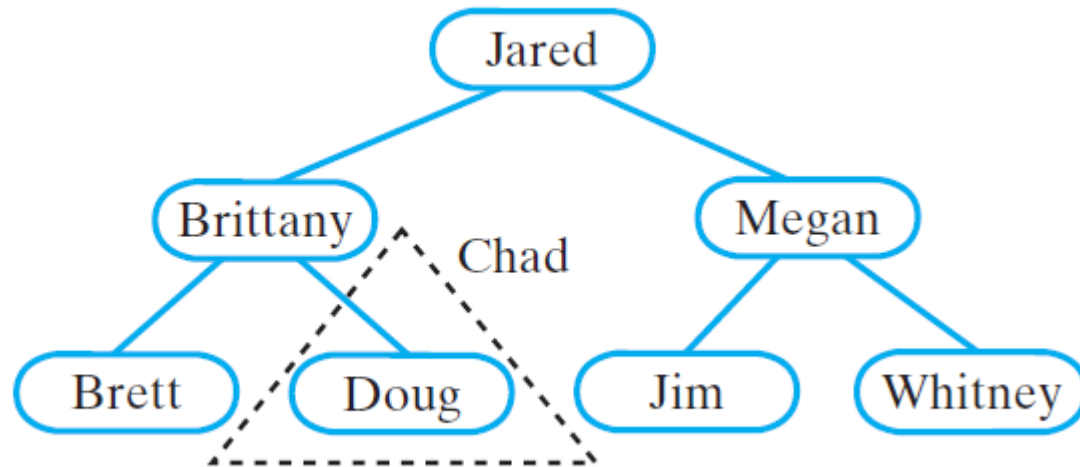


Figure 25-5 Recursively adding Chad to smaller subtrees of a binary search tree

(d)

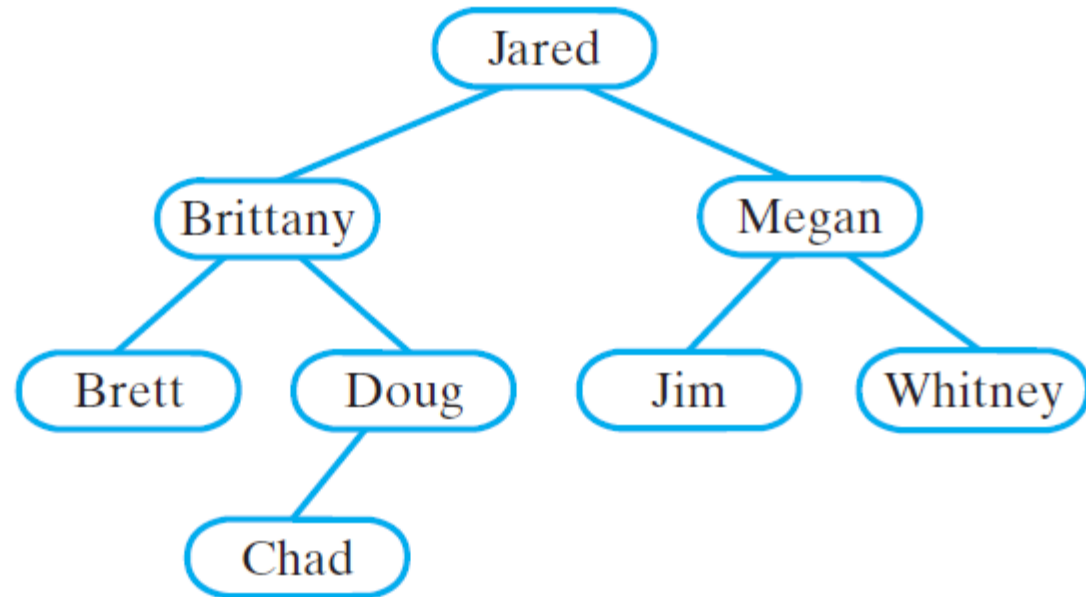


Figure 25-5 Recursively adding Chad to smaller subtrees of a binary search tree

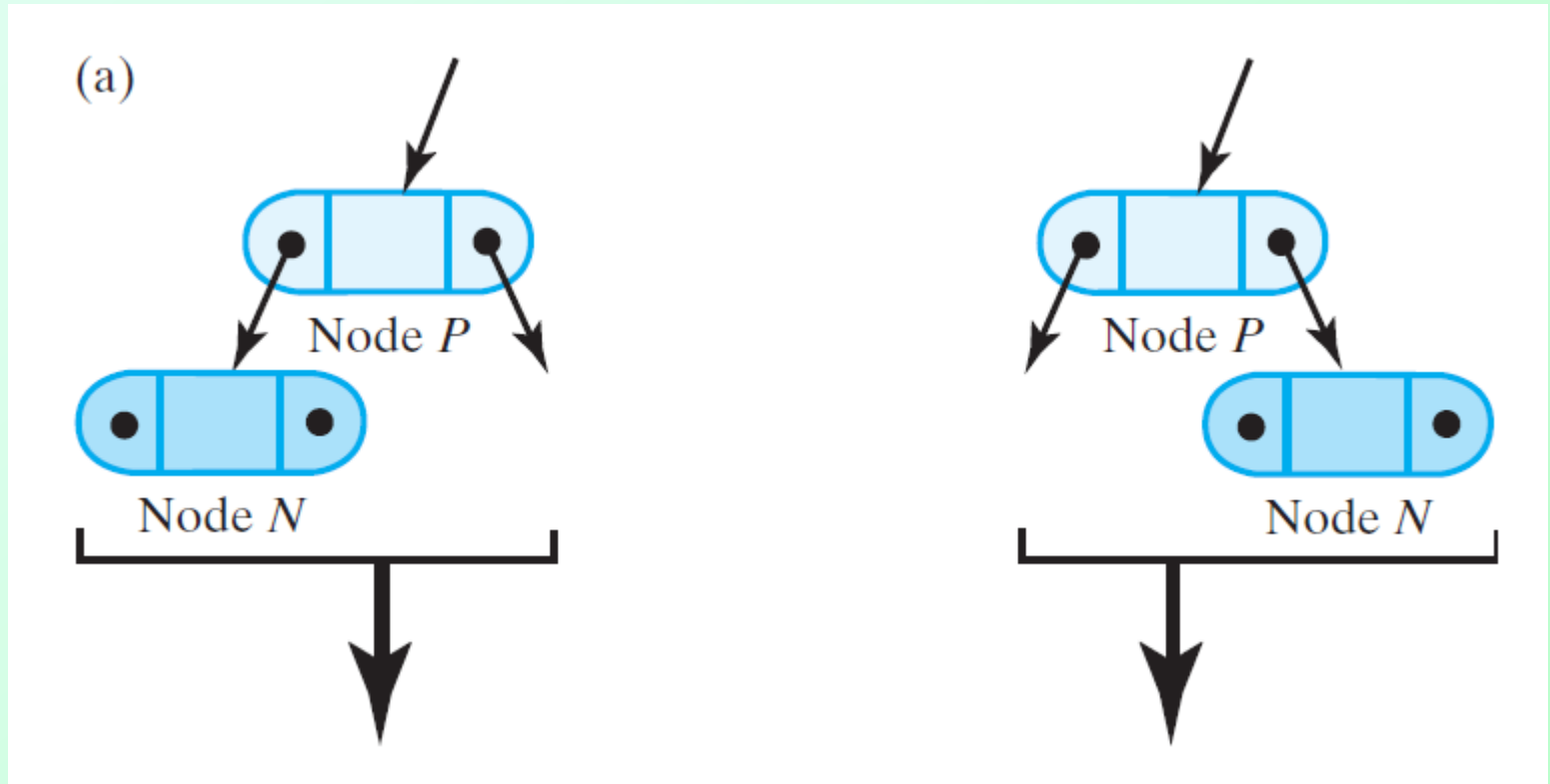


Figure 25-6 (a) Two possible configurations of a leaf node *N*;



(b)

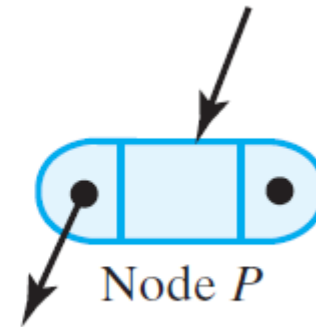
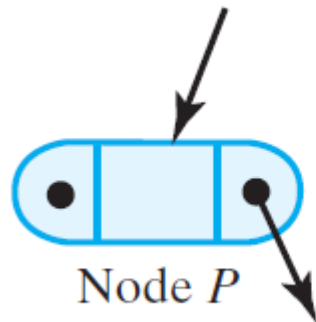


Figure 25-6 (b) the resulting two possible configurations after removing node  $N$

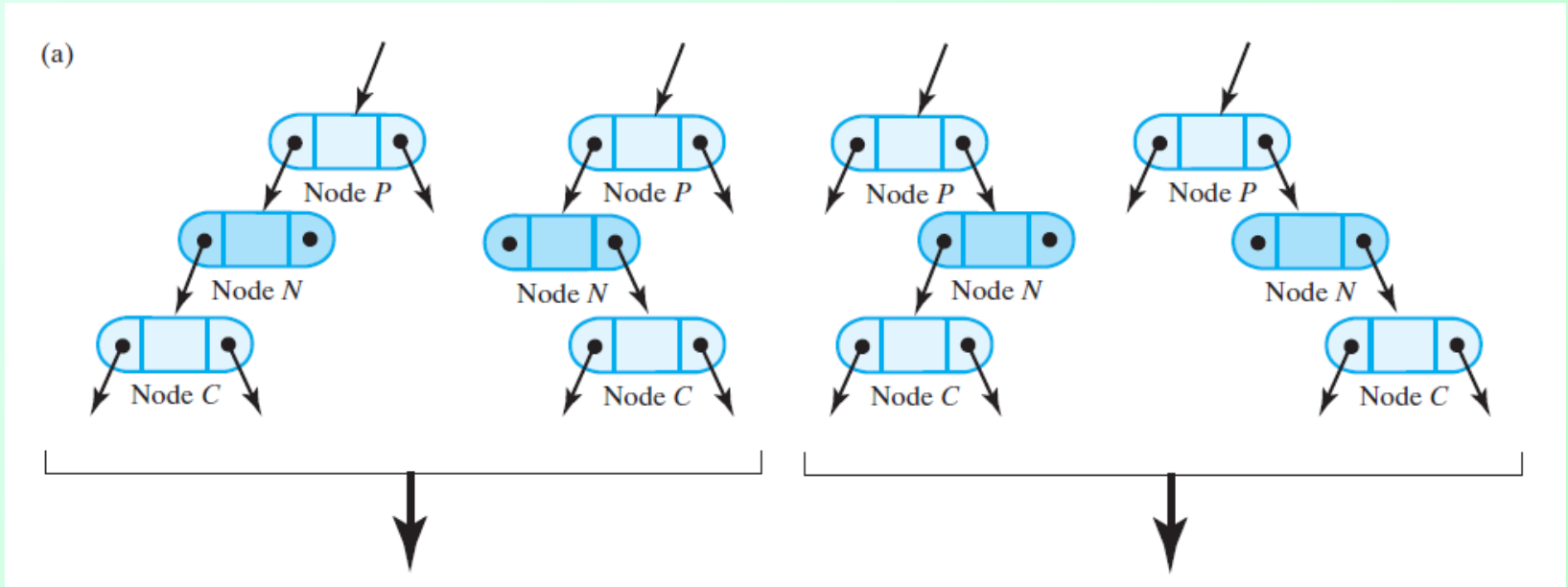


Figure 25-7 (a) Four possible configurations of a node N that has one child;

(b)

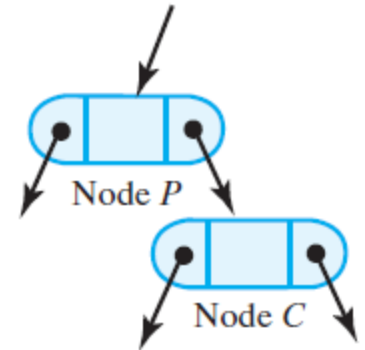
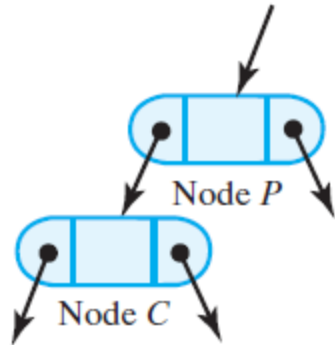
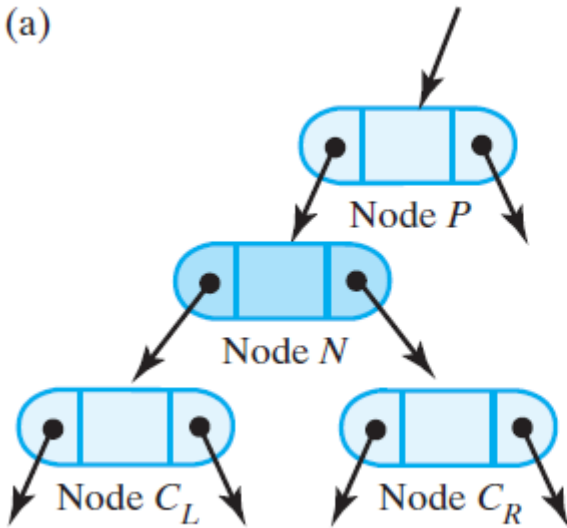


Figure 25-7 (b) the resulting two possible configurations after removing node  $N$

(a)



(b)

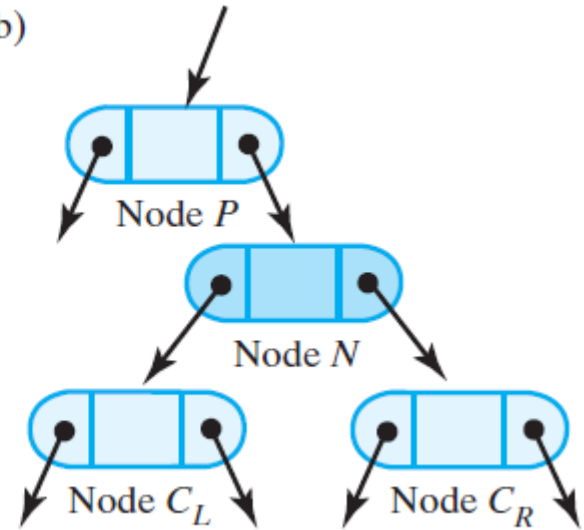


Figure 25-8 Two possible configurations of a node N that has two children

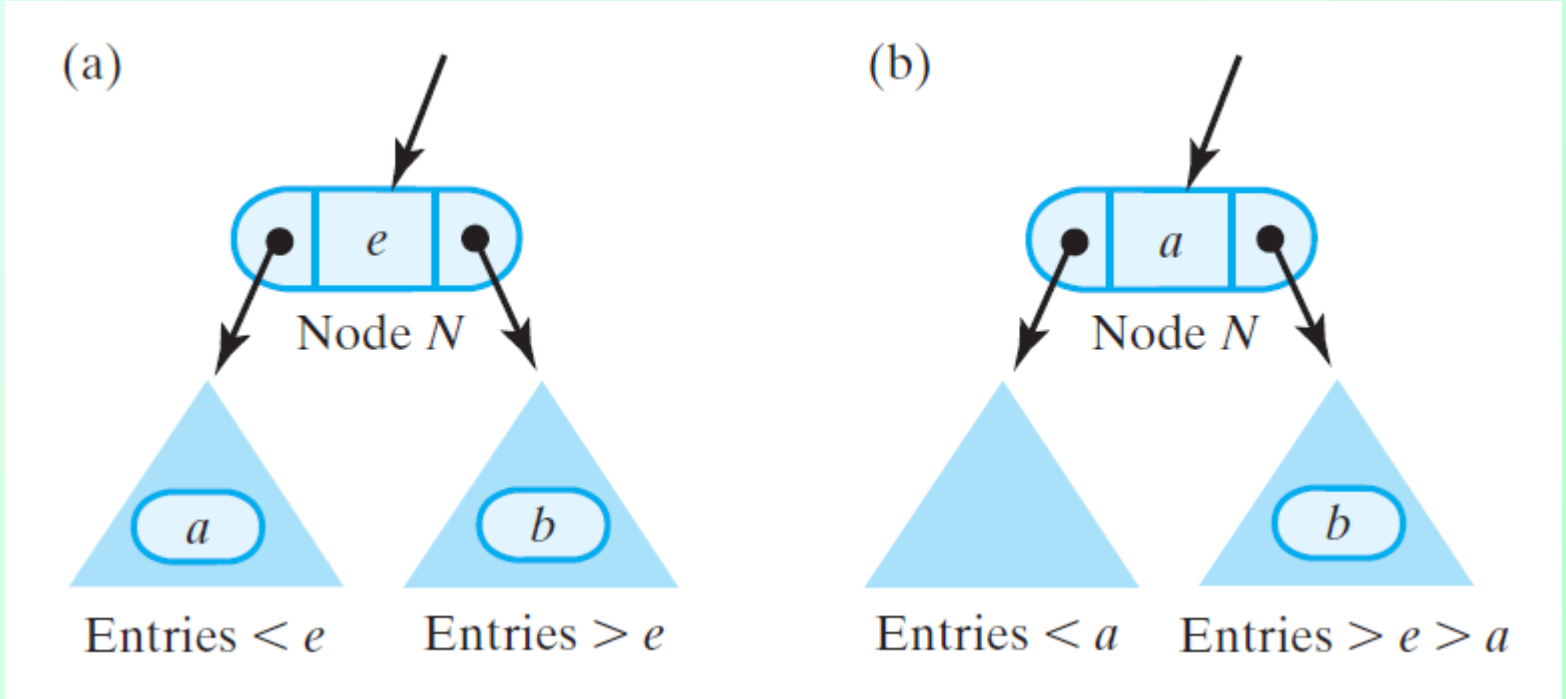


Figure 25-9 Node  $N$  and its subtrees: (a) the entry  $a$  is immediately before the entry  $e$ , and  $b$  is immediately after  $e$ ; (b) after deleting the node that contained  $a$  and replacing  $e$  with  $a$

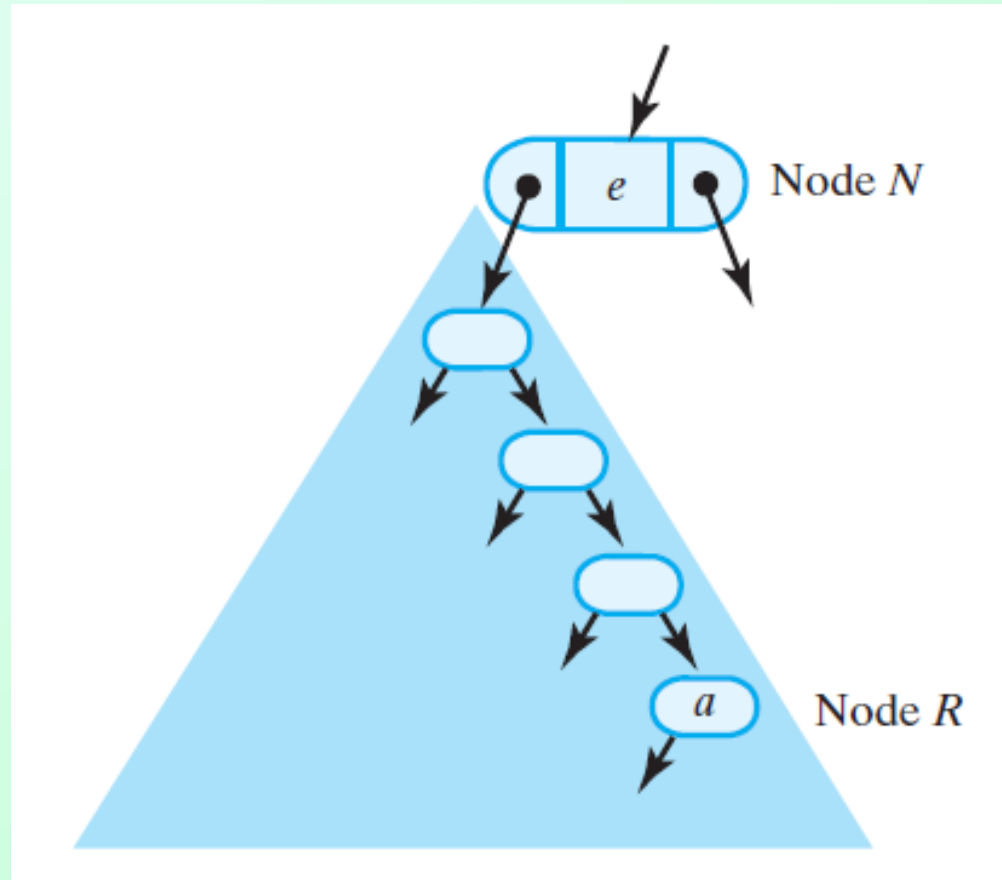


Figure 25-10 The largest entry  $a$  in node  $N$ 's left subtree occurs in the subtree's rightmost node  $R$

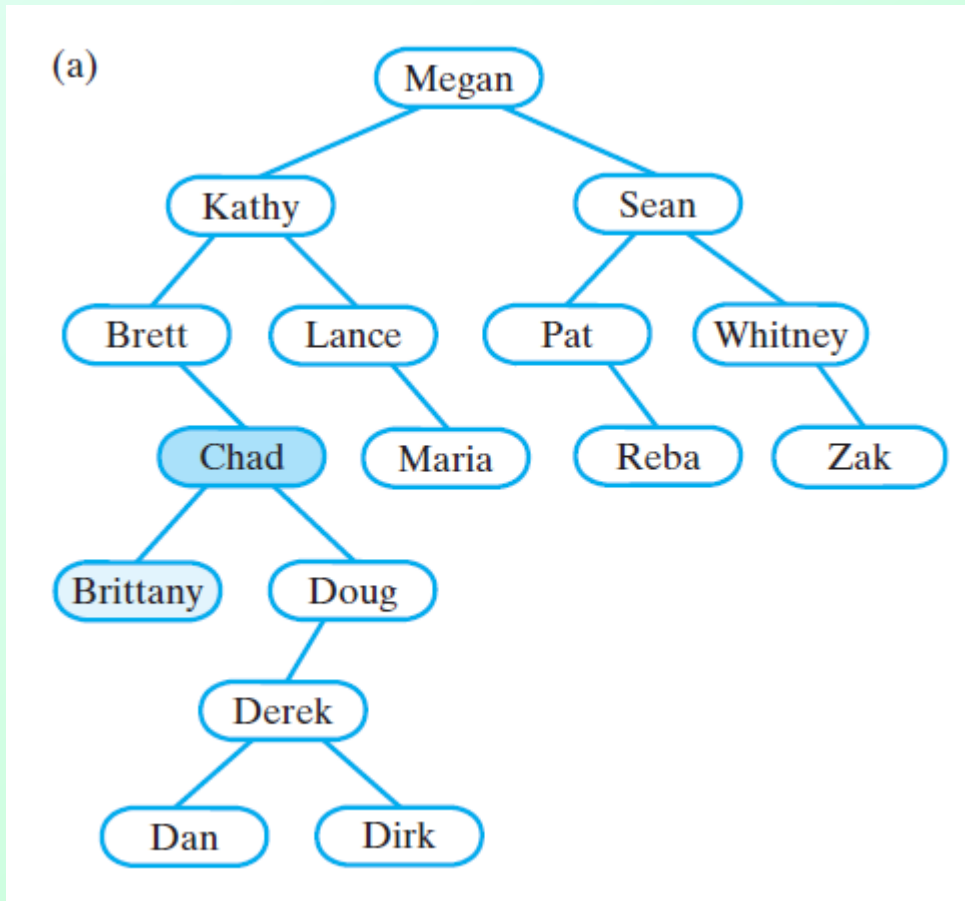


Figure 25-11 (a) A binary search tree

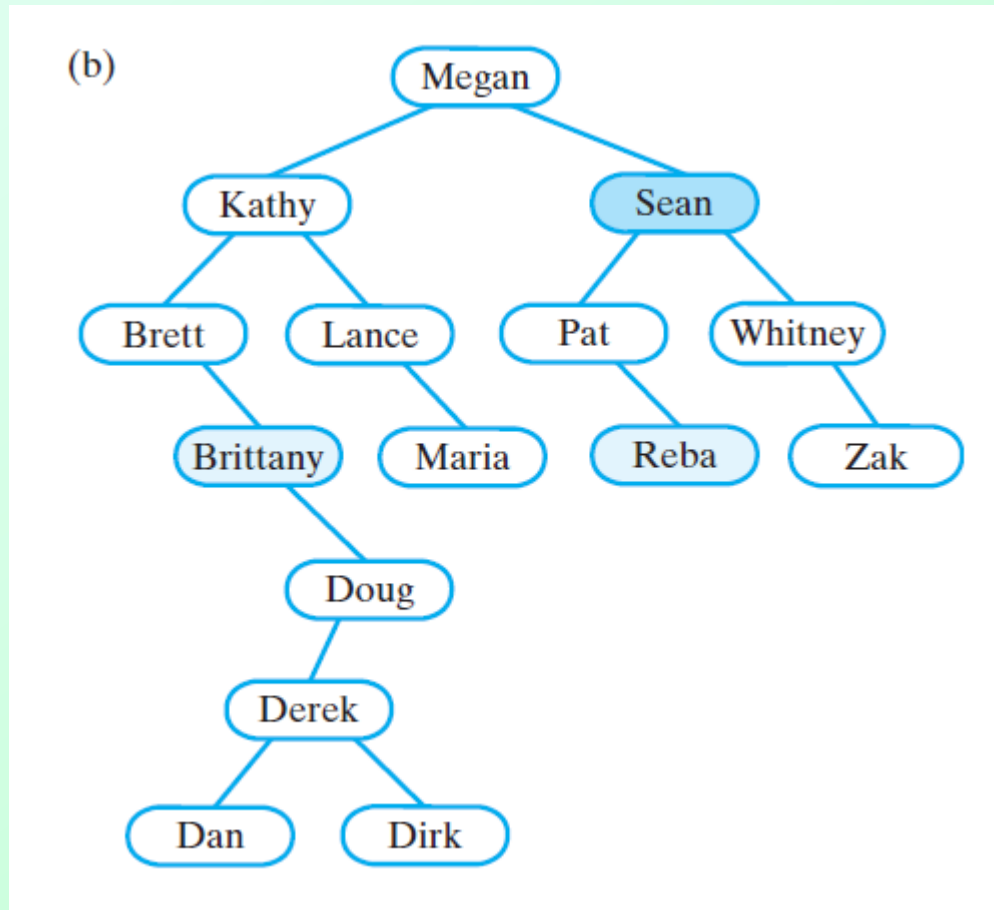


Figure 25-11 (b) after removing *Chad*



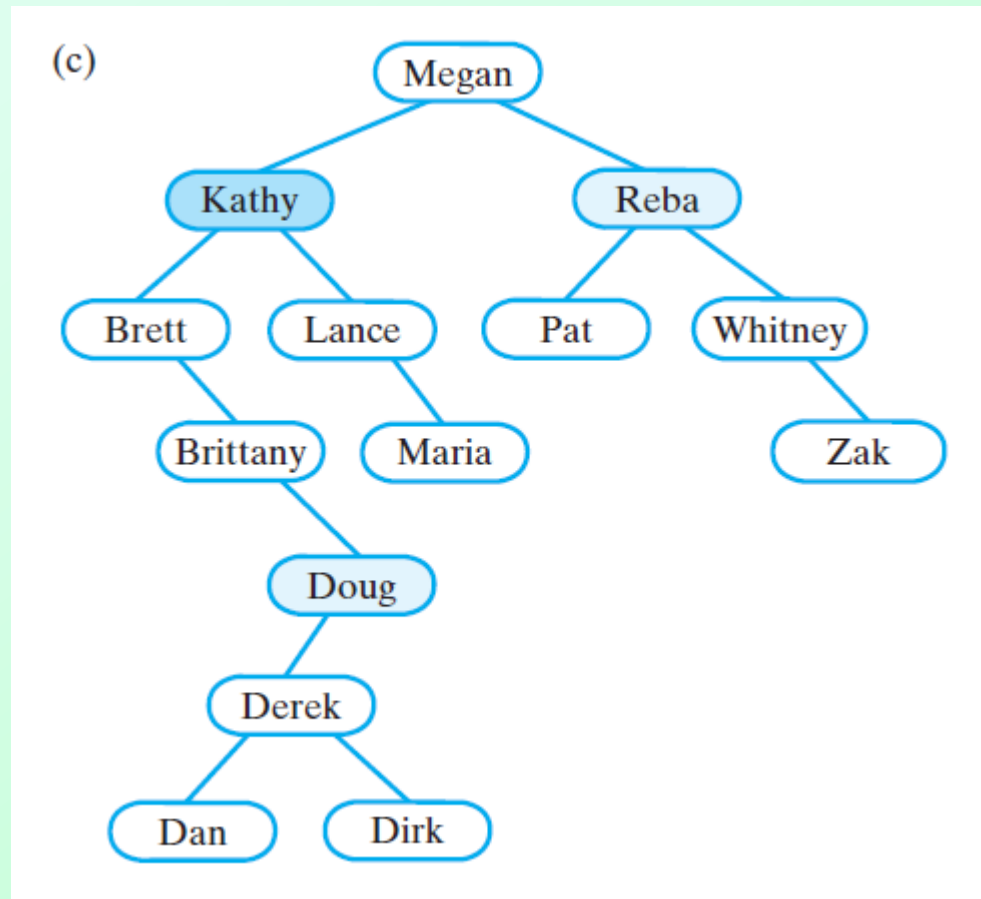


Figure 25-11 (c) after removing *Sean*

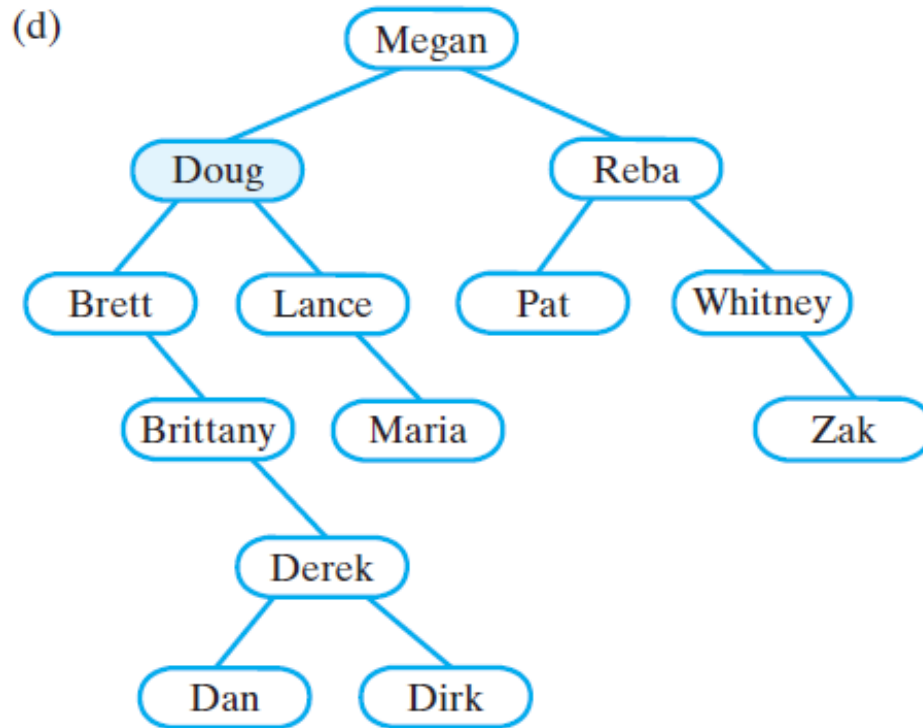
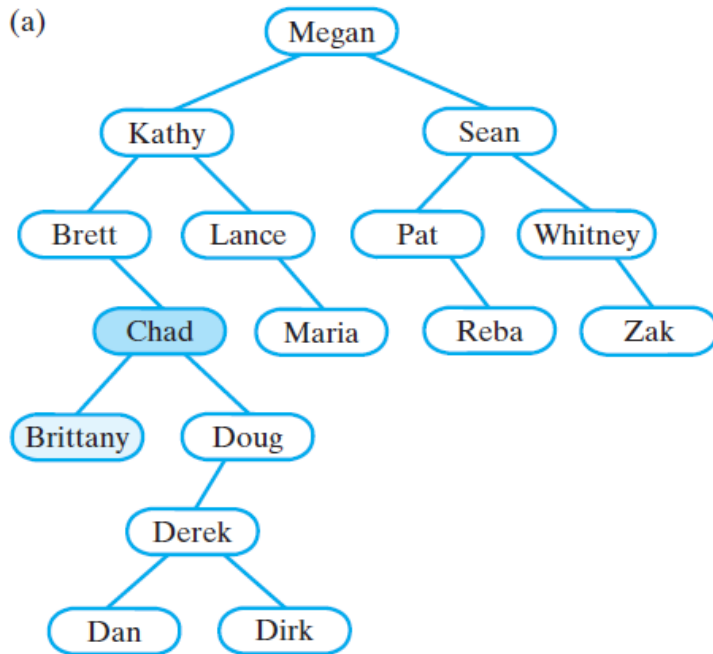
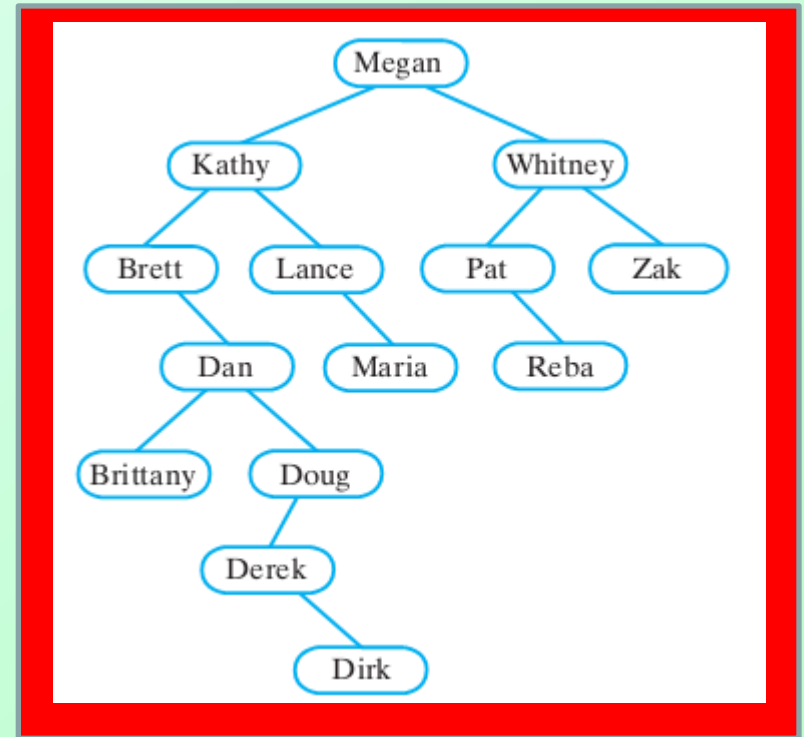
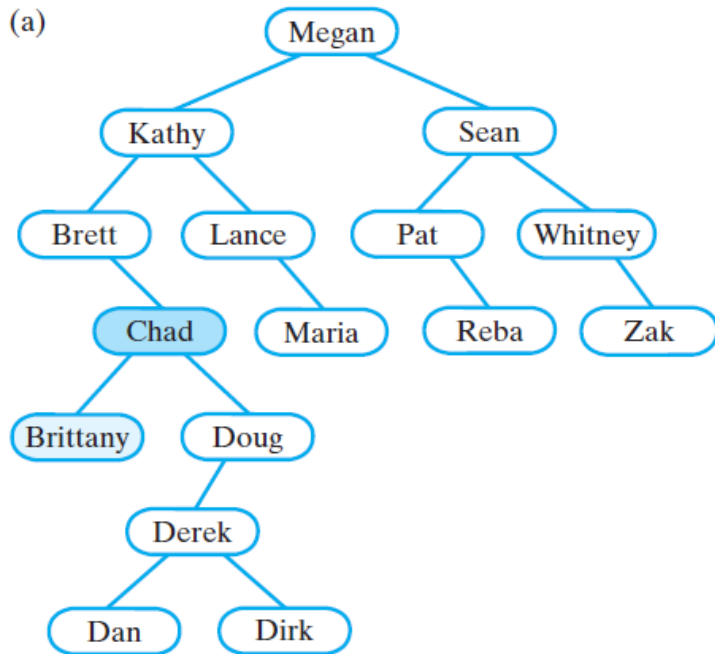


Figure 25-11 (d) after removing *Kathy*

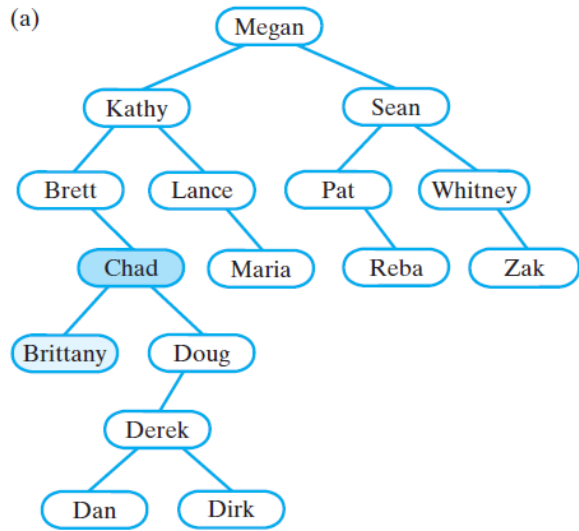
Q 8 The second algorithm described in Segment 25.25 involves the inorder successor. Using this algorithm, remove Sean and Chad from the tree in Figure 25-11a.



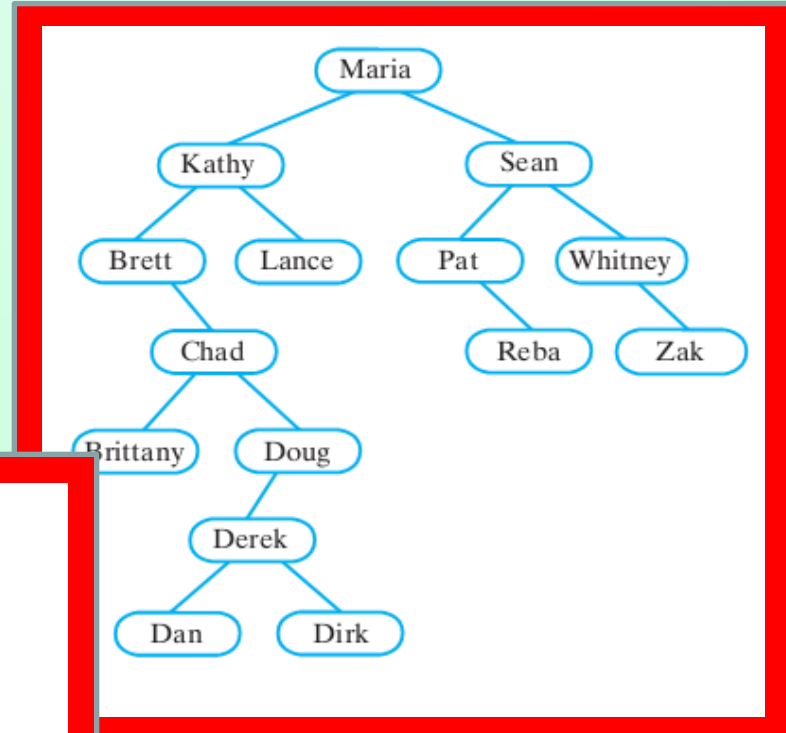
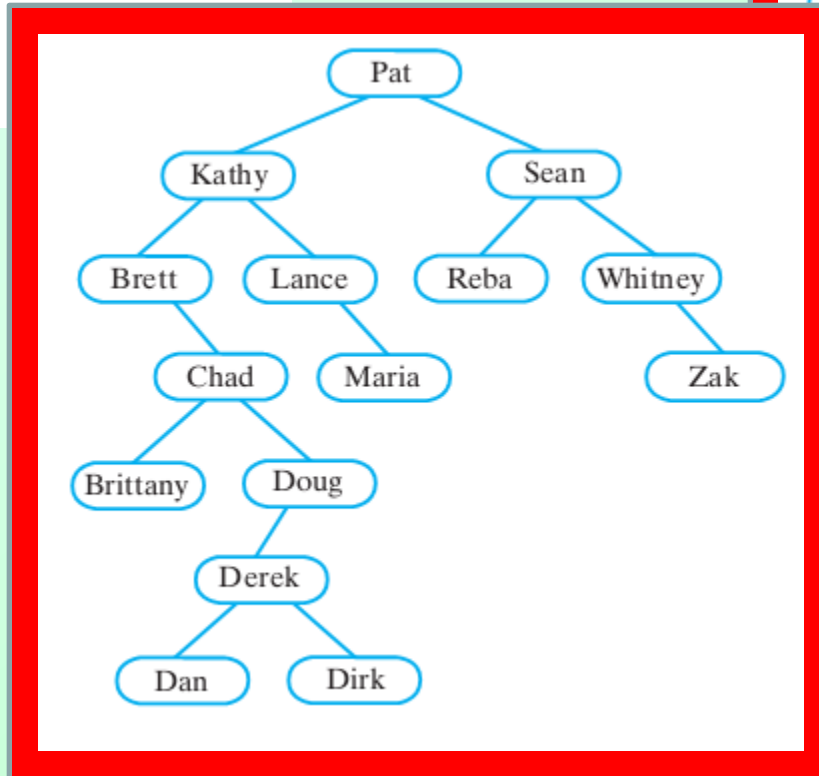
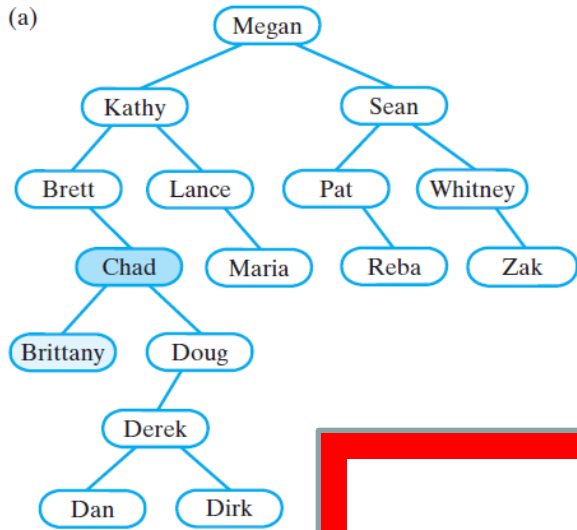
Q 8 The second algorithm described in Segment 25.25 involves the inorder successor. Using this algorithm, remove Sean and Chad from the tree in Figure 25-11a.



Q 9 Remove Megan from the tree in Figure 25-11a in two different ways.



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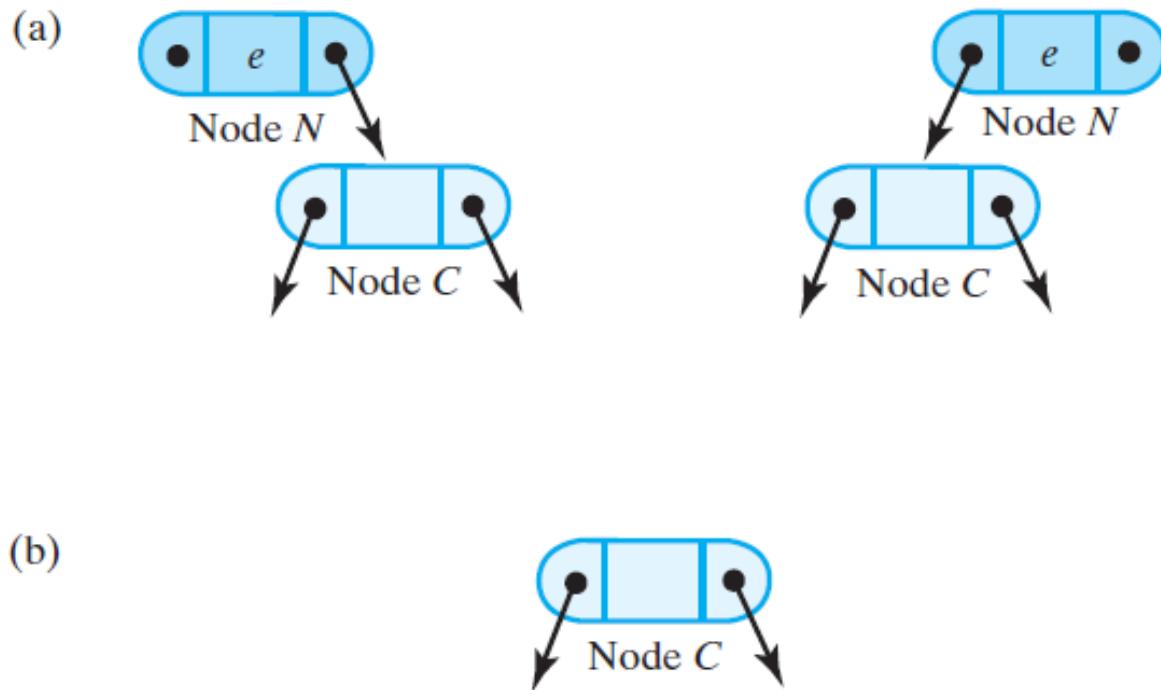


Figure 25-12 (a) two possible configurations of a root that has one child; (b) after removing the root

# Efficiency of Operations

- Operations **add**, **remove**, and **getEntry** require search that begins at root
- Worst case:
  - Searches begin at root and examine each node on path that ends at leaf
  - Number of comparisons each operation requires, directly proportional to height  $h$  of tree



# Efficiency of Operations

- Tallest tree has height  $n$  if it contains  $n$  nodes
- Operations **add**, **remove**, and **getEntry** are  $O(h)$
- Note different binary search trees can contain same data

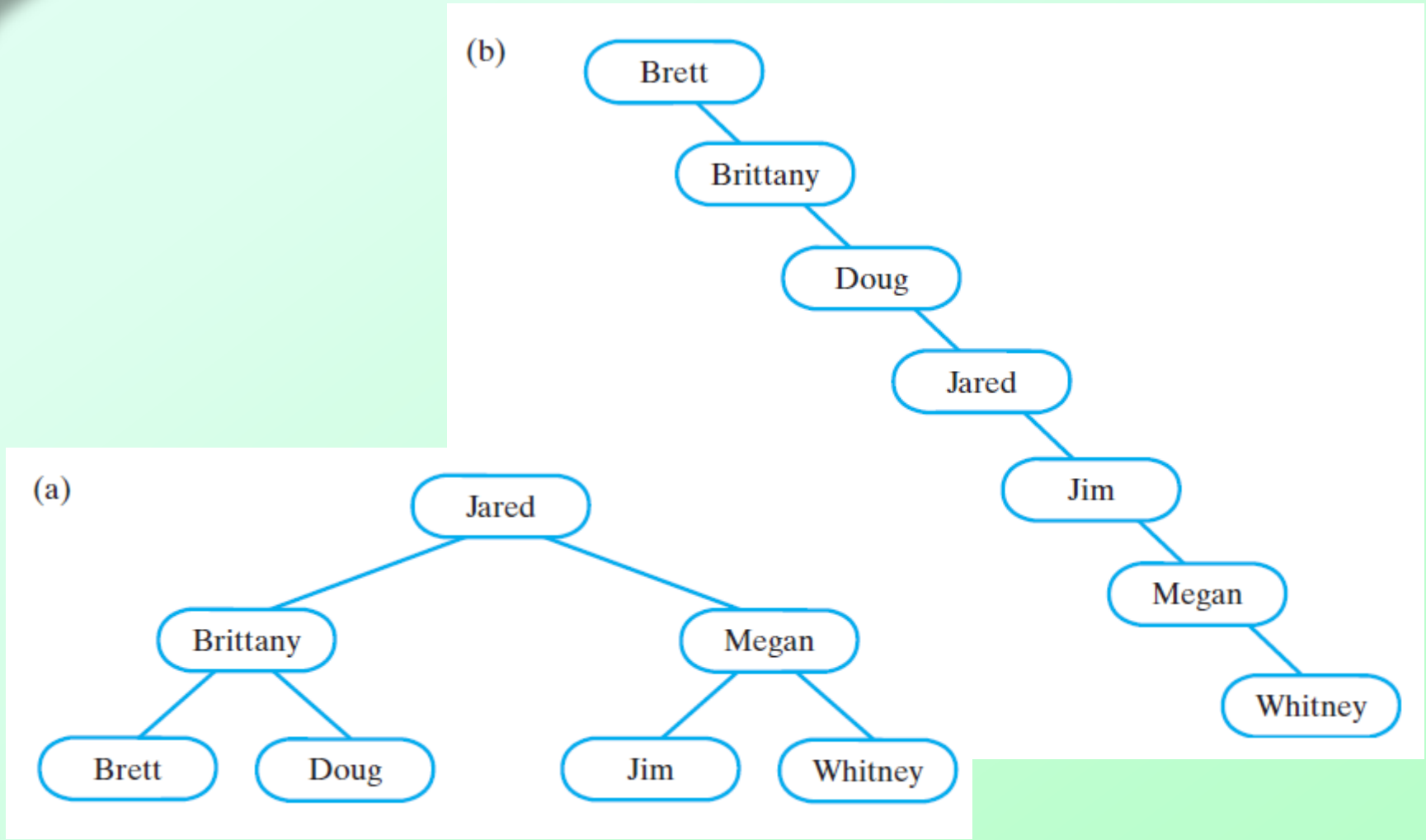


Figure 25-13 Two binary search trees that contain the same data

# Efficiency of Operations

- Tallest tree has height  $n$  if it contains  $n$  nodes
  - Search is an  $O(n)$  operation
- Shortest tree is full
  - Searching full binary search tree is  $O(\log n)$  operation

Q 11 Using Big Oh notation, what is the time complexity of the method contains?

Q 12 Using Big Oh notation, what is the time complexity of the method isEmpty?

Q 11 Using Big Oh notation, what is the time complexity of the method contains?

Since the method contains invokes `getEntry`, the efficiency of these methods is the same. So if the tree's height is as small as possible, the efficiency is  $O(\log n)$ . If the tree's height is as large as possible, the efficiency is  $O(n)$ .

Q 12 Using Big Oh notation, what is the time complexity of the method `isEmpty`?

$O(1)$ .

# Importance of Balance

- Full binary search tree not necessary to get  $O(\log n)$  performance
  - Complete tree will also give  $O(\log n)$  performance
- Completely balanced tree
  - Subtrees of each node have exactly same height
- Height balanced tree
  - Subtrees of each node in tree differ in height by no more than 1

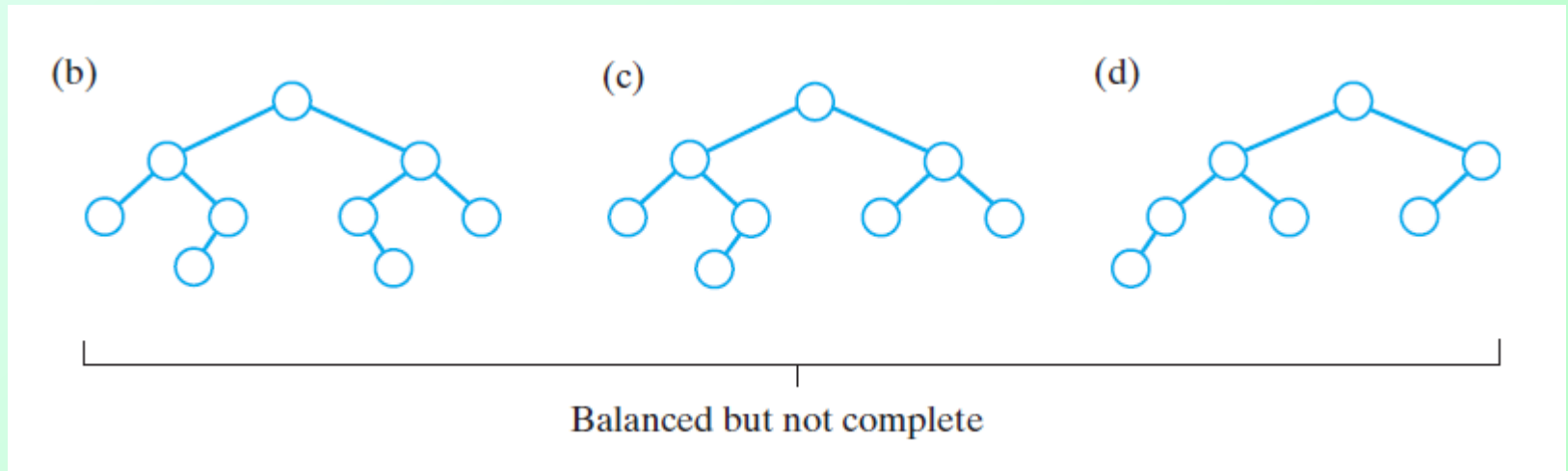
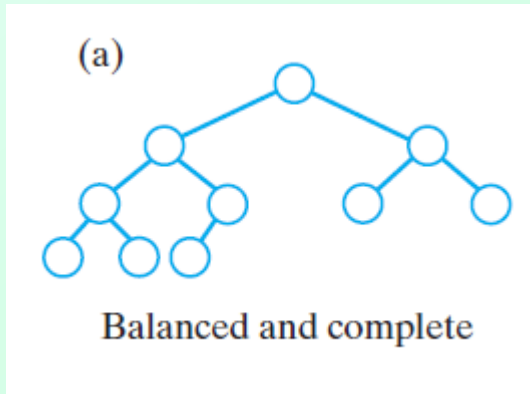
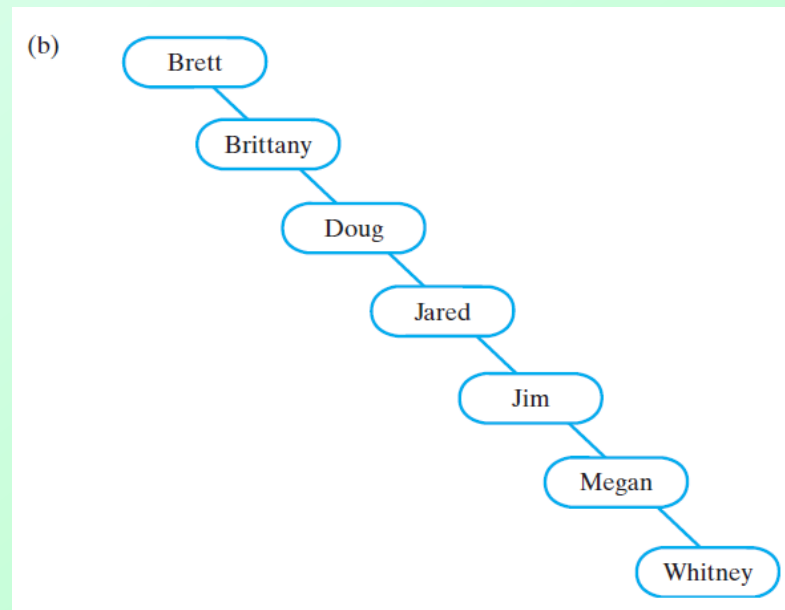


Figure 25-14 Some binary trees that are height balanced

# Order in Which Nodes Added

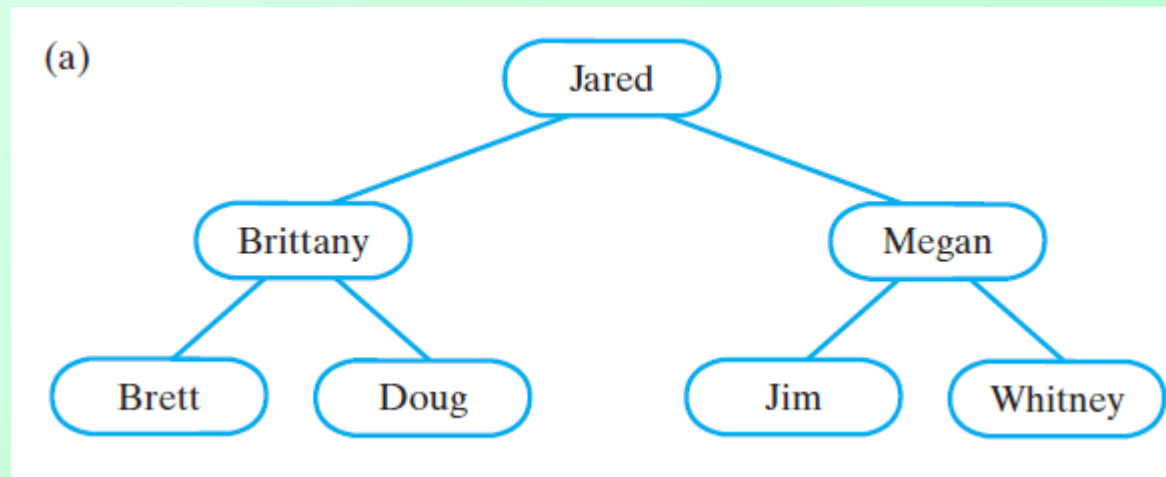
- Adding nodes in sorted order results in tall tree, low efficiency operations





# Order in Which Nodes Added

- Add data to binary search tree in random order
  - Expect tree whose operations are  $O(\log n)$ .



# Implementation of the ADT Dictionary

- Recall interface for a dictionary from Chapter 19, section 4

```
import java.util.Iterator;
public interface DictionaryInterface<K, V>
{
    public V add(K key, V value);
    public V remove(K key);
    public V getValue(K key);
    public boolean contains(K key);
    public Iterator<K> getKeyIterator();
    public Iterator<V> getValueIterator();
    public boolean isEmpty();
    public int getSize();
    public void clear();
} // end DictionaryInterface
```

# Implementation of the ADT Dictionary

- Consider a dictionary implementation that uses balanced search tree to store its entries
  - A class of data objects that will contain both search key and associated value
- Note listing of such a class, [Listing 25-3](#)

# End

## Chapter 25

